

Multi-Criteria Decision-Making Model using Intuitionistic Fuzzy Entropy and Variable Weight Theory

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ABSTRACT – The aim of this research is to develop a new multi-criteria decision-making method that integrates an intuitionistic fuzzy entropy measure and variable weight theory to be implemented in different fields to provide a solution for MCDM problems when the available information is incomplete. A limited number of studies have considered determining decision maker's weights by performing objective techniques, and almost all of these researches detected a constant weights for the decision makers. In addition, most of the MCDM studies were not formulated to perform sensitivity analysis. The new method is based on the TOPSIS model with an intuitionistic fuzzy entropy measure in the exponential-related function form and the engagement of the variable weight theory to determine weights for the decision-makers that vary as per attributes. Lastly, a mathematical model was developed in this research to be as an input for developing the mobile-application based method in future for virtual use of the new MCDM method.

ARTICLE HISTORY

Received: 2nd April 2021

Revised: 12th May 2021

Accepted: 16th June 2021

KEYWORDS

Intuitionistic Fuzzy

Entropy

Exponential Intuitionistic

Fuzzy Entropy

Intuitionistic Fuzzy

TOPSIS

MCDM

INTRODUCTION

In recent complicated life, selecting the best decisions turns out to be a tough part of the management job in both private and government enterprises. Recently, decision-makers have become unwilling to take gut feeling-based decisions, and instead look to adapt quantitative techniques to take and analyze their decisions [1].

Multi-criteria decision-making (MCDM) methods support decision-makers to confront problems with multiple criteria to provide a solution. Usually, a single optimal solution for such problems does not exist. So, preferences from decision-makers differentiate between solutions (alternatives). In other words, MCDM aims to aid decision-makers to shortlist alternatives or choose a single alternative that fulfills the attributes and is aligned with their preferences [2].

The solution of the MCDM problem is derived from the preferences of a group of decision-makers and, due to the vagueness and imprecision in the available information, DMs use intuitionistic fuzzy numbers (IFNs) to give their preferences and build a decision matrix [3-4].

Criteria weights having a significant effect on the final rank of the alternatives [5]. The used techniques to detecting weights for attributes could be grouped into two categories: subjective, based on decision-makers evaluation (like using AHP), and objective, that derived from given decision-makers preferences (like using the Entropy method) [6]. Hatefi [7] claimed that using objective methods would be more robust and rational than using subjective methods.

Entropy measure is used to evaluate the fuzziness and vagueness of the fuzzy sets [8]. Zadeh [9] presented the fuzzy entropy first. Then Deluca and Termini [10] explored the definition of fuzzy entropy. Then, many fuzzy entropy measures were introduced, and the IF entropy measure was introduced by Burillo and Bustince [11].

REVIEW OF LITERATURE

As a summary from the literature review, the challenges in MCDM that this study will try to confront are: assigning objective weights to decision-makers that vary as per attributes, the lack of sensitivity analysis in MCDM methods, and the possibility of performing the MCDM process virtually. Details are as follows:

Assigning decision weights by using objective techniques

Kabak & Ervural [12] mentioned that only 41% of current studies considered the decision-makers' weights and almost all of them assigned weights directly using a subjective rating method, while rarely of them delivered a comprehensive objective method for determining the weights. More objective methods are therefore required [6].

Koksalms & Kabak [13] reviewed the MCDM studies for a 47-year period (1970–2017) and found that 76% of those concerned with determining decision-makers' weights had been published after 2011. They claimed that this subject is still attracting good attention from researchers in the last few years. Although 82% of the studies used constant weights, they expected that dynamic weighting methods would become dominant in future studies.

But the challenge is how to treat the variety and inconsistency within a group of decision-makers [14] when decision-makers' weights are still not normally considered in the MCDM literature [15-16]. Liu et al. [17] added that the

irrationality of the MCDM processes increased when the decision-makers' weights are not considered or are assumed to be known. Hence, defining a process to detect weights is a crucial and motivated research topic [18-19].

An example of the subjective technique frequently used to determine the decision-makers' weights is Equation 1, proposed by Boran et al. [20], where the weights are detected based on the subjective evaluation from senior management, $D = (\mu_k, \nu_k, \pi_k)$:

$$\lambda_k = \frac{\left(\mu_k + \pi_k \cdot \left(\frac{\mu_k}{\mu_k + \nu_k}\right)\right)}{\sum_{k=1}^m \left(\mu_k + \pi_k \cdot \left(\frac{\mu_k}{\mu_k + \nu_k}\right)\right)} \tag{1}$$

The above equation was used in different studies such as [21-23].

Other studies used TOPSIS, statistical variance (SV), and simple additive weighting (SAW) to obtain the decision-makers' weights as an objective weighting process [17, 24]. In both subjective and objective weighting methods, however, still the weights are constant and they do not deal with errors that may arise due to the fact that decision-makers may be biased or that a sudden error may take place [15].

Criteria weights

A group of DMs may find it difficult to agree on assigning exact criteria weights, and a large number of attributes could reduce the accuracy of their subjective weights [25]. For that, Hatefi [26] suggested using objective or semi-objective techniques, but these are few in this field, even though there are techniques like Entropy, Standard Deviation, Ideal Point, and Maximizing Deviation. Therefore there is still a need for analytical techniques to handle situations where there is no preferences information from DMs.

DMs are normally selected from different fields and are characterized by different skills, knowledge, and experience, so it is not rational for DMs to set criteria weights based on their information [3].

As an example of subjective methods to determine criteria weights [22], each DM gives an IFS weight for criterion U_j as $\omega_j^k = [\mu_j^k, \nu_j^k]$, then uses an IFWA operator to determine the final criterion weight:

$$\omega_j(\mu, \nu) = IFWA(\omega_j^1, \omega_j^2, \dots, \omega_j^l) = \left[1 - \prod_{k=1}^l (1 - \mu_j^k)^{\lambda_k}, \prod_{k=1}^l (\nu_j^k)^{\lambda_k} \right] \tag{2}$$

Then, the criteria weights are calculated by using the equation:

$$\omega_j = \frac{\left(\mu_j + \pi_j \cdot \left(\frac{\mu_j}{\mu_j + \nu_j}\right)\right)}{\sum_{j=1}^n \left(\mu_j + \pi_j \cdot \left(\frac{\mu_j}{\mu_j + \nu_j}\right)\right)} \tag{3}$$

where, $\sum \omega_j = 1, j=1, 2, \dots, n$, and $k=1, 2, \dots, l$.

In objective methods, criteria weights are derived from the evaluation rates given by the DMs. As an example, the IF entropy measure $E(e_{ij})$ is used to determine criteria weights [24, 27] by using the equation below:

$$\omega_j = \frac{1 - Q_j}{n - \sum_{j=1}^n Q_j} \tag{4}$$

where $Q_j = \frac{1}{m} \sum_{i=1}^m E(e_{ij})$, $E(e_{ij})$ is the IF entropy of $e_{ij} = (\mu_{ij}, \nu_{ij})$, and $\sum \omega_j = 1$.

Sensitivity analysis

One of the simplest ways to check out how the solution of an MCDM problem varies with changes in the criteria and decision-makers' weights is by performing sensitivity analysis [28]. Well-designed sensitivity analysis determines the inputs that require more care and have a limited effect on the problem solution [28]. There are many pieces of research on this. For example, Y. Chen et al. [29] and Triantaphyllou & Sánchez [30] employed sensitivity analysis to explore the effect of changing either the criteria weights or aggregated methods separately or together. Most studies have not implemented sensitivity analysis on the decision-makers' weights [13].

Aikhuele & Turan [27] used the DMs' attitudinal parameter (β) in an exponential function that could be utilized to check how the MCDM problem solution would vary with different values of the parameters:

$$ER(a) = e^{\left(\frac{1 - \beta(\mu^2 - \nu^2)}{3}\right)} \tag{5}$$

while Liu et al. [15] proposed a new method based on variable weight theory, where a parameter (α) was used for determining the DMs' variable weights, for different values of α , and a variant preference order of alternatives could be yielded. The DMs' variable weights are calculated by using the following equation:

$$\lambda_k(q) = \lambda_k + \alpha \lambda_k \left(\sum_{k=1}^t (\mu^k + \nu^k) \lambda_k - (\mu^k + \nu^k) \right) \tag{6}$$

where: λ_k is the predetermined DM weights for $k=1,2,3,\dots,t$; $\bar{q} = \sum_{k=1}^t (\mu^k + \nu^k) \lambda_k$; $q_k = (\mu^k + \nu^k)$.

MCDM method with Intuitionistic fuzzy set theory

MCDM methods are usually used to select the best alternative according to the different criteria (multi-attributes) [31]. The aim is to provide decision-makers with an efficient and rational decision technique to comprehensively analyze all the objective and subjective criteria of the problem [19, 32]. Researchers started using fuzzy set theory and intuitionistic fuzzy set theory to achieve more accurate results and to deal with imperfect and imprecise data [15], and the fuzzy MCDM area is now a hot research field [33].

A number set (A) is considered as an intuitionistic fuzzy set in the universal set X, if: $A = \{ \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle \mid \forall x_i \in X$, where: $\mu_A(x_i) \rightarrow [0,1]$ is the membership function of $x_i \in X$ in A, and $\nu_A(x_i) \rightarrow [0,1]$ is the non-membership function of $x_i \in X$ in A. Also, $\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i)$, ($0 \leq \pi_A(x_i) \leq 1$) is called the intuitionistic index or hesitancy degree of x_i in A. Since $\pi_A(x_i) = 0$, then A is a fuzzy set.

A virtual MCDM method based on DSS

Decision support systems (DSS) could enhance the effectiveness of the decision-making process by making a better and quick analysis and decision, in addition to dealing with complex and imprecise data [13]. The need for a web- or mobile technology applications-based DSS could become the subject of future studies. The rationale behind using mobile applications to solve MCDM problems is that enhancing the communication will positively affect the MCDM decision, by making the discussion more concentrated on the problem rather than less important issues [34].

METHODOLOGY

The IF-TOPSIS_{EF} method, proposed by Aikhuele [27], utilizes the simplicity of the TOPSIS technique and the exponential-based function using an intuitionistic fuzzy set to introduce the attitudinal parameter to support the product reliability aspect. Meanwhile, the variable weight theory was applied by Liu et al. [15] to objectively determine variable DMs' weights in MCDM problems. For the aim of this research, a combination of these two techniques will help to tackle the aforementioned challenges in the literature review, with consideration to include sensitivity analysis and providing a mobile-based application mathematical model.

The characteristics of IF-TOPSIS_{EF} and variable weight theory techniques are summarized in Table 1:

Table 1. Characteristics of IF-TOPSIS_{EF} and Variable Weight Theory methods.

Aspect	IF-TOPSIS _{EF}	Variable weight theory
Method	Intuitionistic fuzzy set, Exponential related function, TOPSIS	Intuitionistic fuzzy set, Variable weights theory
Aggregation operator	IFWG	IFWA
Decision maker's weights	N/A	Variable weights as per alternatives
Criteria weights	Intuitionistic fuzzy entropy method	N/A
Sensitivity analysis	N/A	N/A
Main equation(s)	Exponential-related function: $ER(a) = e^{\left(\frac{1-\beta(\mu^2-\nu^2)}{3} \right)}$	$\lambda_k(q) = \lambda_k + \alpha \lambda_k \left(\sum_{k=1}^t (\mu^k + \nu^k) \lambda_k - (\mu^k + \nu^k) \right)$
Advantages	<ul style="list-style-type: none"> - The use of the intuitionistic fuzzy set - The simplicity of the TOPSIS method - Introduced attitudinal parameter - Using an objective technique to derive criteria weights 	<ul style="list-style-type: none"> - The use of the intuitionistic fuzzy set - Applying an objective technique to assign variable weights for DMs.

Disadvantages	<ul style="list-style-type: none"> - Deriving DM's weights not considered - The method is designed to test the reliability of product design only - The method is not formed to perform sensitivity analysis 	<ul style="list-style-type: none"> - Determining criteria weights is not considered - Calculated DM's weights vary as per alternatives, not per criteria - The method is not formed to perform sensitivity analysis
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Integrating the IF-TOPSIS_{EF} and variable weight theory methods by utilizing the simplicity of the first one and the logic of deriving variable DMs' weights from the second one, yields a new robust MCDM method that can be implemented in different fields, such as project selection, best product design, site selection, and many other fields. The integration process needs to consider that IF-TOPSIS_{EF} was proposed to tackle product design only, so there is a need to amend this method to be of more general use. On the other hand, the variable weight theory method generates DMs' weights that vary based on the alternatives, which may lead to being unfair with alternatives, and to mitigate this risk, the variable weight theory method needs to be reformulated to generate DMs' weights that vary according to the attribute instead. Once the two baseline methods are modified, theoretically, the new integrated method will utilize variable weight theory to assign variable DMs' weights per attribute, and these weights will be used as an input to the amended IF-TOPSIS_{EF} technique. By doing this, it is expected to have three parameters that can be used for sensitivity analysis: weights parameter (γ), and finally the aggregation operator (IFWA and IFWG).

The inductive approach is selected as the overall methodology in this research because the new combined MCDM method will first be derived from the two existing techniques, then secondary data will be used to validate the results of this new method by comparison with the results of methods from which the data will be taken. Then a deductive approach will be followed to determine the requirements of developing the mathematical model for the new method to be used virtually.

The proposed combined method

A new method is proposed that utilizes the benefit of variable weight theory after being amended and uses the TOPSIS method with the entropy exponential function. The proposed method is presented in Figure 2.

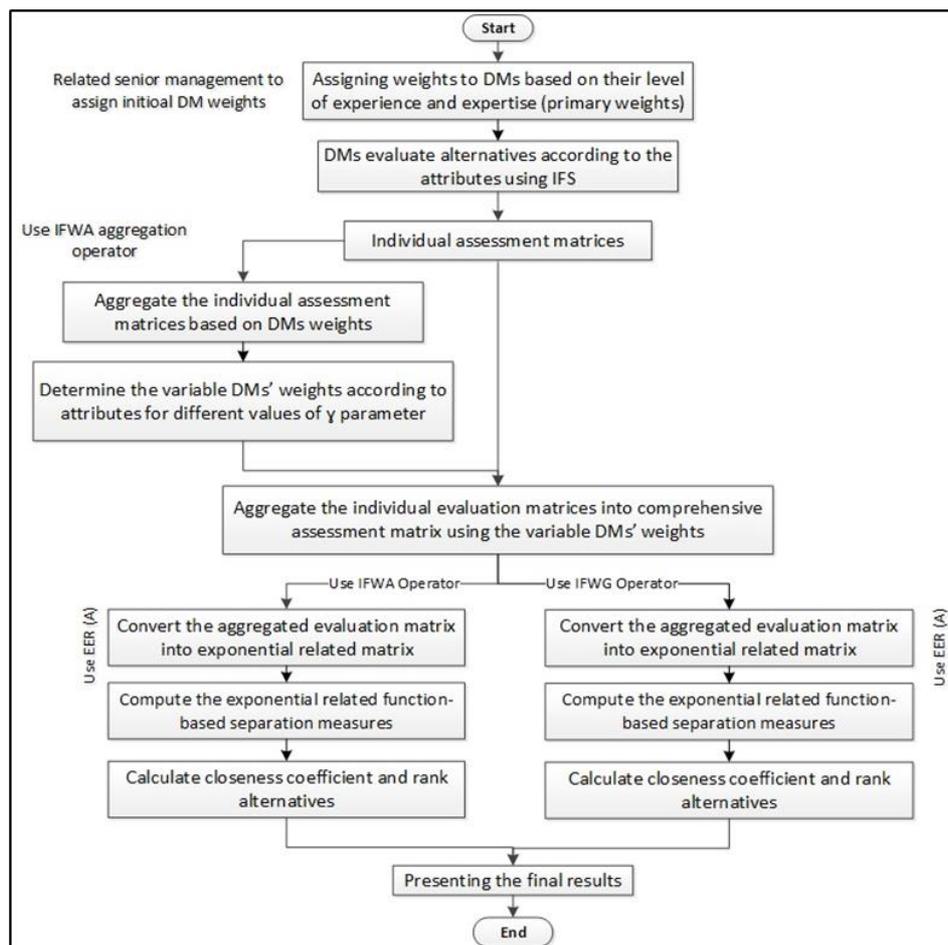


Figure 2. Proposed method layout.

The characteristics of the new method

The characteristics of the IF-TOPSIS_{EF} and variable weight theory techniques are summarized in Table 2.

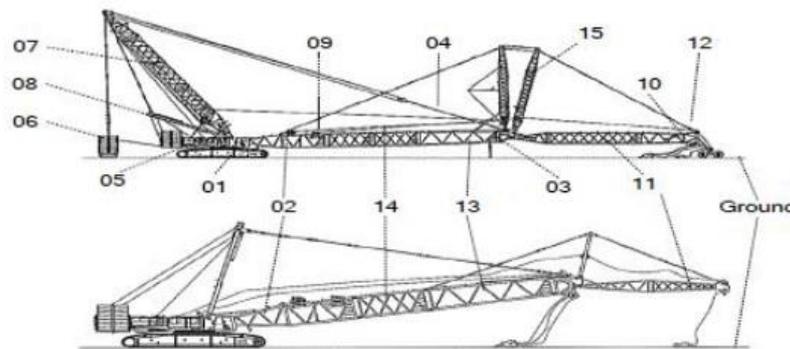
Table 2. Characteristics of the new method.

Aspect	The proposed method
Method	Intuitionistic fuzzy set, IF entropy as exponential-related function, TOPSIS, variable weight theory
Aggregation operator	IFWA and IFWG
Decision maker's weights	Variable weights based on attributes
Criteria weights	Intuitionistic fuzzy entropy method
Sensitivity analysis	Sensitivity analysis by considering: - DMs' weight parameters - Aggregation operators
Main equation(s)	$EE(a_{ij}) = e^{\left(\tan\left(\frac{\pi}{4} - \frac{ \mu^2 - \nu^2 }{4}\right)\frac{\pi}{4}\right)}$ $\lambda_k(q) = \lambda_k + \alpha \lambda_k \left(\sum_{k=1}^t (\mu^k + \nu^k) \lambda_k - (\mu^k + \nu^k) \right)$
Advantages	<ul style="list-style-type: none"> - The use of the intuitionistic fuzzy set - The use of the intuitionistic fuzzy set - The simplicity of the TOPSIS method - Using IF entropy as an exponential-related function - Providing sensitivity analysis - Providing algorithm of the method for virtual use
Disadvantages	<ul style="list-style-type: none"> - The virtual use of the method is not finalized

RESULTS

Case study

Aikhuele (2017) conducted a real case study on a crawler crane to identify the expected root causes of the machine's failure, by evaluating the risk attributes: occurrence (O), detection (D), severity (S), and failure cost (C), considering only the operational components. Figure 3 shows the parts of the crawler crane, and a list of selected operational parts with corresponding failure modes.



01 Slewing gear	06 Super-lift counterweight	11 Luffing jib lubbing device
02 Main boom base (MBB)	07 Super-lift mast	12 Loading limit of jib lubbing device
03 Main boom tip	08 Jib lubbing device of super-lift	13 Fixed jib
04 Load limits device	09 Luffing jib boom	14 Jib insert
05 Hydraulic system of super-lift	10 Luffing jib tip	15 Cantilever mast

Operational part	Failure modes (M)
01	M1 Swing brake valve unable to lock; M2 Slewing gear slow motion; M3 Slewing gear abnormal pressure; M4 Slewing bearing shaking
02	M5 MBB fracture;

	M6 Anchor bolt looseness;
	M7 MBB vibration;
	M8 Anchor bolt breakage
03	M9 Tip distortion;
	M10 Poor lubrication;
	M11 Tip blockage;
	M12 Tip stuck
04	M13 Weight sensor failure;
	M14 No display of amplifier Circuits;
	M15 Actuator damage
05	M16 Hydraulic shock;
	M17 Pressure due to overload of hydraulic;
	M18 High-pressure ball valve spun off;
	M19 Pressure reducing valve stuck;
	M20 HS leakage

Figure 3. The parts of the crawler crane and their related failure modes.

A group of 33 experts was invited to give their preferences on the failure modes. Then, after screening the responses, five experts were selected to continue the evaluation process of the risk assessment. The weight vector $\gamma = \{0.25, 0.15, 0.20, 0.15, 0.25\}^T$ was assigned to the selected experts respectively.

The DMs' preferences while performing the risk assessment of the failure modes were provided by using linguistic variables as shown in Table 3.

Table 3. The intuitionistic fuzzy linguistic variables to express DMs' preferences.

Linguistic term	IF Number
Extremely high (EH)	(0.9, 0.1)
Very high (VH)	(0.8, 0.1)
High (H)	(0.7, 0.2)
Medium high (MH)	(0.6, 0.3)
Medium (M)	(0.5, 0.4)
Medium low (ML)	(0.4, 0.5)
Low (L)	(0.25, 0.6)
Very low (VL)	(0.1, 0.75)
Extremely low (EL)	(0.1, 0.9)

By deploying the new method, the preference order of all the alternatives (failure modes) is shown in Table 4, where M18 (High-pressure ball valve spun off) is the most important failure mode among the others considered, with high stability when using different sensitivity parameters.

Table 4. Preference order after using the new method.

Risk factor	IFWA		IFWG		Risk factor	IFWA		IFWG	
	$\gamma = -10$	$\gamma = 10$	$\gamma = -10$	$\gamma = 10$		$\gamma = -10$	$\gamma = 10$	$\gamma = -10$	$\gamma = 10$
M1	10	11	13	15	M11	20	20	17	17
M2	2	2	6	8	M12	13	13	16	13
M3	6	7	10	11	M13	3	3	2	2
M4	11	9	4	4	M14	8	8	7	6
M5	17	14	19	19	M15	15	15	11	10
M6	7	6	3	3	M16	16	16	18	18
M7	14	17	20	20	M17	18	19	15	14
M8	12	12	9	9	M18	1	1	1	1
M9	19	18	14	16	M19	5	5	12	12
M10	9	10	8	5	M20	4	4	5	7

By comparing the results of the risk assessment for the case study obtained with the new method and with the IF-TOPSIS_{EF} method, the clear result from the new method shows without doubt that M18 is ranked as the first failure mode to be considered by the management to mitigate the risk of failure during the design process. By contrast, if referring to the result from the other mentioned method presented in Table 5, the management will be uncertain about prioritizing the failure modes.

Table 5. Preference order by using IF-TOPSIS_{EF} method.

Risk factor	$\lambda = 0.1$	$\lambda = -0.1$	Risk factor	$\lambda = 0.1$	$\lambda = -0.1$
M1	8	11	M11	5	14
M2	15	5	M12	12	8

M3	3	18	M13	15	5
M4	20	1	M14	17	4
M5	5	14	M15	8	11
M6	4	17	M16	10	10
M7	19	2	M17	5	16
M8	18	3	M18	1	20
M9	2	19	M19	10	11
M10	14	7	M20	12	9

The results from using the new method can be ranked by calculating the average preference order for the different sensitivity analysis parameters used. Therefore, the final ranking of failure modes will be as follows:

M18>M13>M2>M6>M20>M4>M14>M10>M3>M19>M8>M1>M15>M12>M17>M9>M16>M5>M7>M11

Model of the proposed method

The mathematical model of the proposed method for virtual application is shown in Figure 4.

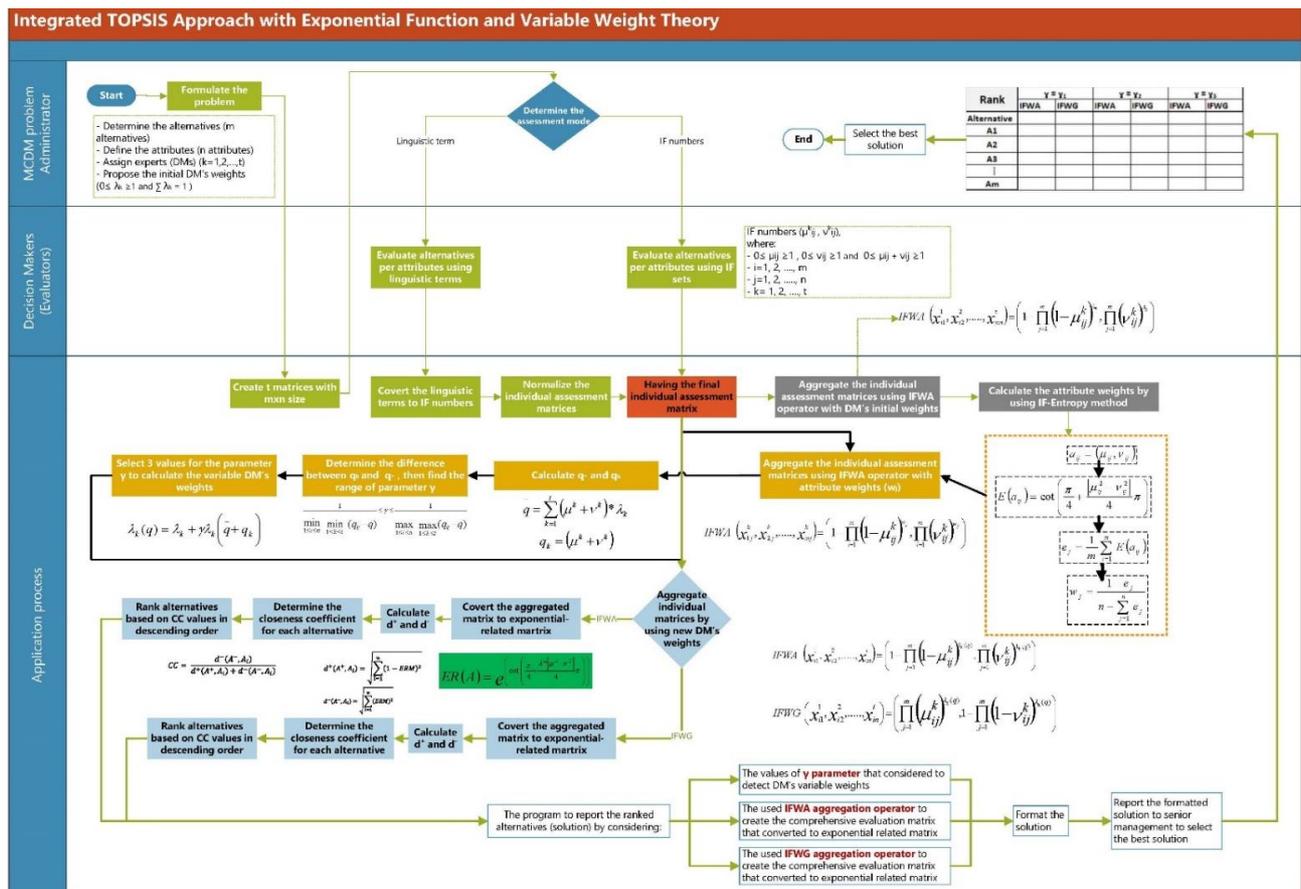


Figure 4. Algorithm of the proposed method.

CONCLUSION

In this paper, a new MCDM is proposed by integrating the IF-TOPSIS method and variable weight theory. This integration took place after enhancing the two base-line methods by emphasizing their advantages and treating the disadvantages of each base-line method separately. Then, the enhanced baseline methods were validated separately by using secondary data (examples) before integration. The final combined method was formulated to provide sensitivity analysis, after which a mobile-based application mathematical model was developed to allow this new method to be used virtually in the future. The use of intuitionistic fuzzy entropy measure in an exponential function form compensated the need to detect attribute's weights, therefore added more simplicity to the new method.

ACKNOWLEDGEMENT

The authors would like to thank the Ministry of Higher Education for providing financial support under Fundamental Research Grant Scheme (FRGS) No. FRGS/1/2019/TK10/UMP/02/10 (University reference RDU1901158) and Universiti Malaysia Pahang for the facilities..

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