

ORIGINAL ARTICLE

Parametric Study of Dual-particle Swarm Optimisation-modified Adaptive Bats Sonar Algorithm on Multi-objective Benchmark Test Functions

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ABSTRACT – An integrated algorithm for solving multi-objective optimisation problems using a dual-level searching approach is presented. The proposed algorithm named as dual-particle swarm optimisation-modified adaptive bats sonar algorithm (D-PSO-MABSA) where the concept of echolocation of a colony of bats to find prey in the modified adaptive bats sonar algorithm is combined with the established particle swarm optimisation algorithm. The proposed algorithm combines the advantages of both particle swarm optimisation and modified adaptive bats sonar algorithm approach to handling the complexity of multi-objective optimisation problems. These include swarm flight attitude and swarm searching strategy. The performance of the algorithm is verified through several multi-objective optimisation benchmark test functions. The acquired results show that the proposed algorithm performs well to produce a reliable Pareto front. The proposed algorithm can thus be an effective method for solving multi-objective optimisation problems.

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Introduction

Nowadays, increasing of real-world problems consist of multiple objectives to be satisfied simultaneously makes multi-objective optimisation become very important research area for academics and engineers [1]. Different from the single objective optimisation, a solution to a multi-objective problem is a concept rather than a definition [1]. Besides, since the last two decade, the most popular multi objective approach is multi-objective evolutionary algorithm [2].

The evolutionary algorithms have been used for the optimisation of real-world problems in many applications instead of conventional techniques [3]. Evolutionary algorithms are promising methods to solve complex optimisation problems because their optimisation approaches depend on population evolution inspired by nature [4]. Besides, they can reach a set of approximated Pareto optimal solutions within a single run due to their population-based search strategy [2].

This paper introduces a new multi-objective evolutionary algorithm by integrating the particle swarm optimisation (PSO) algorithm with a modified adaptive bats sonar algorithm (MABSA). The proposed algorithm also implemented non-Pareto

technique; the weighted sum approach for solving the multi-objective optimisation problem. The results from computer simulations on several multi objective optimisation benchmark test functions prove that this new hybrid algorithm can serve as a practical multi-objective evolutionary algorithm option for solving multi-objective optimisation problems.

A Dual-Particle Swarm Optimisation-Modified Adaptive Bats Sonar Algorithm (D-PSO-MABSA)

A dual-particle swarm optimisation-modified adaptive bats sonar algorithm (D-PSO-MABSA) is a new hybrid algorithm where a dual-level search strategy is adopted through the integration of two algorithms; particle swarm optimisation and modified adaptive bats sonar algorithm for getting the Pareto optimum set of the problem. Figure 1 shows the flowchart of this algorithm works. This algorithm was developed by [5] that uses the weight sum approach to combine all objectives into a single objective where the weights are generated randomly from a uniform distribution as shown in [5]. By doing so, the Pareto optimum set can be attained efficiently as well as Pareto front would be estimated properly.

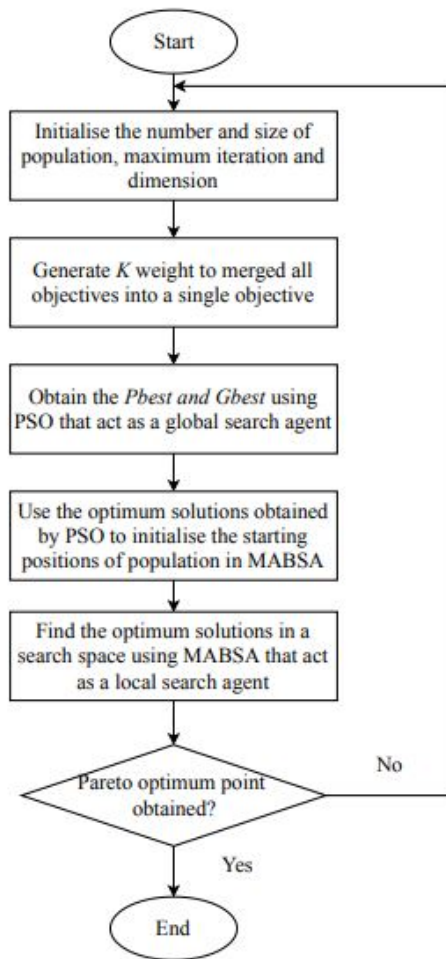


Figure 1. The flowchart of dual-particle swarm optimisation-modified adaptive bats sonar algorithm (D-PSO-MABSA).

Here, the dual-level searching process means that at every time to obtain a Pareto optimum point, there are always two levels of search. During the first level, PSO acts as a global search agent of the algorithm with its embedded global (exploration) and local (exploration) search components. As an explorer, PSO is first to determine and mark a potential location of a solution in the compound of selected search space. The PSO will run according to its standard algorithmic procedures such as locating new velocity and position to obtain the *pbest* and *gbest*.

Then, in the second level search process, the optimum solutions found by the PSO are used to initialize the starting positions of the population in the MABSA. The MABSA considered as a local search agent of the developed algorithm also has its global search (diversification component) and local search (intensification component). Here, MABSA works as a follower to find the optimum solutions starting from the potential location previously marked by the PSO within the designated search space.

However, there are two factors are considered to set PSO as global search agent and MABSA as a local search agent. These factors are inspired by the real behaviour of both swarm groups. As noted, PSO is represented based on a swarm of birds flying in search of food while MABSA is based on a colony of bats flying for capturing preys. The factors are swarm flight attitude and swarm searching strategy.

The first factor is the flight attitude of the swarm. A good global search agent has the capability of viewing and monitoring the search space from the highest position. The broad perspective from the higher ground makes it easier for the agent to mark possible areas within the search space containing potential solutions that would be a true exploration process in swarm intelligence. A local search agent is, on the other hand, needed to verify the location of potential solutions found by a global search agent. To be a good local search agent, the agent must have the ability to observe and inspect the solutions from stone's throw view. This exploration process should be put after the exploration process so that the solutions developed by a global agent could be validated properly by the local search agent.

Looking at the proposed swarm searching strategy, there is a distinct line between the searching strategy of PSO and MABSA. In the PSO, the algorithm utilizes the velocity and positioning of particles to evaluate the obtained solution whereas MABSA depends on the transmission and positioning of sound beams. In the real world, birds can fly with a velocity between 20 to 30 mph [6]. With this fast speed, the searching process of PSO may miss locations of good solutions on their way towards other possible target solutions. Moreover, the velocity of particles in PSO itself makes the particle or bird to move in a single line thus no covering a broad search at one time. The sound beams transmitted in MABSA are multi-line able to disperse and sweep a large search envelope. Thus, the issue of missing good solutions in a smaller area of designated search space does not arise. Hence, the sequence of the searching process as applied in any good swarm intelligence method is followed here where coarse searching (diversification) is done first by PSO followed by fine searching (intensification) by MABSA. In this context, labeling PSO as a global search agent and MABSA as a local search agent in the proposed hybrid algorithm D-PSO-MABSA is a reasonable choice given their characteristics.

Multi-objective Benchmark Test Functions

Six well-known multi-objective benchmark test functions were used in the investigation of D-PSO-MABSA's performance. Each test function consists of two objective functions with or without constraints.

Binh and Korn

This function developed in 1994 by [7] to test their multi-objective evolutionary strategy (MOBES) for multi objective optimisation problem with constraints. The function constitutes a constrained problem and has a convex Pareto front. The function is defined as:

Minimise

$$F_1(x_1, x_2) = 4x_1^2 + 4x_2^2$$

and

$$F_2(x_1, x_2) = (x_1 - 5)^2 + (x_2 - 5)^2$$

subject to

$$g_1(x_1, x_2) = (x_1 - 5)^2 + x_2^2 \leq 25$$

$$g_2(x_1, x_2) = (x_1 - 8)^2 + (x_2 + 3)^2 \geq 7.7$$

where

$$0 \leq x_1 \leq 5$$

$$0 \leq x_2 \leq 3$$

Chakong and Haimes

This function was adapted from [8]. The function as named was developed by Chakong and Haimes in 1983. The function constitutes a constrained problem and has a convex Pareto front. The function is defined as:

Minimise

$$F_1(x_1, x_2) = 2 + (x_1 - 2)^2 + (x_2 - 1)^2$$

and

$$F_2(x_1, x_2) = 9x_1 - (x_2 - 1)^2$$

subject to

$$g_1(x_1, x_2) = x_1^2 + x_2^2 \leq 225$$

$$g_2(x_1, x_2) = x_1 - 3x_2 + 10 \leq 0$$

where

$$-20 \leq x_1, x_2 \leq 20$$

Kursawe

This function is a multimodal function in one component and has pair-wise interactions among the variables in the other component [9]. The function constitutes an unconstrained problem and has a discrete convex Pareto front. This function is defined as:

Minimise

$$F_1(x) = \sum_{i=1}^2 \left[-10e^{-0.2\sqrt{x_i^2 + x_{i+1}^2}} \right]$$

and

$$F_2(x) = \sum_{i=1}^3 \left[|x_i|^{0.8} + 5 \sin x_i^3 \right]$$

where

$$-5 \leq x_1 \leq 5$$

$$1 \leq x_2 \leq 3$$

Osyczka and Kundu

This function was developed by [11]. The function constitutes a constrained problem and has a convex Pareto front. The function is defined as:

Minimise

$$F_1(x) = -25(x_1 - 2)^2 + (x_2 - 2)^2 - (x_3 - 1)^2 - (x_4 - 4)^2 - (x_5 - 1)^2$$

and

$$F_2(x) = \sum_{i=1}^6 x_i^2$$

subject to

$$g_1(x) = x_1 + x_2 - 2 \geq 0$$

$$g_2(x) = 6 - x_1 - x_2 \geq 0$$

$$g_3(x) = 2 - x_2 + x_1 \geq 0$$

$$g_4(x) = 2 - x_1 + 3x_2 \geq 0$$

$$g_5(x) = 4 - (x_3 - 3)^2 - x_4 \geq 0$$

$$g_6(x) = (x_5 - 3)^2 + x_6 - 4 \geq 0$$

where

$$0 \leq x_1, x_2, x_6 \leq 10$$

$$1 \leq x_3, x_5 \leq 5$$

$$0 \leq x_4 \leq 5$$

Constr-Ex

This function was used by [12] after developed by Deb in 2001 as a multi objective benchmark test function. The function constitutes a constrained problem and has a convex Pareto front. The function is defined as:

Minimise

$$F_1(x_1, x_2) = x$$

and

$$F_2(x_1, x_2) = \frac{1 + x_2}{x_1}$$

subject to

$$g_1(x_1, x_2) = x_2 + 9x_1 \geq 6$$

$$g_2(x_1, x_2) = -x_2 + 9x_1 \geq 1$$

where

$$0.1 \leq x_1 \leq 1$$

$$0 \leq x_2 \leq 5$$

CTP1

This function was proposed by [13]. The function constitutes a constrained problem and has a convex Pareto front. The function is defined as:

Minimise

$$F_1(x_1, x_2) = x_1$$

and

$$F_2(x_1, x_2) = (1 + x_2)e^{\left(\frac{-x_1}{1+x_2}\right)}$$

subject to

$$g_1(x_1, x_2) = \frac{F_2(x_1, x_2)}{0.858e^{(-0.541F_1(x_1, x_2))}} \geq 1$$

$$g_2(x_1, x_2) = \frac{F_2(x_1, x_2)}{0.728e^{(-0.295F_1(x_1, x_2))}} \geq 1$$

where

$$0 \leq x_1, x_2 \leq 1$$

Parametric Study of D-PSO-MABSA on Selected Multi Objective Benchmark Test Functions

Binh and Korn

The developed D-PSO-MABSA algorithm will determine sets of 50 Pareto optimum points for this test function by using four different combinations of α and β values respectively. The values are: $\alpha = 0.00$; $\beta = 3.50$, $\alpha = 0.00$; $\beta = 0.00$, $\alpha = 2.50$; $\beta = 0.00$, along with the theoretical values; $0 \leq rand(\alpha, \beta) \leq 1$.

Figure 2 shows the results of Pareto optimum points recorded of Binh and Korn function using four different settings of α and β of the developed D-PSO-MABSA algorithm. As notes, the algorithm was able to converge with each setting to a Pareto front of the test function that was similar to the results recorded by [7]. However, in general, by using theoretical values; $0 \leq rand(\alpha, \beta) \leq 1$, all the points of Pareto optimum set attained are non-dominated vectors. Thus, these solutions perfectly formed a recognisable Pareto front. The stability of final location of non-dominated solutions acquired by D-PSO-MABSA algorithm through theoretical α and β settings show a high prospect of D-PSO-MABSA to solve any multi-objective optimisation problem.

Chakong and Haimes

For this test function, four sets of 50 Pareto optimum points are searched. The developed algorithm operated on three different sets of α and β values in conjunction with the theoretical values; $0 \leq rand(\alpha, \beta) \leq 1$. These three sets considered were: $\alpha = 0.00$; $\beta = 3.50$, $\alpha = 0.00$; $\beta = 0.00$, $\alpha = 2.50$; $\beta = 0.00$.

Figure 3 shows the Pareto optimum sets with different values of α and β . A Pareto front is properly drawn by a set of non-dominated solutions acquired by

the theoretical values of α and β ; $0 \leq rand(\alpha, \beta) \leq 1$. The result comparable to the result acquired by [8]. Even the remaining sets of α and β managed to search the points that settle on the Pareto front, but there were still, few dominated solutions scattered far from the true front. Thus, it is shown that the developed D-PSO-MABSA algorithm with the theoretical parameter values was able to achieve a perfect Pareto front with this test function from the set of Pareto optimum points attained. This performance makes the developed algorithm at par with other multi-objective optimisation algorithms and may be used widely to solve any multi-objective optimisation problems.

Kursawe

For test function involved searching of four sets of 200 Pareto optimum solutions with the developed D-PSO-MABSA algorithm. The theoretical values of α and β ; $0 \leq rand(\alpha, \beta) \leq 1$ were adopted along with another three sets of α and β for performance comparison purposes. The three sets used were: $\alpha = 2.00$; $\beta = 4.00$, $\alpha = 3.00$; $\beta = 2.00$, $\alpha = -2.00$; $\beta = -2.00$.

Figure 4 shows the Pareto optimum sets obtained for Kursawe function using D-PSO-MABSA approach. As noted, the developed algorithm with theoretical values of α and β achieved the best performance compared to when the other three sets of α and β used. Most of the points in the Pareto optimum set were non-dominated solutions, successfully exhibiting a Pareto front of the test function. The pattern of Pareto fronts with the three discontinuous regions also developed nearly matched the result obtained by [10].

With the remaining three sets of α and β the algorithm could not form a Pareto front of this test function, and only a few of the solutions were non-dominated. The Pareto optimum point generated from the set $\alpha = 3.00$; $\beta = 2.00$ is likely to work, but most of the points with this set are dominated solutions and scattered far from the true front. As far as the values of α and β are concerned, negative values do not lead to a Pareto front. When set of $\alpha = -2.00$ and $\beta = -2.00$ was applied, no non-dominated solutions were achieved. Nonetheless, the D-PSO-MABSA algorithm with the right setting of its parameters would be good alternative multi objective algorithm

for solving discrete convex Pareto front-type multi-objective optimisation problems.

Osyczka and Kundu

For this test function, four sets of 500 Pareto optimum points are searched the developed D-PSO-MABSA algorithm. Each set is examined by different value of α and β . The theoretical values $0 \leq rand(\alpha, \beta) \leq 1$ were applied along with the three sets $\alpha = 3.10$; $\beta = 1.50$, $\alpha = -1.70$; $\beta = 5.00$, $\alpha = 2.80$; $\beta = -0.50$.

Figure 5 shows the effect of different values of α and β on the Pareto optimum solutions of Osyczka and Kundu function. When the theoretical values of α and β ; $0 \leq rand(\alpha, \beta) \leq 1$ were used, all the Pareto optimum points were non-dominated vectors. Although the ranges for F_1 and F_2 recorded were wider than the result reported by [11], the shapes of the Pareto front were nearly similar as all the Pareto front were nearly similar as all the Pareto optimum points contributed to form that front. In the meantime, the three sets of α and β produced many dominated vectors of Pareto optimum sets thus unable to form a viable Pareto front. Indeed, the Pareto optimum set gathered by $\alpha = 2.80$; $\beta = -0.50$ was more obvious as the points were scattered outlying from the true front. However, if the theoretical values of α and β are retained by the D-PSO-MABSA, the algorithm will be able to perform well in comparison to available algorithms in solving multi-objective problems.

Constr-Ex

The developed algorithm was evaluated with this function by searching four sets of 50 Pareto optimum solutions. Here, four sets of different values of α and β were used. These included the theoretical values $0 \leq rand(\alpha, \beta) \leq 1$, $\alpha = -4.00$; $\beta = 3.00$, $\alpha = 0.00$; $\beta = -1.70$, $\alpha = 3.50$; $\beta = 3.50$.

As noted in Figure 6, all four sets of 50 Pareto optimum solutions generated from four different values α and β of D-PSO-MABSA were non-dominated vectors. So, the entire sets produced a Pareto front similar to that reported by [12]. It was noted that the convex shape of Pareto front produced by the D-PSO-MABSA algorithm was smoother than that reported by [10]. It is clear that the developed D-PSO-MABSA algorithm generates distinctly better Pareto optimum points in solving multi-objective optimisation problems.

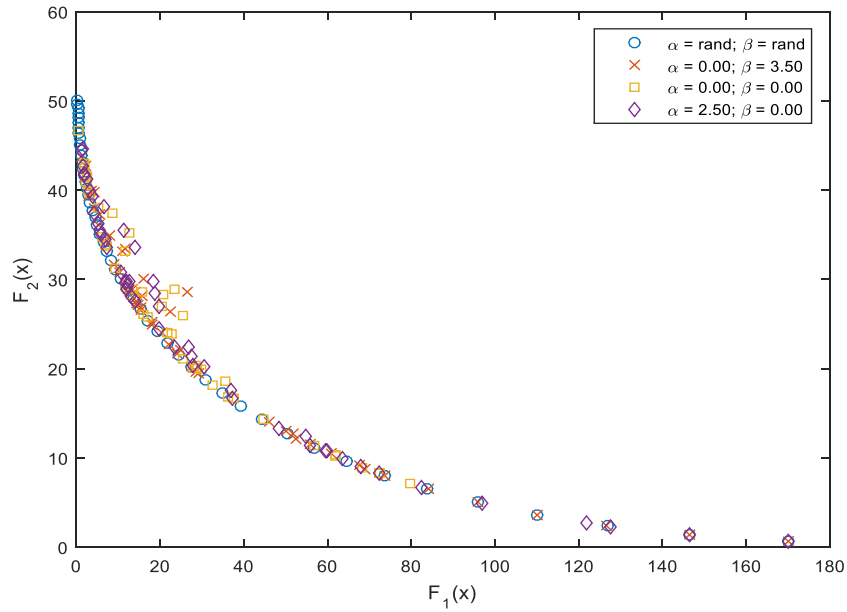


Figure 2. Pareto optimum solutions for Binh and Korn function with different values of α and β .

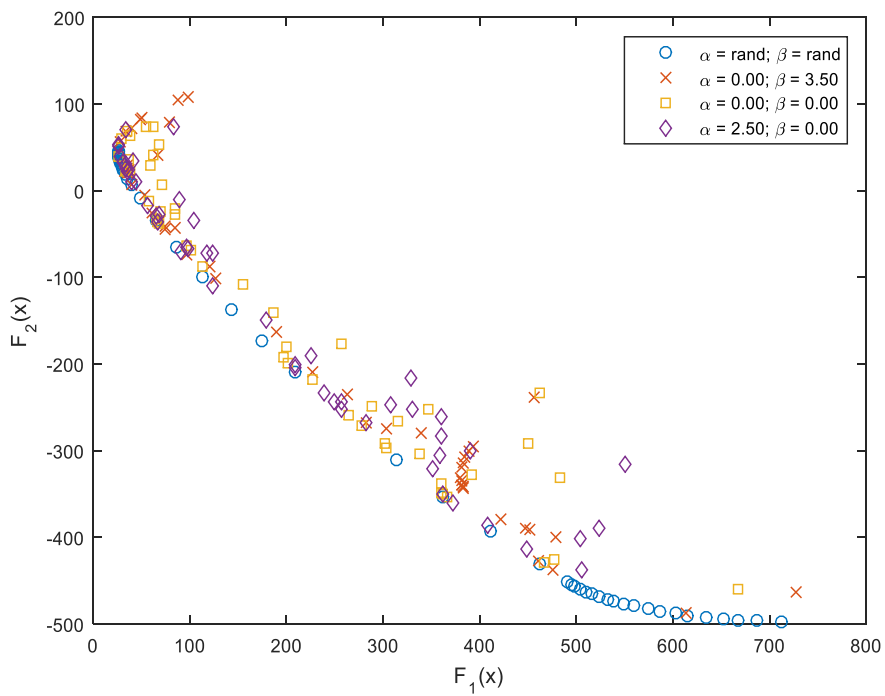


Figure 3. Pareto optimum solutions for Chakong and Haimes function with different values of α and β .

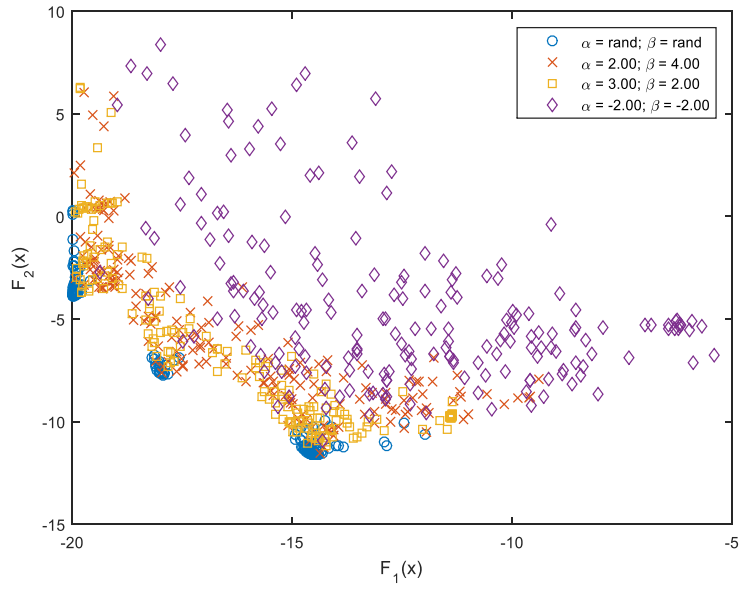


Figure 4. Pareto optimum solutions for Kursawe function with different values of α and β .

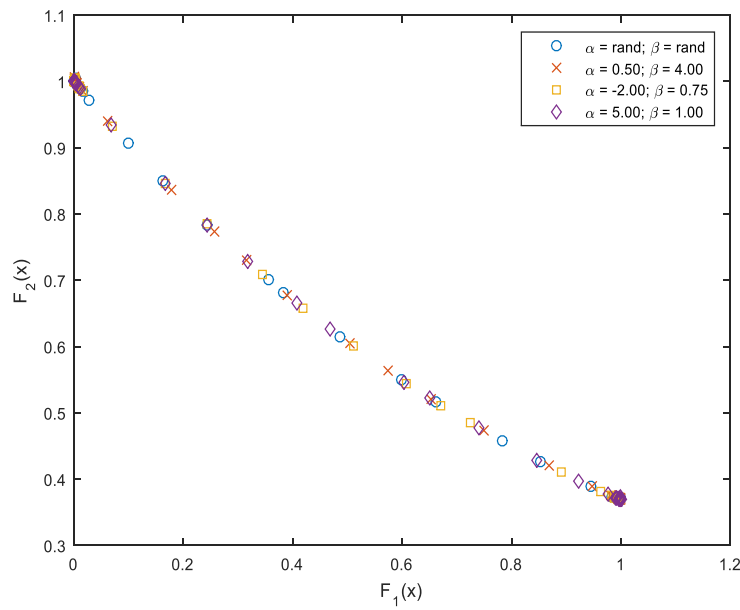


Figure 5. Pareto optimum solutions for Osyczka and Kundu function with different values of α and β .

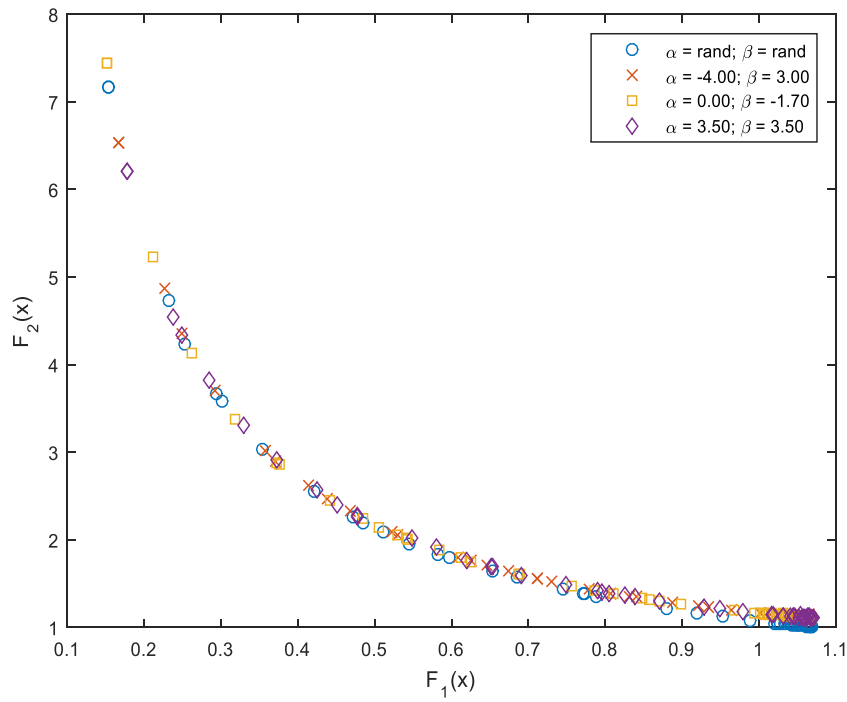


Figure 6. Pareto optimum solutions for Constr-Ex function with different values of α and β .

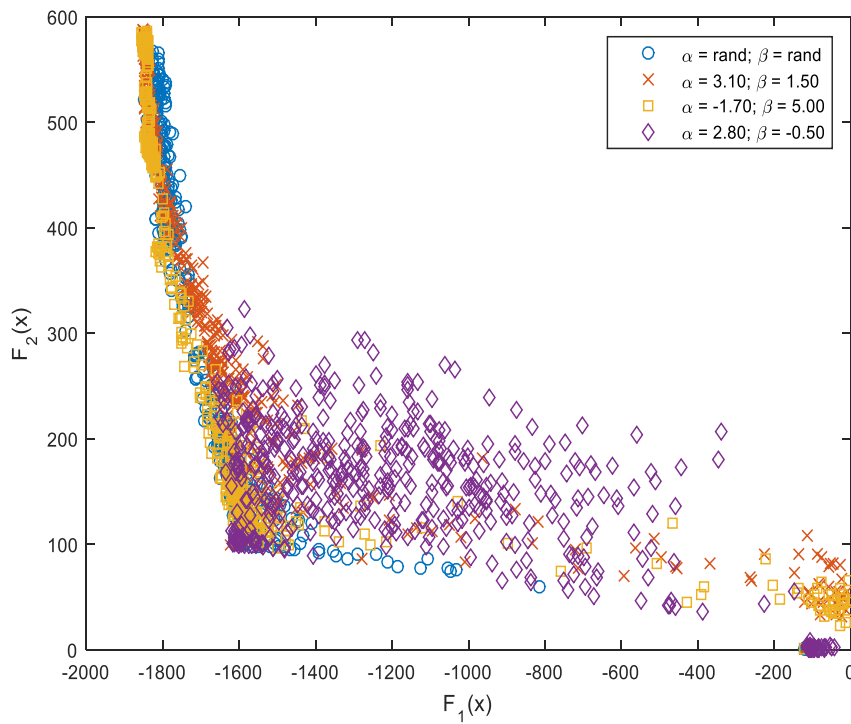


Figure 7. Pareto optimum solutions for CTP1 function with different values of α and β .

CTP1

Here, four sets of 50 Pareto optimum solutions are searched for CTP1 function using the developed D-PSO-MABSA algorithm. These were the theoretical values $0 \leq rand(\alpha, \beta) \leq 1$, $\alpha = 0.50$; $\beta = 4.00$, $\alpha = -2.00$; $\beta = 0.75$, $\alpha = 5.00$; $\beta = 1.00$.

The results of the Pareto optimum solution for the CTP1 are shown in Figure 7. It is noted that all the solutions generated using D-PSO-MABSA algorithm with four different sets of α and β values were non-dominated vectors. The Pareto fronts formed from the solutions were identical to the result reported by [13]. Furthermore, these also reflected the real advantage when using the set of theoretical value; $0 \leq rand(\alpha, \beta) \leq 1$, as the non-dominated solutions produced were uniformly distributed along the front. Hence, the outcomes resulted from a good leverage of minimising both F_1 and F_2 and no one was extremely good while other suffered. The performances shown with the test functions demonstrate the strong ability of the developed algorithm in producing good trade-off solutions for multi-objective optimisation problems.

Conclusions

This paper has introduced a hybridization of particle swarm optimisation with a modified adaptive bats sonar algorithm to solve multi-objective optimisation problems. The multi-objective optimisation problems have been briefly defined with the weighted sum method as an approach to solve the problem. A dual-level searching for multi-objective optimisation problem using PSO and MABSA has been proposed. The proposed approach includes two factors to justify the significance of this hybridisation strategy which are swarm flight attitude and swarm searching strategy.

The parametric study of the proposed algorithm on several multi-objective benchmark test functions has been done to show the ability of the developed algorithm to solve the multi objective optimisation problems. The computer simulation results have showed the ability of the D-PSO-MABSA algorithm to solve a variety of multi-objective benchmark test functions. The application of the proposed algorithm to solve practical multi-objective optimisation problem in other field instead of engineering background will be considered in future works.

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