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ORIGINAL ARTICLE

Improved Inverted Pendulum Control through PID and EPID Controllers

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ABSTRACT – This presentation is on studies of application of combined constant rate reaching law and proportional-integral-derivative (PID) control law (EPID) for the control of inverted pendulum system. The inverted pendulum system is similar to a typical attitude control of booster rocket undergoing the takeoff process. Recent studies indicated records of higher rates of accidents in aircrafts. The records show that about half results due to malfunctions of aircraft systems and close to one-third from propulsion system malfunctions. Others are higher complexities of modern aircraft systems and in trying to reduce cost of maintenance. Therefore, the need enhanced automation, fault detection, faults isolation, faults tolerance, faults diagnosis and faults correction. Linear control techniques may not yield the desired performances in aircraft systems due to high levels of system nonlinearities. Applications of the intelligent control counterparts may not guarantee the generation of mathematical model for in-depth analysis. The major demerit of nonlinear control methods is higher requirement of computational burden making practical implementation difficult. The model of the inverted pendulum system is a linearized analytical model. The system performance of the system was observed with the EPID and PID controllers. Furthermore the effect of sudden changes such as wind, gust or other related variations on the system was also studied using step disturbance. It was a simulation studies using MATLAB/SIMULINK software. Results showed that with the EPID a near zero deviation was achieved. Whereas with PID controller was able to only maintain a deviation of about 40. Results also indicated a near zero disturbance rejection ability with the EPID, while the PID was able to only suppress the disturbance to some extent. It implies a more robust control with the EPID was achieved for aircraft/inverted pendulum control. Hence, it implies enhanced performance and with further improvement it can be used for application in this class of systems as well as similar systems. The work would help to for basic researches in aircraft/inverted pendulum control for beginners as well as experienced researchers in the field.

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INTRODUCTION

The inverted pendulum system has being used as benchmark for testing developed or proposed` control scheme [1]. The system consists of a pendulum which inverted and mounted onto a movable cart which motorized. The cart is moved left and right to ensure that the pendulum is maintained upright. Currently the inverted pendulum system is one of the devices commonly available in control systems books as well as research literature [2, 3]. It is a popular due the fact it is not stable in the absence of control [4]. It is because the pendulum would simply just fall-off if the cart is not moved to make the pendulum balance. It also has nonlinear dynamics and the control system objective is to continuously maintain the inverted pendulum in the balanced state by applying appropriate amount of force to the cart onto which the pendulum is attached [5].

The inverted pendulum system has similarities with a typical attitude control associated with booster rocket that is undergoing the takeoff process. Current studies indicated records of higher rates of accidents occurrences in aircrafts [6]. The records also show that about half of which results due to the malfunctions of aircraft systems and close to one-third are attributed to propulsion system malfunctions [7]. Others are higher complexities of modern aircraft systems resulting from accommodating more features, and also in trying to reduce cost of maintenance for the systems. Therefore, it calls for the need of enhanced automation, fault detection systems, faults isolation, faults tolerance, faults diagnosis and faults correction systems [8].

The linear control methods are applied to the inverted pendulum system by Okubanjo and Oyetola, the explore the PID, pole placement, linear quadratic regulator (LQR) and the combined LQR and estimator techniques [9]. The Linear Quadratic Gaussian (LQG) control method was explored by Priyadarshi in his work [10]. The root locus, frequency response, PID was utilized by Hasan *et al* [11]. In another development the PD with Intermittent Feedback Strategy was used by Morasso *et al* in their works [12]. The PID and LQR methods were presented by Prasad *et al* as well as Jose et al in their works [4, 13]. In the works of Magdy *et al*, the PID controller was tuned using gravitational search algorithm

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[13]. The state feedback control was compared with the PID control scheme in the contributions of Lin and Liu [14]. Cascade form loop with both the inner and outer loop PID controllers with modifications of the inner loop was proposed by Namasivayam. Linear control techniques may not yield the desired performances in inverted pendulum systems due to high levels of system nonlinearities, instability and boundary limit control. The intelligent control method was harnessed by Isa and colleagues where the fuzzy logic adaptive method was utilised [7]. A fuzzy logic controller was proposed by Kumar et al. In the work of Kulikova, the Takagi-Sugeno-Kang first-order type fuzzy controller was applied on the inverted pendulum system [11]. Ping *et al* proposed a controller which is a combination of neural networks and feedforward compensation [7]. Applications of the intelligent control counterparts may not guarantee the generation of mathematical model for in-depth analysis. Nonlinear control methods have also been explored on the inverted pendulum systems. Mishra and Chandra applied the fractional order PID control method for the pendulum systems [4]. In another development Bettayeb et al proposed a fractional order PI state feedback controller for the system [9]. Liu *et al* presented an improved version of active disturbance rejection control technique for stabilization control of the system[15, 16]. Others are the works of Chalupa and Bobál harnessing the model predictive controller [4]. The major demerit of nonlinear control methods is higher requirement of computational burden making practical implementation difficult [14].

METHODS AND MATERIALS

A linearized analytical model of the inverted pendulum system was utilized in the study, but effort would be made to keep upgrading the model in subsequent studies [17]. Combined constant rate reaching law and proportional-integral-derivative (PID) are explored to form the control law referred to EPID controller. The resulting system performance of the system with the EPID was evaluated with those of the PID controller. The effect of wind, gust or a sudden torque changes on the system which are disturbances that may affect the smooth operations of such systems was approximated using step signal. The study was limited to simulations using the MATLAB/SIMULINK software.

Figure 1 is a block diagram illustration of the system. The reference is the upright position to be maintained by the system which is 180^0. The controller is the compensation arrangement for the entire system, this is where the PID and EPID are found. Inverted pendulum is the system under consideration; that is the inverted pendulum system. The response is the output of the system and *e(t)* is the error signal which is the difference between the actual system response and the reference of desired response. The controller always tries to make the error zero in order to achieve the desired goal which in this case is maintaining the inverted pendulum in the upright position always.

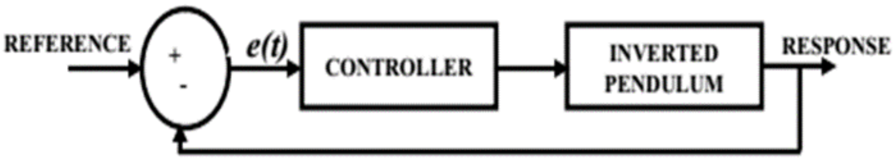

Figure 1. Block diagram of the system

The Inverted Pendulum System

These base metals were welded using a JAFO horizontal universal milling machine in a butt joint configuration. FSW process was conducted using a H13 tool with 9.3 mm pin length and 15 mm shoulder diameter. The pin is fabricated with an 8º tapered profile and 6 mm pin root diameter, which was threaded and includes three flats. Figure 1 shows the tool used at different angles. Welding parameters such as tool tilt angle, rotational speed and weld speed was kept constant at 2.5°, 1120 rpm and 90 mm/min, respectively. The inverted pendulum system basically contains an inverted pendulum mounted on a movable cart powered by electric motors. The cart moves sideways in order keep the pendulum in the upright inverted condition. Consider figure 2 which is a simplified form of the system. Making it a two-dimensional problem, the pendulum is forced to only move in the vertical plane as illustrated. The force is the control input which moves the cart sideways horizontally. The outputs are: (1) the angular position of the pendulum (2) the horizontal position of the cart.

The summation horizontal forces on the cart in figure (2) becomes as given by equation (1) in the time domain. Where F is the horizontal force acting on the cart, M is the mass of the cart, x is the horizontal distance moved, b is the coefficient of friction and R is the horizontal component of the vertical reaction force on the cart. Resolving relevant forces R becomes as given by equation (2). Where m is the mass of the pendulum, l is the position of the centre of mass and is the angle of the pendulum from zero; that is it would be 180^0 at upright position. Substituting equation (2) in equation (1), the horizontal force becomes as given by equation (3). The summation of forces in the vertical position is given by equation (4). Where N is the vertical component of the vertical reaction force on the cart and g is the acceleration due to gravity.

$$F = M\ddot{x} + b\dot{x} + R \tag{1}$$

$$R = m\ddot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}\text{^}2sin\theta \tag{2}$$

$$F = (M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta \tag{3}$$

$$Nsin\theta + Rcos\theta - mgsin\theta = ml\ddot{\theta} - m\ddot{x}cos\theta \tag{4}$$

The summation of moments about the centre of the pendulum gives equation (5). Where I is the moment of inertia of the inverted pendulum structure. Substituting equation (5) into equaton (4) yields equation (6).

$$-Nlsin\theta - Rlcos\theta = I\ddot{\theta} \tag{5}$$

$$(I + ml^2)\ddot{\theta} + mglsin\theta = -ml\ddot{x}cos\theta \tag{6}$$

We now linearize the nonlinear system. The pendulum is desired to remain at 180^0 that is pi (rad). The steady state error tolerance in control system is between 2-5% that means the maximum allowable amount of deviation for our system should not exceed a deviation of 9^0 from the vertical position. Hence the deviation should then be very small. If the deviation is represented by $\emptyset$, therefore the angle θ should not be more than pi + $\emptyset$; that is $\theta = pi + \emptyset$. Assuming small deviation, the small angle approximations can be applied to the system.

$$cos\theta = \cos(pi + \emptyset) \approx -1 \;\; and \;\; sin\theta = \sin(pi + \emptyset) \approx \emptyset \tag{7}$$

$$\dot{\theta}^2 = \dot{\emptyset}^2 \approx 0 \tag{8}$$

Equations (9) and equation (10) below are obtained after substituting the above in equation (3) and equation (6). The equation are then written in the s domain and the transfer function is obtained.

$$F = (M + m)\ddot{x} + b\dot{x} - ml\ddot{\emptyset} \tag{9}$$

$$(I + ml^2)\ddot{\emptyset} + mgl\emptyset = ml\ddot{x} \tag{10}$$

$$\frac{\emptyset(s)}{F(s)} = \frac{\frac{ml}{G}s}{s^3 + \frac{b(I + ml^2)}{G}s^2 + \frac{(M + m)mgl}{G}s + \frac{bmgl}{G}} \tag{11}$$

$$G = (M + m)(I + ml^2) - ml^2 \tag{12}$$

Equation (11) and equation (12) give the transfer function of the system in the frequency domain. The table below shows the parameters of the system, their designation as well as values as used in the study.

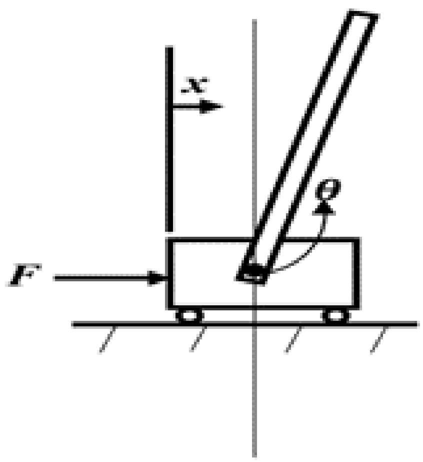

Figure 2. Simplified diagram of the inverted pendulum system..

Table 1. The parameters of the inverted pendulum system.

Parameter	Symbol	Value
Mass of the cart	M	0.5 kg
Mass moment of inertia of the pendulum	I	0.006 kgm^2
Mass of the pendulum	m	0.2 Kg

The control schemes

As mentioned earlier enhanced PID controller EPID was achieved by combining the PID and the constant rate reaching law. Therefore starting with the PID the control law is as given by equation (13). Where Kp, Ki, Kd are the proportional, integral and derivative gains respectively. Where as e(t)is the resulting error signal in the system.

$$u(t) = K_p e(t) + \frac{K_i}{s} e(t) + K_d s e(t) \tag{13}$$

Therefore the EPID control law is simply obtained by adding the PID control law and the reaching law. Equation (14) is the reaching law. Where c and η are constants which are all carefully selected and are greater than zero. The control law is simply equation (13) plus equation (14). The control is hence as given by equation (16).

$$ox_i = 2x_{co} - x_i \tag{14}$$

$$ss = c\, e(t) + se(t) \tag{15}$$

$$u(t) = K_p e(t) + \frac{K_i}{s} e(t) + K_d s e(t) + \eta sign(ss) \tag{16}$$

EXPERIMENTAL RESULTS

Presented are results obtained in the study. It is categorized into; without any disturbance and with disturbance, starting with the first followed by the later.

Results without any disturbance

The results presented here are without the effect of any disturbance. Figure 3 shows the angular position response of the system with both PID and the EPID. It can be seen that the amount of error at steady state is higher with the PID. Figure 4 shows the error plots of the system with the different schemes which portrayed a RMSE of 4^0 with the PID and 1^0 with the EPID.

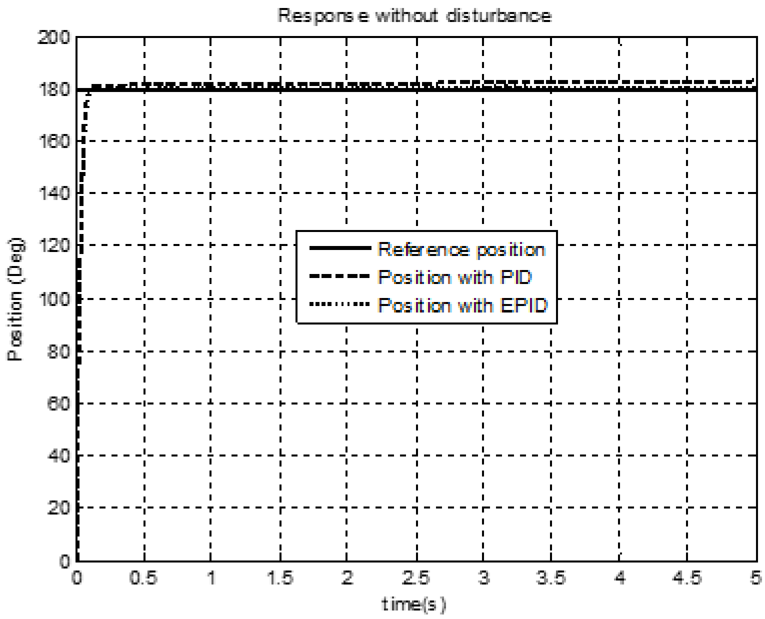

Figure 3. Pendulum position without disturbance

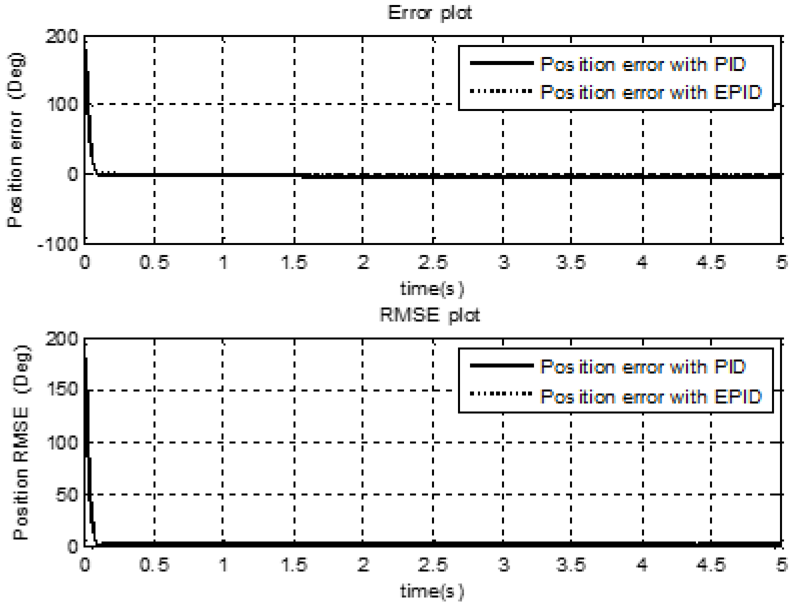

Figure 4. Position error without disturbance

Results with disturbance

The effect of step disturbance was considered starting at 3 seconds which mimics the effect of sudden gust, wind or changes. Figure 5 showed the angular position response of the system with both schemes. The amount of error at steady state is also higher with the PID. Figure 6 is the error plots of the system which showed resulted in an RMSE of 5^0 with the PID and maintaining 1^0 with the EPID.

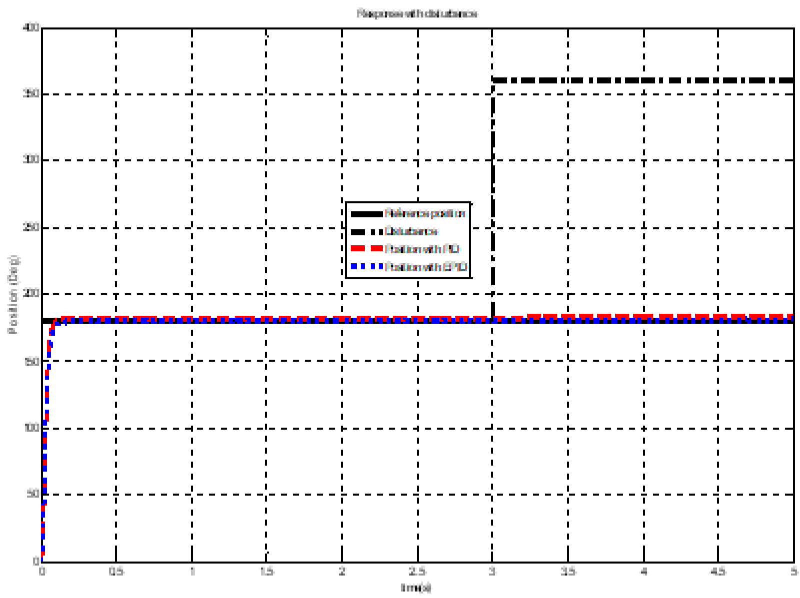

Figure 5. Pendulum position with disturbance

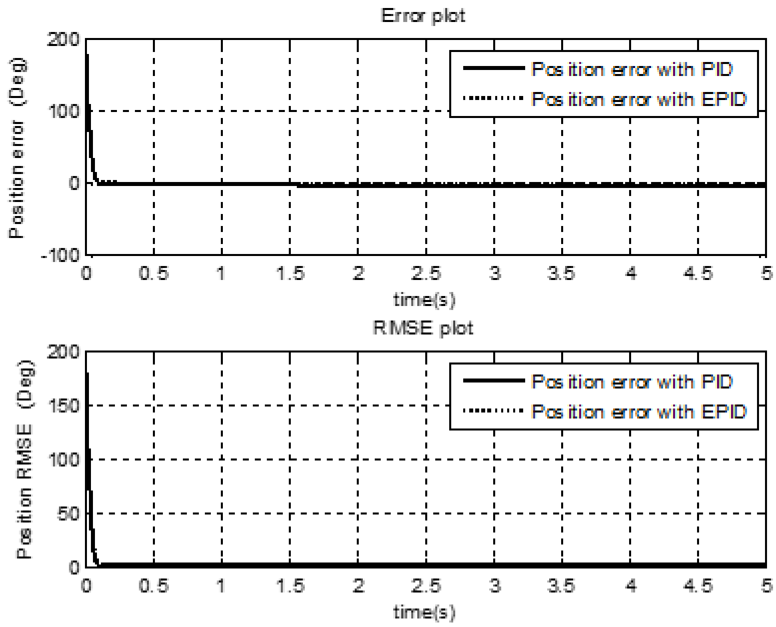

Figure 6. Position errors with disturbance

DISCUSSION

Effect of tool offsetting

Presented results obtained in the study are categorized into; without the effect of disturbance and with disturbance. The discussion started with the first followed by the other.

Figure 3 shows the angular position response of the system with both PID and the EPID without the effect of any disturbance. It shows the amount of errors at steady state for the control schemes and it is higher with the PID. It is about 4^0 with the PID, while it is about 1^0 with the EPID. Figure 4 gives the error plots of the system which revealed an RMSE value of 4^0 with the PID and 1^0 with the EPID. The EPID control scheme becomes superior over the PID because of its accuracy which is within the 2% steady state error usually specified in control systems. The amount of error reduced with time. Hence, this indicates the likelihood stability for both methods.

The effect of step disturbance was considered starting at 3 seconds which mimics the effect of sudden gust, wind or sudden torque changes. Figure 5 showed the angular position response of the system with both schemes. The amount of error at steady state is also higher with the PID which was even increased by about 1^0 due to the introduced disturbance. There was no change with the EPID even though the disturbance was introduced, this made it more robust compared to the PID. Figure 6 is the error plots of the system which showed resulted in an RMSE of 5^0 with the PID and maintaining about 1^0 with the EPID.

Hence, the new explored control scheme which has the property of being nonlinear acquire the high robustness ability popular with such kind of control schemes. It also has the simplistic nature of the linear/PID control method which an advantage or an effort to provide solution to the computational difficulties lingering the normal nonlinear control techniques. In a nutshell the presented scheme is characterized by being simple and robust. It is expected to be beneficial not only for inverted pendulum application but for other similar systems with proper and careful management. It is also a novel idea expected solve systems control problems.

CONCLUSIONS

The study on the application of a combined constant rate reaching law and proportional-integral-derivative (PID) control law (EPID) controller for inverted pendulum system control was accomplished successfully. The inverted pendulum system has properties similar to a typical attitude control of booster rocket undergoing the takeoff process. However, studies indicated increased rates of accidents in aircrafts of which about half are due to malfunctions of systems and one-third from propulsion system malfunctions. Others are higher complexities of aircraft systems and reducing cost of maintenance. Hence, the need for enhanced automation for operation as well as for faults diagnosis and correction. The different control schemes have their merits and demerits. Therefore, the simplicity of the linear control method and robustness of the nonlinear counterpart are harnessed. Combining the constant rate reaching law which is nonlinear with the PID which is linear enhances the PID scheme which is referred to as EPID in the work. A linearised analytical model of the inverted pendulum system was utilized and The performance of the system was observed together with the effect of wind, gust or a sudden torque changes on the system which was represented using step signal starting at 3 seconds.

The simulation studies using MATLAB/SIMULINK software showed that the response with the EPID produced a near zero deviation from the desired position. The result with the PID controller tries to maintain a deviation of about 4^0. The Results further indicated a near zero disturbance rejection ability of the EPID despite the introduced disturbance, while the PID was able to only suppress the disturbance to certain limit. The implication is that a more robust control was achieved with the EPID for aircraft/inverted pendulum system considered. Hence, the results portrayed the likelihood of enhanced performance with the more complex version of the system. Therefore, with further improvement it can be applicable for not only this class of systems but even for similar systems. The work would be beneficial for basic researches for beginners as well as experienced researchers in the area. The study is novel in the sense that to the best of authors has never been applied for such systems.

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