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ORIGINAL ARTICLE

Enhancing simulated Kalman filter algorithm using current optimum opposition-based learning

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ABSTRACT – Simulated Kalman filter (SKF) is a new population-based optimization algorithm inspired by estimation capability of Kalman filter. Each agent in SKF is regarded as a Kalman filter. Based on the mechanism of Kalman filtering, the SKF includes prediction, measurement, and estimation process to search for global optimum. The SKF has been shown to yield good performance in solving benchmark optimization problems. However, the exploration capability of SKF could be further improved. From literature, current optimum opposition-based learning (COOBL) has been employed to increase the diversity (exploration) of search algorithm by allowing current population to be compared with an opposite population. By employing this concept, more potential agents are generated to explore more promising regions that exist in the solution domain. Therefore, this paper intends to improve the exploration capability of SKF through the application of COOBL. The COOBL is employed after the estimation process of SKF. Experimental results over the IEEE Congress on Evolutionary Computation (CEC) 2014 benchmark functions indicate that current optimum opposition-based simulated Kalman filter (COOBSKF) improved the exploration capability of SKF significantly. The COOBSKF also has been compared with five other optimization algorithms and outperforms them all.

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Introduction

The main goal of an optimization problem is to obtain the best combination of variables of a fitness function such that the value of the fitness is maximum or minimum. This can be done effectively by using a population-based optimization algorithm.

A new population-based optimization algorithm termed as simulated Kalman filter (SKF) is inspired by the estimation capability of Kalman filter [1]. Designed from the procedure of Kalman filtering, which incorporates prediction, measurement, and estimation, the global minimum or maximum can be estimated. Measurement process, which is needed in Kalman filtering, is mathematically modelled and simulated. Agents interact with each other to update and optimize the solution during the search process.

The concept of opposition-based learning (OBL) can be used to improve the performance of population-based optimization algorithm [2]. The important idea behind the OBL is the concurrent consideration of an estimate and its corresponding opposite estimate which is closer to the global optimum. OBL was initially implemented to improve learning and back propagation in neural networks [3], and until now, it has been employed in various optimization algorithms, such as differential evolution [4], particle swarm optimization [5] and ant colony optimization [6].

In this research, inspired by the concept of current optimum opposition-based learning (COOBL) [7], we propose a modified SKF which is called as current optimum opposition-based simulated Kalman filter (COOBSKF) to enhance the performance of SKF. From the SKF perspective, this is the first attempt to improve its performance through COOBL strategy. The COOBSKF compares the fitness of an individual to its opposite and maintain the fitter one in the population. Experimental results show that the proposed algorithm can achieve better solution quality.

The remainder of this paper is organized as follows: Section 2 briefly presents an overview of optimization algorithms and opposition-based learning application. Section 3 explains the standard simulated Kalman filter algorithm, the concept of opposition-based learning and the proposed enhance version of SKF. Section 4 provides the experimental settings and discusses the experimental results. Section 5 concludes the paper.

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Related Work

This part provides a brief overview of optimization algorithms followed by the application of OBL in optimization algorithms.

Some of optimization algorithms are based on population-based where the search process is perform with multiple agents. One example of population-based optimization algorithm is particle swarm optimization (PSO) [8]. In PSO, a swarm of agent searches for the global optimum solution by velocity and position updates, which are depending on current position of agent, personal best, and global best of the swarm. They move towards those particles which have better fitness values and finally attain the best solution.

Another population-based optimization algorithm is gravitational search algorithm (GSA) [9]. GSA was designed according to the Newtonian gravity law and mass interactions. In the algorithm, agents and their performance is evaluated by their masses which rely on fitness function values. The location of each agent in the search space indicates a problem solution. The heaviest mass is the optimum solution in the search space and by lapse of time, masses are attracted by the heaviest mass and converged to the better solution.

The concept of opposition-based learning is applicable to a wide range of optimization algorithms. Even though the proposed approach is originally embedded in differential evolution (DE), it is universal enough to be employed in other optimization algorithms. In [4], the OBL has been used to accelerate the convergence rate of DE. The proposed opposition-based DE (ODE) implements the OBL at population initialization and also for generation jumping. Besides that, a comprehensive investigation was conducted by using 58 benchmark functions with a purpose to analyze the effectiveness of ODE. Various sets of experiments are performed separately to examine the influence of opposite points, dimensionality, population size and jumping rates on the ODE algorithm.

Opposition-based differential evolution using the current optimum (COODE) was introduced for function optimization [7]. In the COODE, the optimum agent in the current population is dynamically functioned as the symmetry point between an estimate and its respective opposite estimate. The distance between opposite numbers and the global optimum is short enough to maintain a significant rate of applying OBL throughout the search process.

Opposition-based particle swarm optimization (OPSO) is proposed by employing OBL to population initialization, generation jumping and the swarm's best particle [10]. Initially, swarms are initialized with random velocities and positions. The opposite swarm is determined by calculating the opposite of velocity and position, and then the fittest of swarm and opposite swarm is chosen as the next population. The similar approach is used in current generations by applying jumping rate and dynamic constriction factor, which is used to improve the convergence rate.

In other report, the OBL technique has been used to enhance the quality of solutions and convergence rate of an ant colony system (ACS) [6]. Five versions of implementing opposition idea have been proposed to extend the solution construction phase of ACS, known as, free opposition, free quasi-opposition, synchronous opposition, opposite pheromone per node (OPN) and opposite pheromone per edge (OPE). Results of these algorithms on TSP problems indicate that only OPN technique shows significant improvement.

Current optimum opposition-based simulated Kalman filter

Simulated Kalman filter

The simulated Kalman filter (SKF) [1, 11] algorithm is shown in Figure 1. The algorithm started with initialization of n agents, in which the positions of each agent are initialized randomly in the search space. The maximum number of iterations, t_{max}, is defined as the stopping condition for the algorithm. The initial value of error covariance estimate, $P(0)$, the process noise value, Q, and the measurement noise value, R, which are needed in Kalman filtering, are also determined during initialization stage. After that, each agent is subjected to fitness evaluation to generate initial solutions. The fitness values are checked and the agent having the best fitness value at every iteration, t, is recorded as $\boldsymbol{X}_{\mathbf{best}}(t)$. For function minimization problem,

$$\boldsymbol{X}_{\boldsymbol{best}}(t) = \min_{i \in 1,\ldots,n} fit_i(\boldsymbol{X}(t)) \tag{1}$$

and for function maximization problem,

$$\boldsymbol{X}_{\boldsymbol{best}}(t) = \max_{i \in 1,\ldots,n} fit_i\big(\boldsymbol{X}(t)\big) \tag{2}$$

The best so far solution in SKF is named as $\boldsymbol{X}_{\mathbf{true}}$. The $\boldsymbol{X}_{\mathbf{true}}$ is updated only if the $\boldsymbol{X}_{\mathbf{best}}(t)$ is better ($\boldsymbol{X}_{\boldsymbol{best}}(t) < \boldsymbol{X}_{\boldsymbol{true}}$ for minimization problem, or $\boldsymbol{X}_{\boldsymbol{best}}(t) > \boldsymbol{X}_{\boldsymbol{true}}$ for maximization problem) than the $\boldsymbol{X}_{\mathbf{true}}$.

The subsequent computations are basically identical to the prediction, measurement and estimation procedures in Kalman filter. In the prediction stage, the following time-update equations are calculated:

$$\boldsymbol{X}_{\boldsymbol{i}}(t|t) = \boldsymbol{X}_{\boldsymbol{i}}(t) \tag{3}$$

$$P(t|t) = P(t) + Q \tag{4}$$

where $\boldsymbol{X}_i(t)$ and $\boldsymbol{X}_i(t|t)$ are the previous state and predicted state, respectively, and $P(t)$ and $P(t|t)$ are previous error covariant estimate and predicted error covariant estimate, respectively. Note that the error covariant estimate is influenced by the process noise, Q.

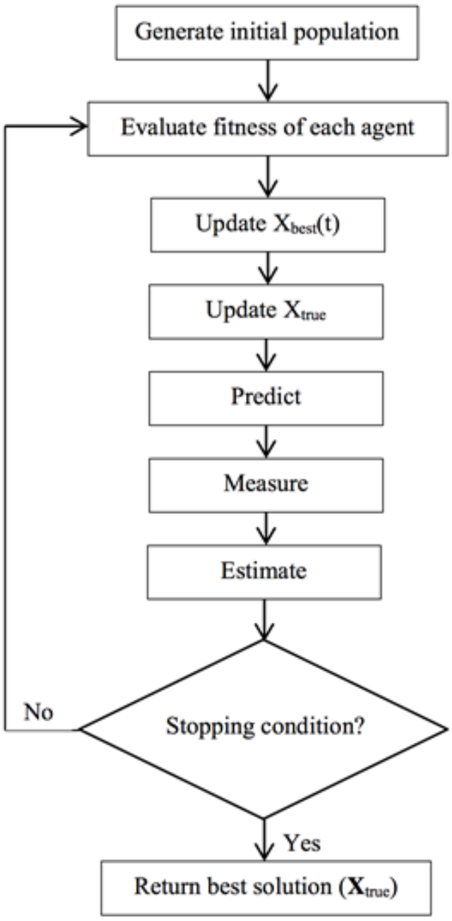


Figure 1. Simulated Kalman filter (SKF) algorithm.

The next step is measurement, which is a feedback to estimation process. Measurement is modelled such that its output may take any value from the predicted state estimate, $\boldsymbol{X}_i(t|t)$, to the true value, $\boldsymbol{X}_{true}$. Measurement, $\boldsymbol{Z}_i(t)$, of each individual agent is simulated based on the following equation:

$$\boldsymbol{Z}_i(t) = \boldsymbol{X}_i(t|t) + \sin(rand \times 2\pi) \times |\boldsymbol{X}_i(t|t) - \boldsymbol{X}_{true}| \tag{5}$$

The $\sin(rand \times 2\pi)$ term provides the stochastic aspect of SKF algorithm and $rand$ is a uniformly distributed random number in the range of [0,1].

The final step is the estimation. During this step, Kalman gain, $K(t)$, is computed as follows:

$$K(t) = \frac{P(t|t)}{P(t|t)+R} \tag{6}$$

Then, the estimation of next state, $\boldsymbol{X}_i(t+1)$, is computed based on Equation 7 and the error covariant is updated based on Equation 8. Finally, the algorithm will continue the search process until the maximum number of iterations, t_{max}, is reached.

$$\boldsymbol{X}_i(t+1) = \boldsymbol{X}_i(t|t) + K(t) \times (\boldsymbol{Z}_i(t) - \boldsymbol{X}_i(t|t)) \tag{7}$$

$$P(t+1) = (1 - K(t)) \times P(t|t) \tag{8}$$

Since the introduction of the SKF algorithm, fundamental studies [12-15] have been reported to understand the potentials of the SKF algorithm. Furthermore, fundamental modifications also been done to enhance the performance of the SKF [16-22], to enable the SKF to operate in discrete domains [23-28], and to solve multi-objective optimization problems [29]. The SKF has also been applied to solve engineering problems. For example, the SKF have employed as feature selector [30-33], algorithms in adaptive beamforming [34-37], routing algorithm in manufacturing process [38-40] and airport gate allocation [41], tuning algorithm in control engineering [42-45], and matching algorithm in image processing [46-48].

Opposition-based learning

The concept of Opposition-based learning (OBL) is to concurrently assess the current solutions and its opposite solutions in order to obtain a better approximation of the current candidate solutions. Figure 2 illustrate the opposite point which is determined in domain [a,b]. Let $x \in [a, b]$ be a minimum and maximum values of variable in current population. The opposite number ox is determined as:

$$ox = a + b - x \tag{9}$$

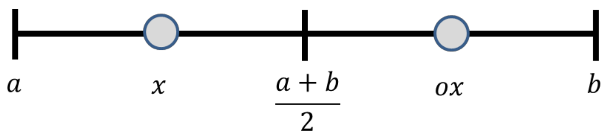


Figure 2. Opposite point defined in domain [a,b].

Current optimum opposition-based learning

In the original OBL concept, the agents and their opposite agents are asymmetric on the midpoint within the range of variables' current interval. This opposite agents might possibly flee from the global optimum, which leads to decrease the contribution of opposite points. Therefore, opposition-based learning using the current optimum (COOBL) was proposed in [7] to address this drawback. So this approach is used to enhance the effectiveness of the SKF. The proposed algorithm is known as current optimum opposition-based simulated Kalman filter (COOBSKF).

The significant difference is the formation of opposite population in COOBSKF is depends on the

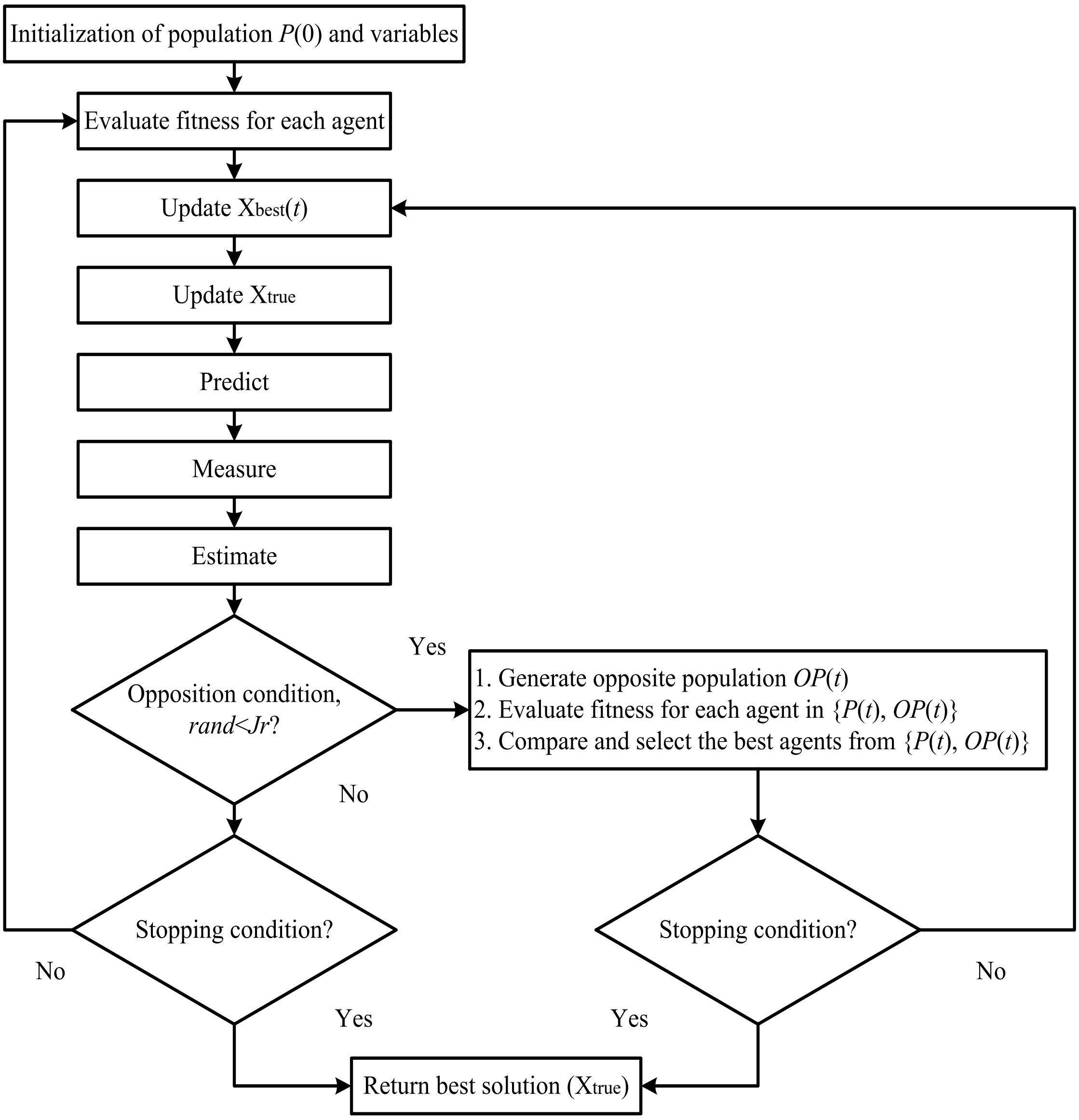


Figure 3. Flowchart of COOBSKF algorithm.

best agent so far which is identified by fitness calculation on particular objective function. The opposite population is generated using Equation 10.

$$ox_i = 2x_{co} - x_i \tag{10}$$

where x_{co} is the best agent so far or current optimum agent.

Enhancing SKF using current optimum opposition-based learning

The original SKF is selected as a parent algorithm and the COOBL strategies are embedded in SKF to boost its performance. COOBL is employed at one stage of SKF which is after estimation process of SKF. This implementation generated opposite population which is potentially fitter compared to the current ones. Figure 3 shows the flowchart of the proposed algorithm.

Initially, COOBSKF generates randomly initial population or candidate solutions. The initial value of error covariance estimate, $P(0)$, the process noise value, Q, the measurement noise value, R, and jumping rate value, *Jr*, are also determined during initialization stage. Then, the fitness of agents in the population is calculated based on the objective function. Next, $\boldsymbol{X}_{\mathbf{best}}(t)$ and $\boldsymbol{X}_{\mathbf{true}}$ are updated based on SKF algorithm steps. The algorithm continues with prediction, measurement and estimation similar to SKF algorithm using Equation 3 to Equation 8.

After that, COOBL is applied to the current solution in order to check a potential solution on opposite side. This action is performed probabilistically influenced by a parameter known as the jumping rate, $Jr \in [0,1]$. *Jr* is a control parameter to form or ignore the formation of opposite population at specific iteration. The following jumping condition is considered:

Table 1. CEC 2014 benchmark functions.

Types	No	Functions	Ideal Fitness
Unimodal Functions	1	Rotated High Conditioned Elliptic Function	100
	2	Rotated Bent Cigar Function	200
	3	Rotated Discus Function	300
Simple Multimodal Functions	4	Shifted and Rotated Rosenbrocks Function	400
	5	Shifted and Rotated Ackleys Function	500
	6	Shifted and Rotated Weierstrass Function	600
	7	Shifted and Rotated Griewanks Function	700
	8	Shifted Rastrigins Function	800
	9	Shifted and Rotated Rastrigins Function	900
	10	Shifted Schwefels Function	1000
	11	Shifted and Rotated Schwefels Function	1100
	12	Shifted and Rotated Katsura Function	1200
	13	Shifted and Rotated HappyCat Function	1300
	14	Shifted and Rotated HGBat Function	1400
	15	Shifted and Rotated Expanded Griewanks plus Rosenbrocks Function	1500
	16	Shifted and Rotated Expanded Scaffers F6 Function	1600
Hybrid Functions	17	Hybrid Function 1 (N=3)	1700
	18	Hybrid Function 2 (N=3)	1800
	19	Hybrid Function 3 (N=4)	1900
	20	Hybrid Function 4 (N=4)	2000
	21	Hybrid Function 5 (N=5)	2100
	22	Hybrid Function 6 (N=5)	2200
Composition Functions	23	Composition Function 1 (N=5)	2300
	24	Composition Function 2 (N=3)	2400
	25	Composition Function 3 (N=3)	2500
	26	Composition Function 4 (N=5)	2600
	27	Composition Function 5 (N=5)	2700
	28	Composition Function 6 (N=5)	2800
	29	Composition Function 7 (N=3)	2900
	30	Composition Function 8 (N=3)	3000

if *rand* < *Jr*
then
apply COOBL
else
check stopping condition
else

where *rand* is a random number in the range of [0,1]. Within this stage, if opposition condition is met, the respective opposite population is formed according to Equation 10. Then, the best agents for next generation will be selected as follows:

$$x_i(t+1) = \begin{cases} ox_i(t), fit\left(ox_i(t)\right) < fit\left(x_i(t)\right) \\ x_i(t), fit\left(ox_i(t)\right) > fit\left(x_i(t)\right) \end{cases} \tag{11}$$

Finally, the process of searching for optimum solution continued until the maximum number of function evaluation is reached.

Experimental and results

In order to make a fair comparison of the SKF and the proposed COOBSKF, we used a test suite of 30 standard benchmark functions and the same settings.

Benchmark functions

A comprehensive list of 30 benchmark global optimization functions (CEC 2014) [49] has been employed for performance verification of the proposed algorithms. The description of the benchmark functions and their global optimum (ideal fitness) are listed in Table 1. All the functions used in this experiment are minimization problem. It comes with 3 unimodal functions, 13 simple multimodal functions, 6 hybrid functions and 8 composition functions. The search space for all the test functions is between -100 to 100 for all dimensions.

Table 2. SKF parameters.

SKF Parameters	Values
Initial error covariance estimate, $P(0)$	1000
Process noise, Q	0.5
Measurement noise, R	0.5

Table 3. Experimental parameters.

Experimental Parameters	Values
Number of agent	100
Number of dimension	50
Number of run	50
Number of function evaluations	10000

Table 4. Mean value comparison of COOBSKF with SKF.

Function	SKF	COOBSKF (Jr=0.1)	COOBSKF (Jr=0.3)	COOBSKF (Jr=0.5)	COOBSKF (Jr=0.7)	COOBSKF (Jr=0.9)
1	4702013.17	**1076066.9455**	1229553.6314	1295419.0349	1539599.2793	1709933.4488
2	24498691.66	**3734.4152**	4364.2133	5191.9996	5896.5624	5252.6651
3	18147.70	3576.0458	**2508.6555**	3106.0056	3433.6582	3283.6733
4	532.77	**490.7681**	509.5353	497.4695	521.1160	503.8873
5	520.01	**520.0000**	520.0000	520.0000	520.0000	520.0000
6	633.44	629.1087	627.1215	625.8455	**625.6539**	626.4995
7	700.25	700.0191	700.0172	700.0140	700.0141	**700.0130**
8	807.98	802.6068	801.9899	801.3332	801.6516	**801.2735**
9	1059.14	1054.8748	1053.5615	**1051.9098**	1054.7753	1058.2776
10	1335.18	1269.6576	1211.2699	1211.5394	1202.4715	**1174.6086**
11	6249.37	6058.5216	6089.7465	6154.4400	**5909.7128**	5964.2966
12	1200.24	1200.1665	1200.1602	1200.1510	1200.1523	**1200.1509**
13	1300.56	1300.5692	1300.5758	1300.5593	1300.5460	**1300.5406**
14	**1400.30**	1400.3079	1400.3219	1400.3350	1400.3479	1400.3294
15	1551.66	1523.9582	1520.5517	1520.1406	**1518.5534**	1519.0694
16	1619.13	1619.1377	1619.0205	1618.8370	**1618.6569**	1618.9107
17	908272.09	**142202.2586**	162138.3897	193156.7732	184132.1261	223642.6456
18	6941389.77	2889.1810	2900.4436	2943.6296	**2805.0259**	2928.3890
19	1950.22	**1917.8190**	1918.1752	1920.2604	1923.5501	1924.6712
20	34799.06	3952.7988	2760.8150	2919.4987	2723.6052	**2574.2365**
21	1186640.91	151359.6270	**120710.0306**	138331.9163	160142.6513	163087.1809
22	3429.11	3346.0273	3291.2296	**3268.7018**	3298.4063	3329.7652
23	2645.69	**2644.0045**	2644.0045	2644.0045	2644.0045	2644.0045
24	2667.25	**2663.7930**	2664.1926	2664.5113	2664.2735	2665.1893
25	2730.40	2725.1857	2719.1827	**2716.7524**	2718.2685	2717.4820
26	2766.39	2772.2931	2760.3367	2760.3725	**2749.5381**	2750.4254
27	3883.34	3764.6612	3763.2763	3760.4055	3742.2586	**3703.4934**
28	7223.37	6131.1664	5720.3915	5386.0278	5293.3785	**5244.8513**
29	5997.83	104245.4826	882355.6217	824794.4430	826387.6088	**5448.3110**
30	19753.29	**16472.8260**	17281.6365	16704.7432	17416.0645	17128.4940

Settings for the experiments

In order to compare the performance of COOBSKF with the original SKF, all the experiments were executed in the same platform and subjected to the similar parameter settings in order to get a fair competition. Table 2 and Table 3 show the SKF parameters and experimental parameters respectively.

The stopping condition is defined to be the maximum number of function evaluations for all algorithms. Besides that, these experiments also have been conducted to explore the effect of jumping rate (*Jr*) upon the overall performance of COOBSKF algorithm. The performance can vary for different *Jr* values. To identify appropriate *Jr* value, the different numbers from 0 to 1 (0.1, 0.3 0.5, 0.7 and 0.9) was applied. Zero *Jr* means that opposition-based technique is totally removed from the algorithm. The *Jr* value is an important control parameter in which, if optimally set, will attain better results.

The performance of COOBSKF over SKF

This experiment investigates the performance of COOBSKF over the SKF. Based on Table 4, the results obtained show that the proposed COOBSKF has a significant improvement.

According to Friedman Test, the average rankings of these algorithms are shown in Table 5. These algorithms can be sorted by average ranking into the following order: COOBSKF (*Jr*=0.9), COOBSKF (*Jr*=0.5), COOBSKF (*Jr*=0.7), COOBSKF (*Jr*=0.3), COOBSKF (*Jr*=0.1) and SKF. The best average ranking is obtained by the COOBSKF (*Jr*=0.9). The Friedman statistic for this experiment is 47.295. Since this value is greater than 11.070 (based on 5 degree of freedom at a 0.05 level of significance according to Chi-square table), hence, significant difference exists in term of performance among these algorithms.

Therefore, to compare the performance differences between these algorithms, the Friedman Post Hoc Test was performed. Post Hoc Test using Holm's procedure is chosen to evaluate the significant difference between the algorithms' performance [50]. Table 6 shows the resultant p-values when comparing between SKF and COOBSKF. Holm's procedure rejects those hypotheses that have p-value lower than 0.005. The rejection of these hypotheses indicates a significant difference exists between the performances of the compared algorithms. The p-values below 0.005 are shown in bold. According to the results, COOBSKF is significantly better than SKF.

Table 5. Average rankings of COOBSKF and SKF.

Algorithms	Ranking
COOBSKF (Jr=0.9)	1
COOBSKF (Jr=0.5)	2
COOBSKF (Jr=0.7)	3
COOBSKF (Jr=0.3)	4
COOBSKF (Jr=0.1)	5
SKF	6

Figure 4 to Figure 7 show the diversity analysis of COOBSKF and its impact on algorithm's performance for some CEC 2014 benchmark functions. Based on these figures, it shows that the higher *Jr* value is set, the higher diversity of population generated by COOBSKF. The high value of population diversity corresponds to good exploration and vice versa.

Table 6. Algorithms p-values table for $\alpha = 0.05$.

Algorithms	z	p	Holm
SKF vs. COOBSKF (Jr=0.9)	5.7275	**0.0000**	0.0033
SKF vs. COOBSKF (Jr=0.5)	5.5205	**0.0000**	0.0036
SKF vs. COOBSKF (Jr=0.7)	5.4515	**0.0000**	0.0038
SKF vs. COOBSKF (Jr=0.3)	4.8305	**0.0000**	0.0042
SKF vs. COOBSKF (Jr=0.1)	4.1404	**0.0000**	0.0045

Based on those figures and statistical analysis performed previously, the higher diversity appears to be helpful to improve the search efficiency. This is due to the strength of current optimum opposition-based learning (COOBL) technique when generating the opposite agents. The formation of opposite agents is depended on the best-agent-so-far. The best-agent-so-far is used as symmetry point between agents and their opposite agents, which increase the chance to locate the global optimum solution.

To conclude, the jumping rate is an important control parameter in which, if optimally set, can achieve even better results. Based on statistical analysis (considering all CEC2014 benchmark functions), the optimal *Jr* value for COOBSKF is 0.9.

The performance of COOBSKF over other optimization algorithms

This experiment investigates the performance of COOBSKF in comparison with other optimization algorithms such as particle swarm optimization (PSO), grey wolf optimizer (GWO), genetic algorithm (GA), gravitational search algorithm (GSA) and black hole (BH). The experimental parameters used in this experiment are shown in Table 7. For COOBSKF, the *Jr* value used is 0.9. For GSA, α is set to 20 and initial gravitational constant, G_0 is set to 100. For PSO, cognitive coefficient, c_1, and social coefficient, c_2, are set to 2. The inertia factor is linearly decreased from 0.9 to 0.4. For GWO, components of a are linearly decreased from 2 to 0. Lastly, for GA, the probabilities of selection and mutation are set to 0.5 and 0.2, respectively.

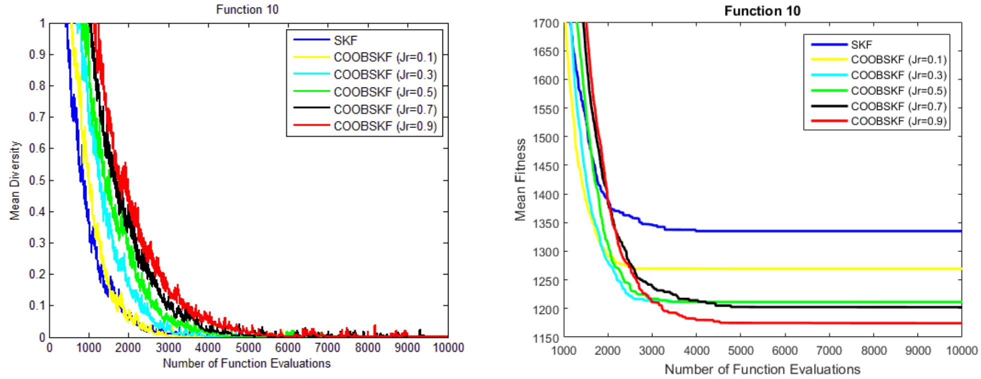
Table 7. Experimental parameters.

Experimental Parameters	Values
Population size	100
Number of dimensions	50
Number of runs	50
Number of function evaluations	10000
Jumping Rate, Jr	0.9

The results of each algorithm are presented in Table 8. In general, the COOBSKF and GSA show excellent performance on many test functions. According to the Friedman Test, the average rankings of these algorithms are shown in Table 9. These algorithms can be sorted by average ranking into the following order: COOBSKF, GSA, SKF, BH, PSO, GWO and GA. The best average ranking is obtained by the COOBSKF. The Friedman statistic for this experiment is 66.482. Since this value is greater than 12.592 (based on 6 degree of freedom at a 0.05 level of significance according to Chi-square table), significant difference exists in term of performance among these algorithms. Therefore, to compare the performance differences significantly between these algorithms, the Friedman Post Hoc Test was performed. Table 10 shows the resultant p-values when comparing between COOBSKF and the other optimization algorithms. Holm's procedure rejects those hypotheses that have p-value lower than 0.0045. The rejection of these hypotheses indicates a significant difference exists between the performances of the compared algorithms. The p-values below 0.0045 are shown in bold. From the results, it can be seen that COOBSKF is significantly better than GA, GWO, PSO, BH, and SKF. The COOBSKF obtains the best result and the GSA has the second-best performance. Figure 8 to Figure 11 show the convergence curve for Function 3, Function 6, Function 20 and Function 27, respectively. Based on these figures, the COOBSKF has good convergence performance than the other compared algorithms.

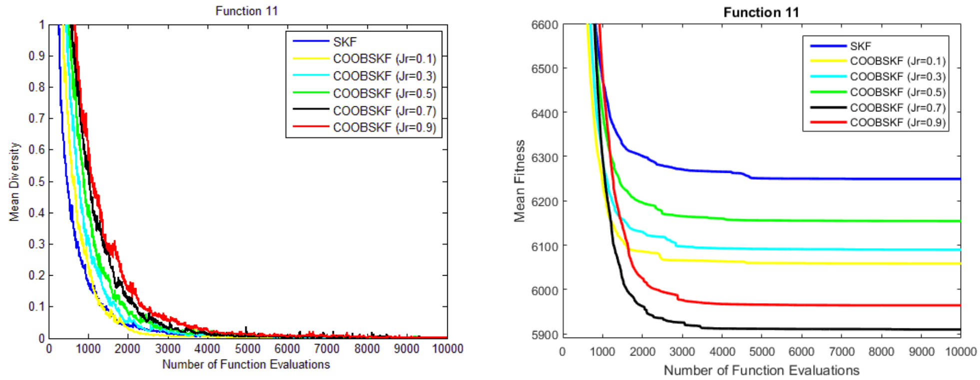
Conclusion

This paper reports the first attempt to enhance the exploration capability of SKF by applying COOBL technique. In addition, jumping rate is also integrated in the proposed method. Once the jumping rate condition is met, the opposite solution is selected if the solution is better than the current one. The analysis confirmed that the proposed COOBSKF is superior to SKF and better than GA, GWO, PSO and BH. For future research, different OBL techniques shall be considered to enhance further the SKF.



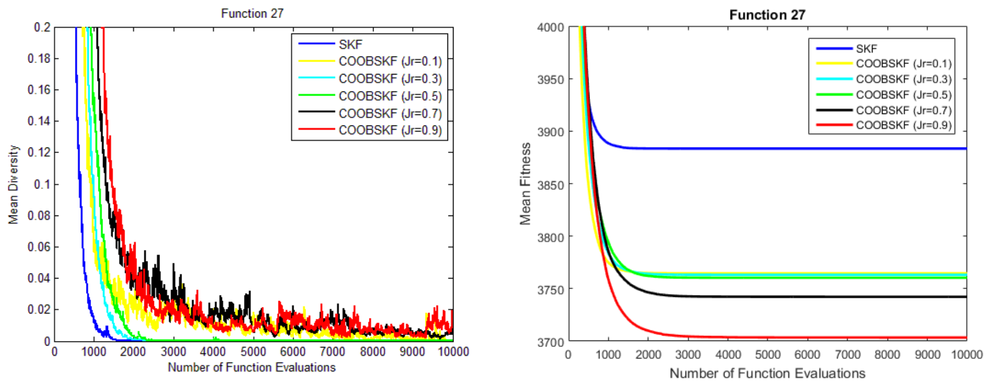
(a) Diversity (b) Fitness

Figure 4. Analysis on Function 10.



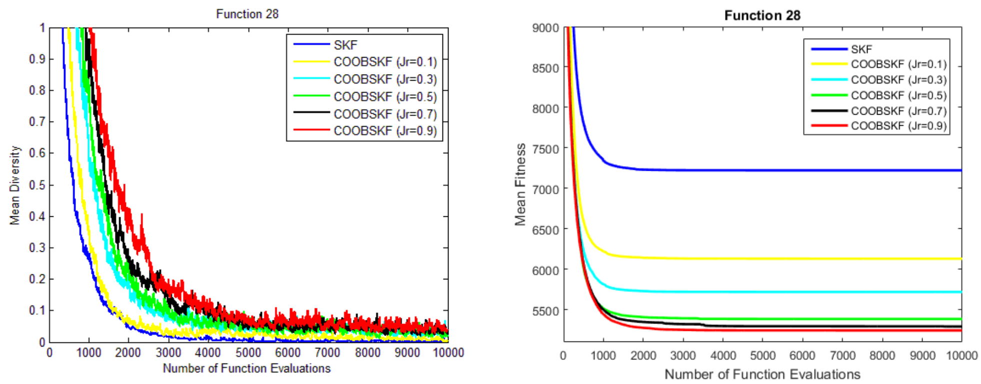
(a) Diversity (b) Fitness

Figure 5. Analysis on Function 11.



(a) Diversity (b) Fitness

Figure 6. Analysis on Function 27.



(a) Diversity (b) Fitness

Figure 7. Analysis on Function 28.

Table 8. Mean value comparison of COOBSKF with other optimization algorithms.

Function	COOBSKF	SKF	PSO	GSA	BH	GA	GWO
1	1709933.4488	4702013.17	43464224.92	**1400195.11**	4111807.31	339979486.97	56283362.12
2	**5252.6651**	24498691.66	11404043.34	7166.79	193491.14	23639977763.22	5273423032.81
3	**3283.6733**	18147.70	9934.12	64249.05	11557.54	62699.82	49773.52
4	**503.8873**	532.77	1062.06	653.46	564.79	3008.49	958.42
5	520.0000	520.01	521.06	**520.00**	520.01	521.01	521.11
6	626.4995	633.44	631.49	636.34	658.13	655.83	**625.95**
7	700.0130	700.25	700.02	**700.00**	700.13	924.79	745.23
8	**801.2735**	807.98	858.72	1074.39	922.25	1067.94	975.30
9	1058.2776	1059.14	**1051.65**	1222.23	1212.14	1400.33	1078.85
10	**1174.6086**	1335.18	1644.00	7456.32	3121.04	6254.17	6381.00
11	**5964.2966**	6249.37	11965.59	8637.64	8051.04	12793.02	6582.83
12	1200.1509	1200.24	1202.61	**1200.00**	1200.73	1202.23	1201.98
13	1300.5406	1300.56	1300.60	**1300.37**	1300.55	1302.82	1300.58
14	1400.3294	1400.30	1400.33	1400.30	**1400.26**	1461.55	1407.27
15	1519.0694	1551.66	1528.76	**1504.41**	1787.84	35513.58	1965.26
16	**1618.9107**	1619.13	1621.68	1622.57	1621.54	1621.85	1619.53
17	223642.6456	908272.09	3591382.02	**161088.84**	552079.20	16798809.69	3226248.21
18	2928.3890	6941389.77	30740.67	3731.21	**2433.40**	5476926.38	48192233.30
19	1924.6712	1950.22	1962.25	**1923.75**	1952.80	2004.46	1979.52
20	**2574.2365**	34799.06	6513.63	26574.24	8499.56	35020.04	14603.11
21	**163087.1809**	1186640.91	710017.59	187636.63	395411.76	5296958.82	1923398.21
22	3329.7652	3429.11	3421.42	3857.96	3708.16	3429.02	**2873.34**
23	2644.0045	2645.69	2661.52	**2500.00**	2649.46	2714.77	2708.26
24	2665.1893	2667.25	2672.80	2600.03	2666.42	2777.24	**2600.00**
25	2717.4820	2730.40	2729.83	**2700.00**	2750.48	2761.21	2725.34
26	2750.4254	2766.39	**2700.50**	2800.03	2792.16	2702.69	2769.23
27	3703.4934	3883.34	3843.02	4577.53	4654.81	4473.23	**3672.93**
28	5244.8513	7223.37	9891.73	6261.32	11047.63	6288.68	**4647.03**
29	5448.3110	5997.83	23201.80	**3100.15**	10361.49	6716062.26	3276290.32
30	17128.4940	19753.29	194616.38	**8695.44**	58613.31	161845.14	112029.89

Table 9. Average rankings of COOBSKF and others.

Algorithms	Ranking
COOBSKF	1
GSA	2
SKF	3
BH	4
PSO	5
GWO	6
GA	7

Table 10. Algorithms p-values table for $\alpha = 0.05$.

Algorithms	z	p	Holm
COOBSKF vs. GA	7.7989	**0.0000**	0.0024
COOBSKF vs. GWO	4.8706	**0.0000**	0.0026
COOBSKF vs. PSO	4.4522	**0.0000**	0.0028
COOBSKF vs. BH	4.0638	**0.0000**	0.0031
COOBSKF vs. SKF	3.4960	**0.0005**	0.0036
COOBSKF vs. GSA	2.7191	0.0065	0.0045

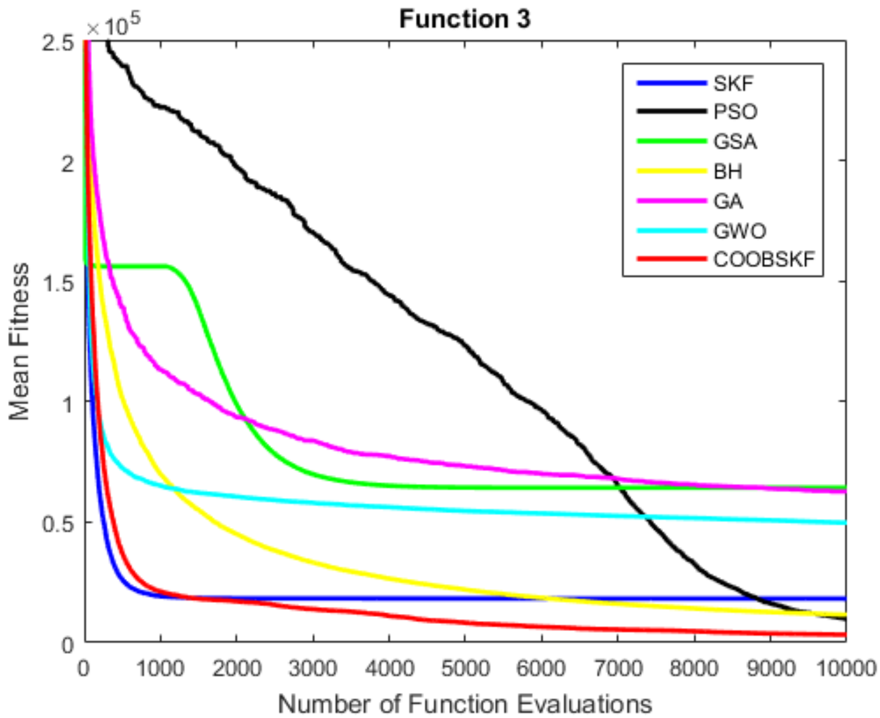


Figure 8. Analysis on Function 3.

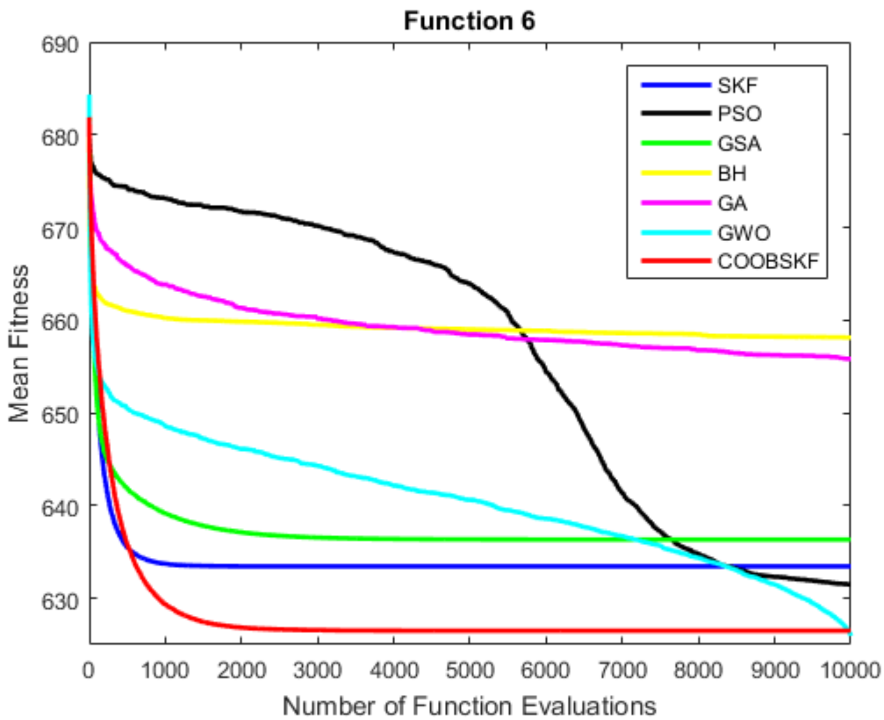


Figure 9. Analysis on Function 6.

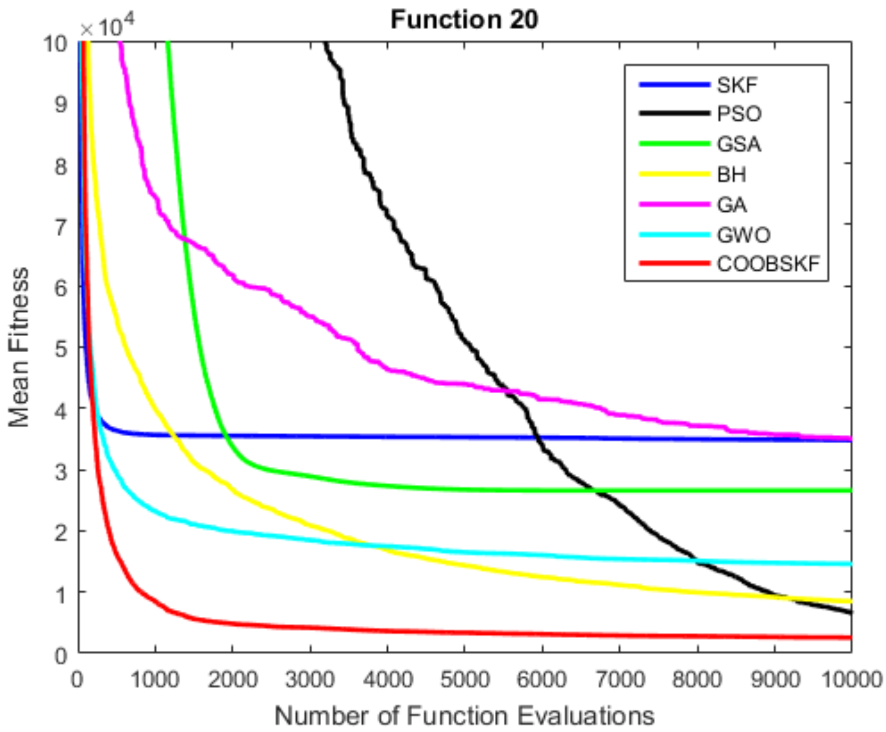


Figure 10. Analysis on Function 20.

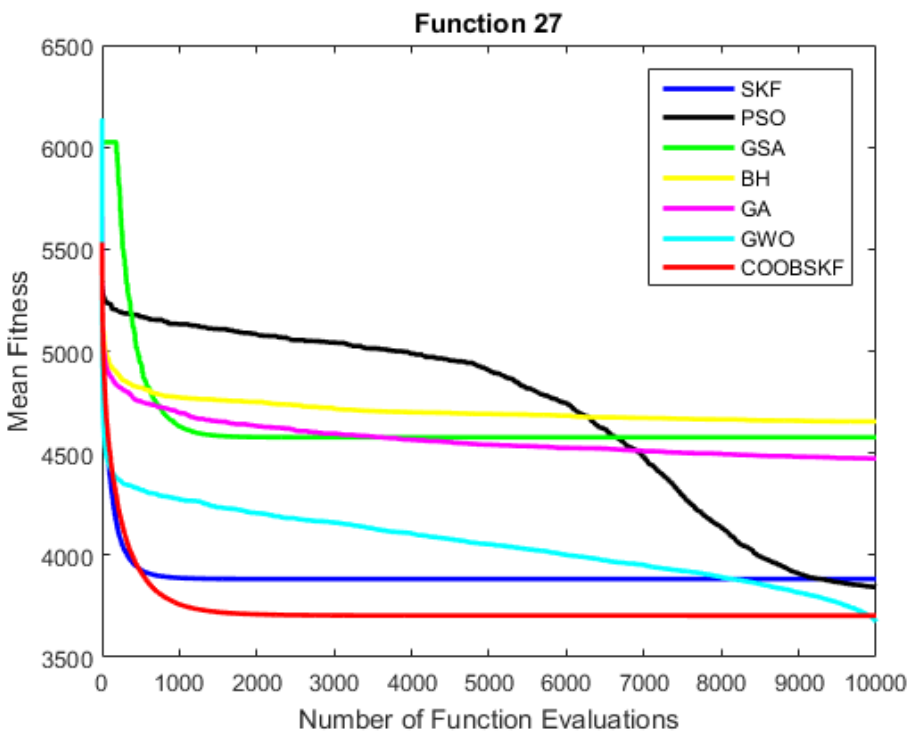


Figure 11. Analysis on Function 27.

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