

RESEARCH ARTICLE

Reduced Sliding Mode Control Algorithm for Two-Link Planar Robot System

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ABSTRACT - Robots have become popular these days due to their ability accuracy and precision. Areas of applications include welding, machining, moving items, storage, retrieval and other precise surgical operations. Controllers are continuously being improved in design simplicity, robustness, and performance accuracy. High-accuracy trajectory tracking is a challenging area in robot control due to nonlinearities and input couplings. To achieve high accuracy and precision, robots need accurate, fast-processing controllers. This study there focused on application of the sliding mode controller (SMC) but in the reduced or simplified algorithm form (SSMC) for the control of a two-link planar robotic system. The MATLAB/Simulink software was employed. Result was evaluated using the root mean square error (RMSE) for both links. The proposed SSMC was compared with the PID and SMC schemes. The RMSE values of the tracking errors with disturbance for the SSMC were 0.0751 and 0.0814 rad, for the SMC were 0.0755 and 0.0817 rad and for the PID were 0.2784 and 0.1062 rad. The RMSE values with no disturbance for SSMC they were 0.07435 and 0.0811 rad, for the SMC were 0.0752 and 0.08153 rad and finally with the PID were 0.2579 and 0.1021 rad. The length of algorithm as shown in the text was shorter for the SSMC compared to the SMC and close to that of the PID. It is clear that the proposed control scheme SSMC is simpler than the SMC. It also had the property of improved performance with robustness.

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1. INTRODUCTION

Robotics is an area which is rapidly developing and form the backbone in automation of industries across the world. In order to achieve enhanced efficiency and uniform quality. Many industrial processes are being automated some of which are done using robots. These devices can be utilized in handling hazardous materials in dangerous environments like an atomic plant. Robots are applied in building and maintaining satellites as well as space stations, moving cameras for recording of videos from elevations, crowds, and other dangerous environments, installation of power cables, building of roads and buildings, performance of repetitive tasks in industries, loading and unloading in transits, in drilling processes, picking of tools, in mining operations and for picking and placing of items and tools, handling of special medical equipment and for critical surgery operations in hospitals. Presentation of the entire list of applications robotics cannot be exhausted. The robotic manipulator or robotic arm is a kind of robot which is mostly utilized in industries. It comprises of a chain of rigid links interconnected by moveable joints which can either be opened or closed while moving [1-5].

Robots are specifically designed for accomplishing specific tasks to make the work of humans easier. Hence, saving time as well as improved productivity. Mostly of these activities have high accuracy and precision requirement [6-7]. The nature of the two-link robotic manipulator being highly nonlinear such that special control systems are mostly requirement in maintaining position during carrying-out tasks [8]. Many studies have proven the effectiveness of the SMC over the PID [9-10]. Other researchers have applied the PID feedback loops together with holographic neural networks as well as artificial neural networks for performance of the tasks of paper folding [11]. Improved the sliding mode controller (SMC) performance in tracking the target joints angular locations in the presence of uncertainty parameters. Lagrange technique was utilised for the dynamic model coupled with the least square based Self tuning mechanism. The Self tuned SMC was compared with conventional SMC and outperformed it for both Signum function as well as Saturation function [12]. The double pendulum system using the Lagrange approach for dynamic modelling and was controlled by PID controllers. The proposed control methods performance was better than expectation [13]. Therefore, it is clear that the sliding mode control has good performance and suitable for complex systems, but it associated with the issue of requiring higher computational time. Hence, the need to relax some of its components if possible for very complex systems [14-17].

2. METHODS AND MATERIAL

2.1 The Two-Link Robot

The generalized robotic model system was a second order system as given by equation (1). Equation (2) is the inverse model of the system. The gravity matrix is represented by g , C is the Coriolis matrix and H represent the inertia matrix. The torques for the system is given by (3), (4) and (5) were the definition of the Coriolis matrix. Equation (6) defines the gravity matrix, (7)-(11) give the definition of the inertia matrix [18-21]. The nominal values used for the dynamic model are $q_1 = q_2 = \sin(2\pi t)$, $t = 10\text{seconds}$, $m_1 = m_2 = 1\text{kg}$, $I_i = \frac{1}{12}\text{kgm}^2$, $l_1 = l_2 = 1\text{m}$ and $l_{c1} = 0.5\text{ (kg)}$ [19,21].

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \quad (1)$$

$$\ddot{q} = H^{-1}\tau - H^{-1}C\dot{q} - H^{-1}g \quad (2)$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (3)$$

$$C = \begin{bmatrix} h\dot{q}_2 & h\dot{q}_2 + h\dot{q}_1 \\ -h\dot{q}_1 & 0 \end{bmatrix} \quad (4)$$

$$C(\dot{q}, q) = \begin{bmatrix} -m_2 l_1 l_2 \sin\theta_2 (2\dot{\theta}_1 \dot{\theta}_2 + \dot{\theta}_2^2) \\ m_2 l_1 l_2 \cos\theta_2 \end{bmatrix} \quad (5)$$

$$g(q) = \begin{bmatrix} -(m_1 + m_2) l_1 g \sin\theta_1 - m_2 l_2 g \sin(\theta_1 + \theta_2) \\ -m_2 l_2 g \sin(\theta_1 + \theta_2) \end{bmatrix} \quad (6)$$

$$H(q)\ddot{q} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \quad (7)$$

$$d_{11} = (m_1 + m_2) l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos\theta_2 \quad (8)$$

$$d_{12} = m_2 l_2^2 - m_2 l_1 l_2 \cos\theta_2 \quad (9)$$

$$d_{21} = m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_2 \quad (10)$$

$$d_{22} = m_2 l_2^2 \quad (11)$$

2.2 Sliding Mode Controller Application for the Two-Link Robot

The plant is given by (12) which can be used to determine the control law and (13) was the inverse. The equation (14) is the sliding surface for the SMC. The constant c has to be chosen so that the Hurwitz criterion is obeyed; c has to be greater than zero ($c > 0$). Equation (15) gives the tracking error, (16) it derivative and (17) its second derivative. Differentiating (14) and putting in (17) gives (18). Equation (19) gives the reaching law η and k are chosen so that they are all greater than zero. Substituting (19) in (18) yields (20). Putting (13) into (20) and making u the subject makes the control law as given by (21). [10-16].

$$H(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) = u(t) \quad (12)$$

$$\ddot{\theta}(t) = H^{-1}u(t) - H^{-1}C(\theta, \dot{\theta})\dot{\theta} - H^{-1}g(\theta) \quad (13)$$

$$s(t) = ce(t) + \dot{e}(t) \quad (14)$$

$$e(t) = \theta_d(t) - \theta(t) \quad (15)$$

$$\dot{e}(t) = \dot{\theta}_d(t) - \dot{\theta}(t) \quad (16)$$

$$\ddot{e}(t) = \ddot{\theta}_d(t) - \ddot{\theta}(t) \quad (17)$$

$$\dot{s}(t) = c\dot{e}(t) + \ddot{e}(t) = c\dot{e}(t) + \ddot{\theta}_d(t) - \ddot{\theta}(t) \quad (18)$$

$$\dot{s}(t) = -\eta \text{sgn}(s(t)) - ks(t) \quad (19)$$

$$-\eta \text{sgn}(s(t)) - ks(t) = c\dot{e}(t) + \ddot{\theta}_d(t) - \ddot{\theta}(t) \quad (20)$$

$$u(t) = Hc\dot{e}(t) + H\ddot{\theta}_d(t) + H^{-1}c(\theta, \dot{\theta})\dot{\theta} \quad (21)$$

$$+g(\theta) + H\{\eta \text{sgn}(s(t)) + ks(t)\} \quad (21)$$

2.3 SSMC Application for the Two-Link Robot

The plant was given by (22), (23) is the sliding surface, (24) the tracking error and (25) the derivative. The reduced or simplified control law is given by (26).

$$H(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + g(\theta) = \tau \quad (22)$$

$$s(t) = ce(t) + \dot{e}(t) \quad (23)$$

$$e(t) = \theta_d(t) - \theta(t) \quad (24)$$

$$\dot{e}(t) = \dot{\theta}_d(t) - \dot{\theta}(t) \quad (25)$$

$$u(t) = -H\{\eta \operatorname{sgn}(s(t)) + ks(t)\} \quad (26)$$

2.4 PID Application for the Two-Link Robot

The tracking error is given by (27) and (28) the derivative. The PID control law is given by (29) which is a PD version.

$$e(t) = \theta_d(t) - \theta(t) \quad (27)$$

$$\dot{e}(t) = \dot{\theta}_d(t) - \dot{\theta}(t) \quad (28)$$

$$u(t) = k_p e + k_d \dot{e} \quad (29)$$

3. RESULTS OBTAINED

The root mean square of error (RMSE) was used as the index for the quantitative analysis of the tracking performance of the robot. It was carried out by comparing the performances of all the three control schemes for the trajectory tracking.

3.1 Study Results of Link 1

Results obtained are tabulated in Table 1 which was presented further using bar charts in Figure 1, the RMS errors of 0.2579, 0.0752 and 0.0743 rads with the PID, SMC and the SSMC respectively in the absence of disturbance for link 1. The results were 0.2784, 0.0828 and 0.0753 rads with the PID, SMC and the SSMC accordingly. The PID, SMC and the SSMC give changes in errors of 0.0206, 0.0076 and 0.0009 respectively. The PID control scheme had most amount of error in both without disturbance and with disturbance, so also the levels of the difference in the errors without and with disturbance. Next was with the SMC and then the SSMC. It can therefore be seen that the PID gives more amount error in the tracking of trajectory errors than the SMC, while the SSMC gives the minimum error. This meant that the SSMC had best tracking performance with regards to the resulting error on link 1.

Table 1. Results of link 1

Condition	Control Scheme		
	PID	SMC	SSMC
No Disturbance Error (rad)	0.2579	0.0752	0.0743
Error with Disturbance (rad)	0.2784	0.0828	0.0753
Change in Error (rad)	0.0206	0.0076	0.0009

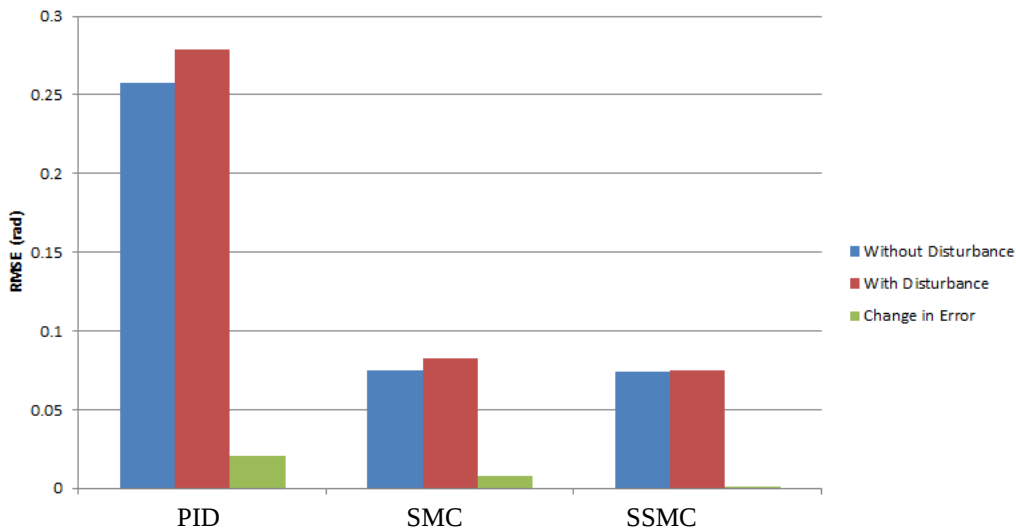


Figure 1. Barcharts of the response errors of link 1

Table 1 indicated that the error on link 1 showed that the SSMC had the minimum amount of error, then it is clear that the SMC and PID are not accurate as the SSMC. Although the difference was not very high between the SSMC and SMC, but it was very high compared to the PID. Since, the PID had highest magnitude of change in error it was the least robust, followed by the SMC and followed by the SSMC which can be inferred as the most robust among all the other controllers applied on link 1.

3.2 Study Results of Link 2

Presented below were the results of the link 2 response of the two-link planar robot Figure 2 and Table 2 presented the resulted RMSE on link 2. The tracking errors of 0.1021, 0.0815 and 0.0811 rads resulted with the PID, SMC and the SSMC respectively. The errors were 0.1062, 0.0852 and 0.0817 rads accordingly. Regarding the changes in the tracking errors without and with disturbances were 0.0041, 0.0036 and 0.0006 respectively. The system with the PID had the most amount of error, both with and without disturbance. The amount of error with the PID was the highest followed by the SMC and with the SSMC; which has the minimum error. It implied that the SSMC has best tracking performance with regards to the amount of the errors on link 2 tabulated in Table 2 and depicted by Figure 2 It meant that the SSMC had the minimum error as well on link 2. It meant as well that both RSMC and PD are less accurate compared to the SSMC due to their higher magnitudes of errors. The resulting least amount of the change in the tracking error without and with disturbance using the SSMC made it the most robust among them; maintaining the lead as in the case of link 1. The system with the PID was the worst. The results were as shown in Table below.

Table 2. Results of link 2

Condition	Control Scheme		
	PID	SMC	SSMC
No Disturbance Error (rad)	0.1021	0.0815	0.0811
Error with Disturbance (rad)	0.1062	0.0852	0.0817
Change in Error (rad)	0.0041	0.0036	0.0006

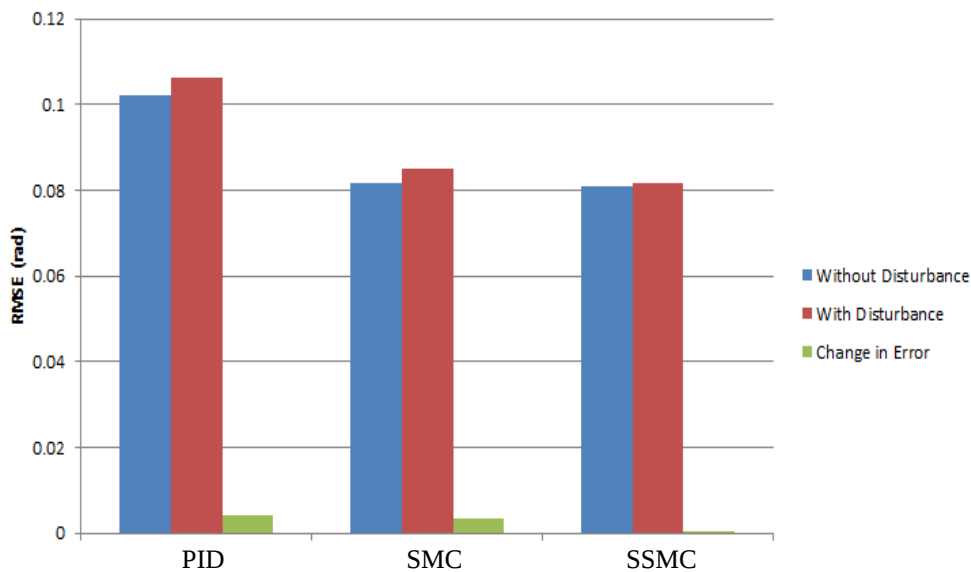


Figure 2. Bar charts of the response errors of link 2

4. RESULTS AND DISCUSSION

In the study effort was made to ease the complexity of the SMC so that it has reduced computational time. That was achieved by reducing the algorithm length resulting to the proposed SSMC controller. It was also found to have a slightly improved response in comparison to the SMC. The position tracking responses revealed that the tracking errors are least with the SSMC followed by the SMC and then the PID. The results are similar both without and with disturbance and also for link 2. Meaning that the system was best with the SSMC; the proposed scheme. The difference in the tracking errors between the different control schemes as well revealed that of the SSMC was the least, followed by the SMC and the PID. These results are similar for both links 1 and 2 as well. Hence, it indicated that the SSMC was the most robust followed by the SMC and the least was the PID. The error was highest with the PID making it the worst and justifying it least robustness.

5. CONCLUSION

The reduced sliding mode controller (SSMC) was successfully applied for the two-link planar robot. The capability of the conventional sliding mode controllers (SMC) on such systems was remarkable but a major problem hindering its practical implementation was the complexity of the algorithm. In the case of the PID is the need for better accuracy sensors. The synthesized SSMC controller had the simplicity of the PID and robustness of the SMC. These are illustrated by the length of the control algorithms realized as well as the results revealing accuracy of tracking and how well the presence of disturbance was reduced. Novelty of the study lies in the infancy of the proposed control scheme for the two-link robotic system application. The presentation indicated the likelihood of the SSMC that with further research could lead to fruitful results for these systems as well as similar systems.

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CONFLICT OF INTEREST

The authors to the best of their knowledge declare that there are no conflicts of interest.

AUTHORS CONTRIBUTION

The paper was synthesized based on adequate collaborative efforts of the authors. are encouraged to provide an author statement file describing their specific contributions to the article using the appropriate author contribution roles to increase transparency. The corresponding author is responsible for ensuring that all authors agree o

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