

# DYNAMIC STRUCTURAL ANALYSIS OF A PAIR OF MESHED SPUR GEARS

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## ABSTRACT

Design of gears must be précised for accurate power transmission. A pair of meshed spur gear of 2:1 ratio has been designed for the angular displacement to the driven gear in order to apply in a practical setup and finite element analysis was performed to determine various phenomena such as stress, strain, displacement, contact pressure, reaction force, factor of safety etc. in order to observe the durability and sustainability of the designed gears. The materials, geometrical constraints and load were applied considering the setup in real application. From the simulation result, it is evident that, the durability and sustainability of the designed gear pair is beyond the requirement.

## INTRODUCTION

Gears are the most important tools in industrial machinery, automotive applications, and to transmit power as well as change the direction of motion. Considering gears' health monitoring and durability analysis is very important. As is known, gear tooth failure can cause removal and/or plastic deformations on the contacting tooth surfaces or even presence of fatigue crack. The severity of tooth damage is usually assessed by the reduction of the stiffness. Some related research works are found closed to this topic. Gears are mainly used for torque conversion, changing rotational direction, transmitting power to machine elements etc. For any kind of mechanical system gears are very commonly used tools because almost every mechanical along with some electrical elements have at least one power transmitting section thus gears are must for the specific field of selection. Though the design criterion of gear manufacturing is very old, but some studies and experiments conducted last 20 years showed very flawless results on gear designing. Flodin, A., & Andersson, S. (1997) developed a numerical model for wear prediction of spur gears that may be caused by unfavorable changes of the surface topography, giving non-uniform gear rate, increasing dynamic effects and perhaps more severe forms of tooth failures [1]. Hassan, A. R. (2009) analyzed the contact stress between two spur gear teeth in different contact positions, representing a pair of mating gears during rotation. He developed to plot a pair of teeth in the contact [2].

G. Hyatt et al. 2014, made a compact review on new methods of gearing generation which involves a detailed mathematical and materialistic approach on InvoMill software which is widely used for gear production and simulation [3]. This research also analyzed the quality of some specific compact shaped gear designs generated from InvoMill software along with fabrication. H. L. Bartlett et al. 2018 designed, modelled and fabricated a spur gear train for power transmission where the basic validation parameters were selected on basis of stress, normal force acting on teeth meshing, lubrication along with thermal stress [4]. The researchers had mathematically modelled the validation parameters into some functions which can experimentally and numerically calculated and on further analysis the results showed very interesting points such as: stress-strain relationship faultiness on excessive heating produced by various lubrications, gear train total power reduction due to rough surface finish and etc. This numerical investigation is mainly targeted to design compact gear trains with lower complexity and high compatibility by using the AutoDesk 360 Inventor Pro environment.

**METHODOLOGY**

The design of the spur gear set was done using Autodesk Inventor Pro 2017. The dynamic structural simulation was undertaken and linear displacement, strain; contact pressure etc. parameters were investigated for the designed gear mechanism. The 3D model of the gear set is given in the following Fig. 1. The driver and driving gear have 18 and 36 teeth, thus the gear ratio is 1/2, pressure angle 20 degree and face width 1 inch. The simulations were accomplished using Finite Element Analysis (FEA) using Autodesk Fusion 360 simulation environment. The meshed view of the design is represented in the Fig. 3.

Table 1: Mesh properties

Solids	10
Scale Mesh Size Per Part	No
Element Order	Parabolic
Create Curved Mesh Elements	Yes
Max. Turn Angle on Curves (Deg.)	60
Max. Adjacent Mesh Size Ratio	1.5
Max. Aspect Ratio	10
Minimum Element Size (% of average size)	20

For linear dynamic structural analysis, displacements can be termed in a form of matrix given below:

$$[Constant]\{s\} = \{F\} \tag{1}$$

Where, s – Displacement, F – Dynamically applied force. No time varying forces along with zero inertial effect has been considered. Here, the system was considered as a 2-DoF along Z-axis, thus finite planar rotational transformation takes place. Fig. X denotes the angular position of the driver gear. Let, A is a point on any driver gear tooth going through a rotation of 15° deg/s circularly from A<sub>1</sub> to A<sub>2</sub> on 2-D coordinate. Thus, the rotational operator can be mathematically given by;

$$R_{1,2} = \begin{bmatrix} \cos \theta_{1,2} & -\sin \theta_{1,2} & 0 \\ \sin \theta_{1,2} & \cos \theta_{1,2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

So, the positional matrix is;

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = R_{1,2} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \tag{3}$$

The contact between two gear teeth is approximated by two cylinders of different radii. These radii vary along the line of action as [Larsson, R. (1997). Transient non-Newtonian elasto-hydrodynamic lubrication analysis of an involute spur gear. Wear, 207(1-2), 67-73.]:

$$R_1(\varepsilon) = r_{w1} \sin \theta_1 + \varepsilon \dots \dots \dots (4)$$

$$R_2(\varepsilon) = r_{w2} \sin \theta_2 - \varepsilon \dots \dots \dots (5)$$

Where,  $R_1, R_2$ = radii of curvature of gear teeth,  $\varepsilon$  = Action line coordinate,  $\theta_1, \theta_2$  = Angular speeds of gear (rad/s),  $r_{w1}, r_{w2}$  = Pitch circle radii of gear. The mathematical formula of maximum shear stress is [2]:

$$\text{Contact Stress, } \sigma_c = \sqrt{\frac{T(1+r_{w1}/r_{w2})}{r_{w1} \cdot \pi \delta \left[ \frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2} \right] \sin \theta}} \dots \dots \dots (6)$$

$$\text{Contact Ratio, } C_r = \frac{q_t}{P_c} \dots \dots \dots (7)$$

$T$  = Load,  $E_1$  and  $E_2$  = Modulus of Elasticity,  $\nu_1$  and  $\nu_2$ = Poisson's Ratio,  $\delta$  = Face width,  $P_c$ = Circular pitch and  $q_t$ = Arc produced from action of load. The shafts of the gears were dynamic, and the gears can rotate along the Z-axis to both positive and negative direction. The other axes were constrained as fixed. Angular velocity of magnitude  $15^\circ/sec$  was applied to the driving gear.

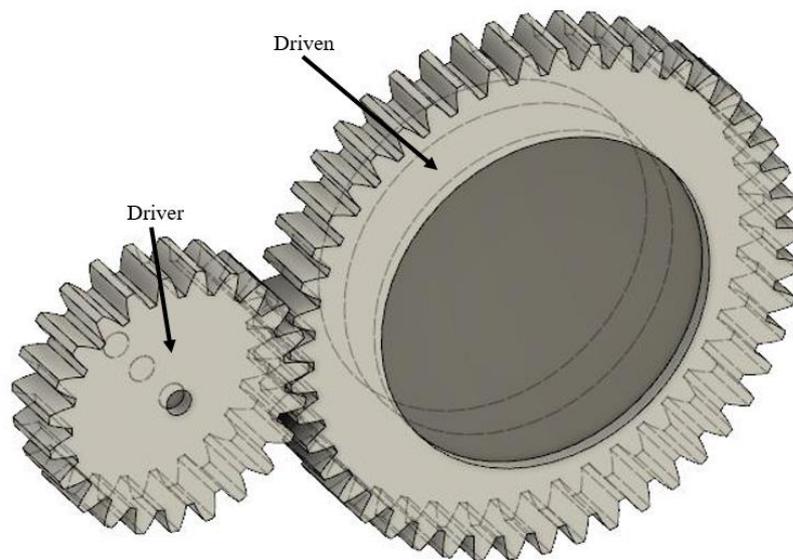


Fig. 1: 3D model of the gear set.

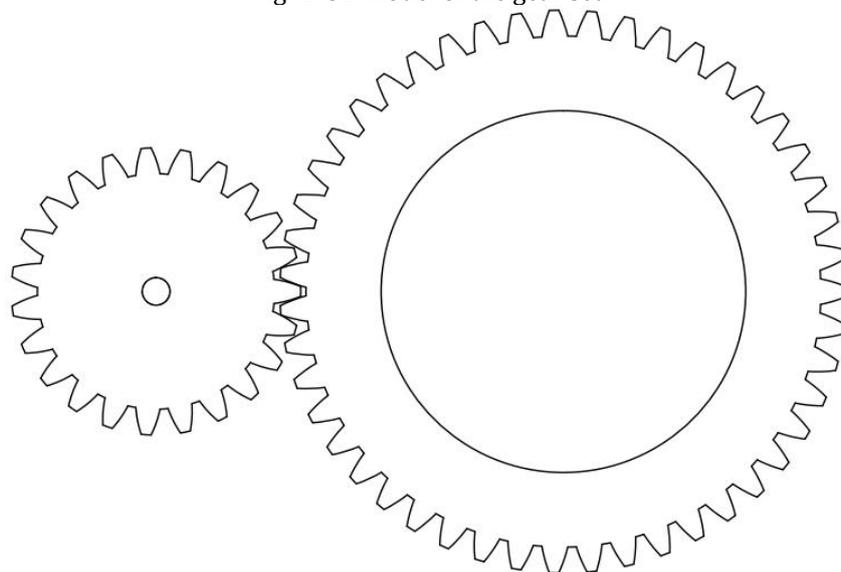


Fig. 2: Contact Region of the Spur gears.

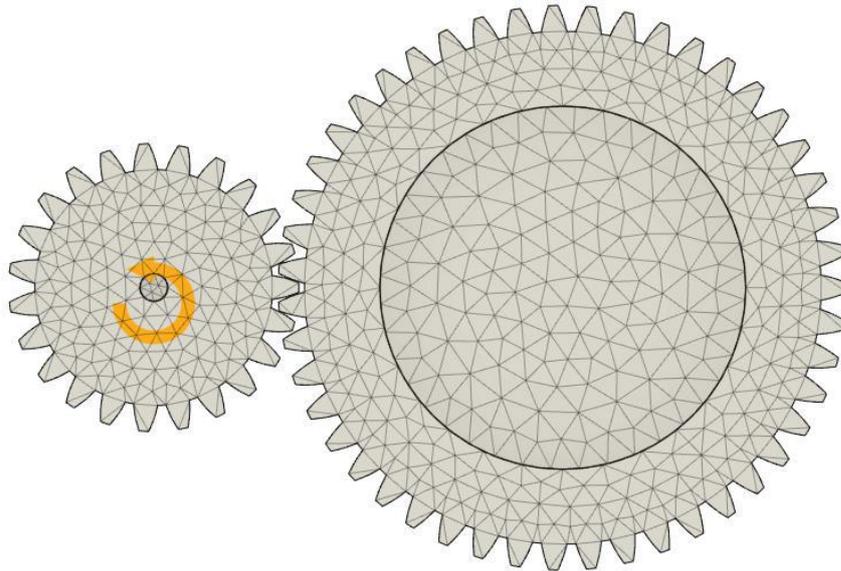


Fig. 3: Meshed view of the Gear set for simulation.

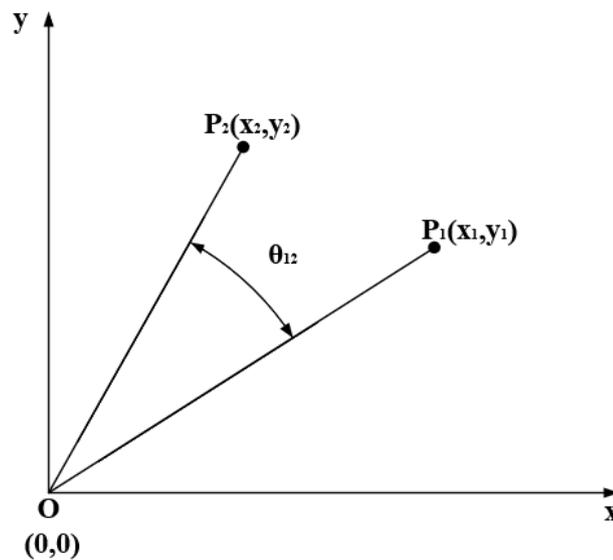


Fig. 4: Angular position of designed driver gear with respect to time (per sec).

## RESULT AND DISCUSSION

After setting the material as ABS plastic (Gears can be 3D printed for real application) of both gears and shafts the simulation was ready to start solving phase. The following results were obtained from the simulation. For better understanding, maximum and minimum values including the unit are represented in the Fig. 5-13.

The factor of safety is very high here because the load is very low for this model and material. The contact region of the shaft and gear affects much. The displacement, stress, strain, contact pressure and reaction force were very low which will not affect the gears for a long time. The load on the gears will be constant and may vary into the  $\pm 2\%$  tolerance limit. The load is the output of a stepper motor that will be connected to the driver gear shaft.



Fig. 5: Maximum and Minimum Factor of safety

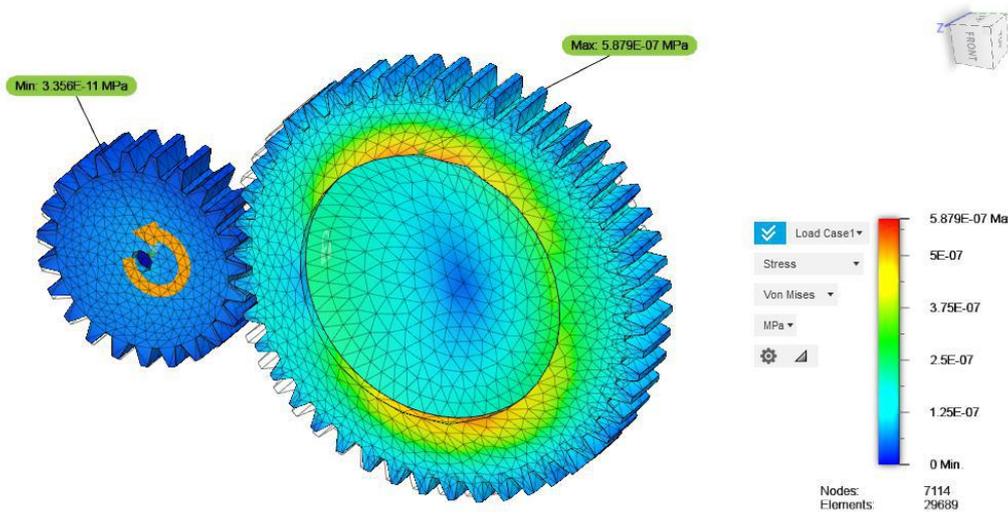


Fig. 6: Stress (Von Mises) including Maximum and Minimum.

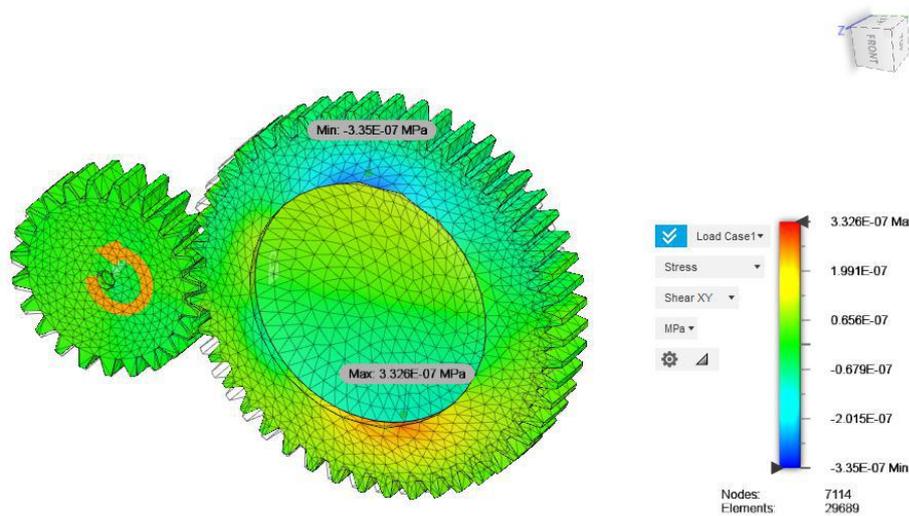


Fig. 7: Shear Stress along XY plane including Maximum and Minimum.

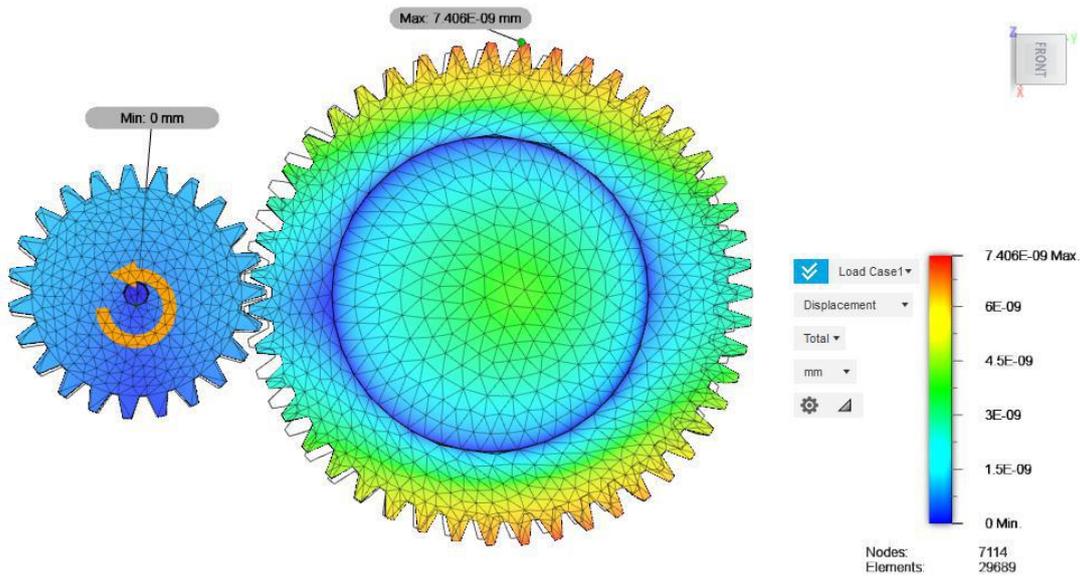


Fig. 8: Total Displacement including Maximum and Minimum.

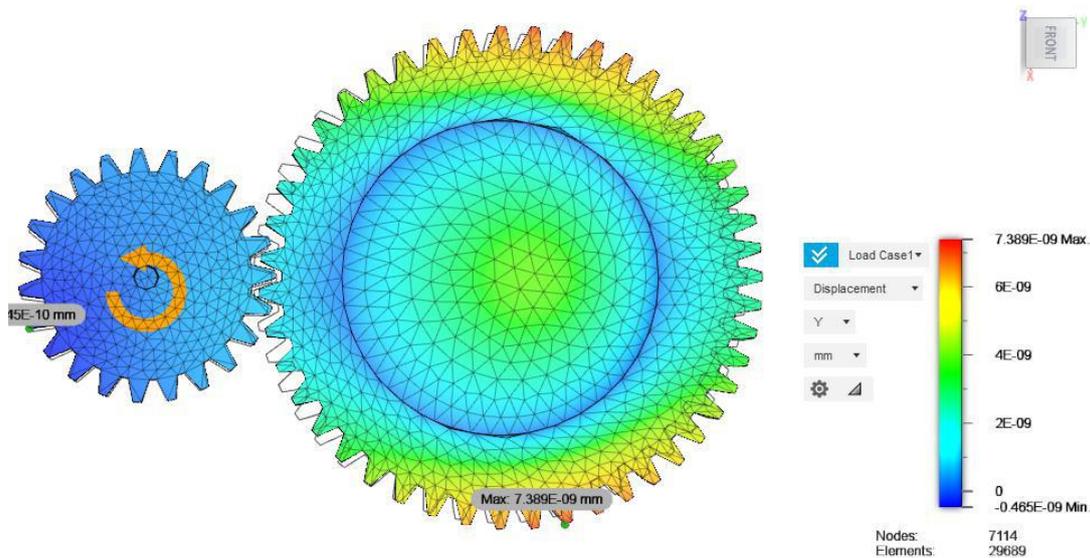


Fig. 9: Displacement along Y Axis.

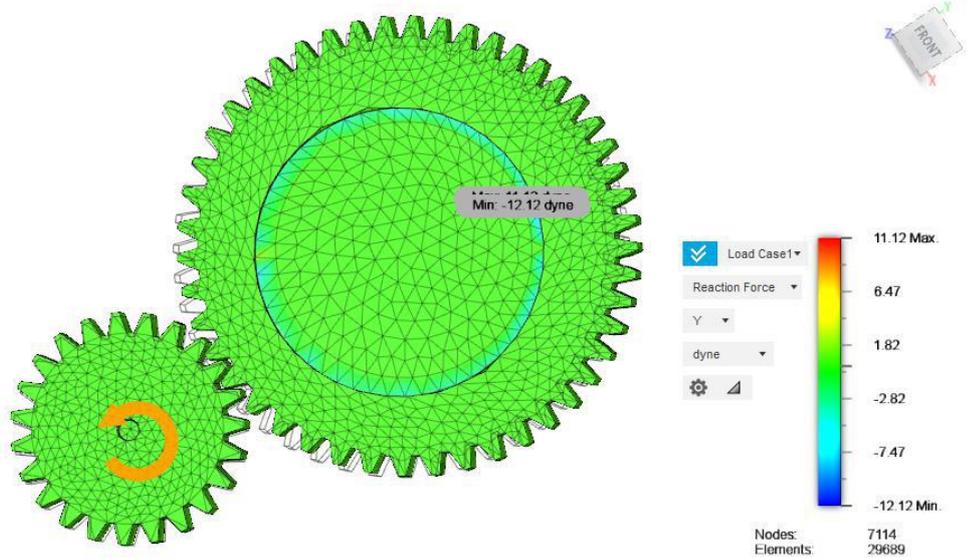


Fig. 10: Reaction Force along Y- Axis.

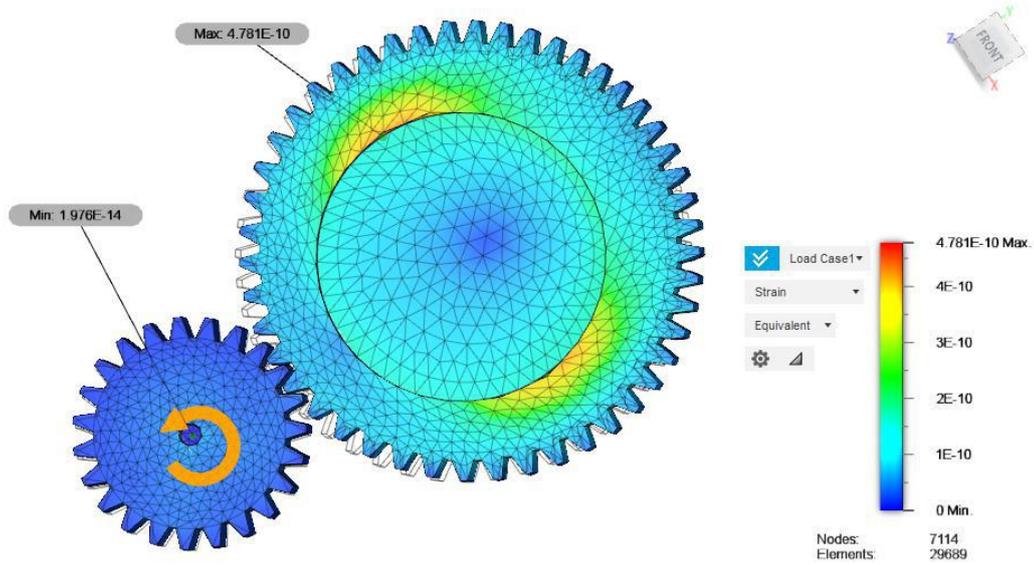


Fig. 11: Equivalent strain.

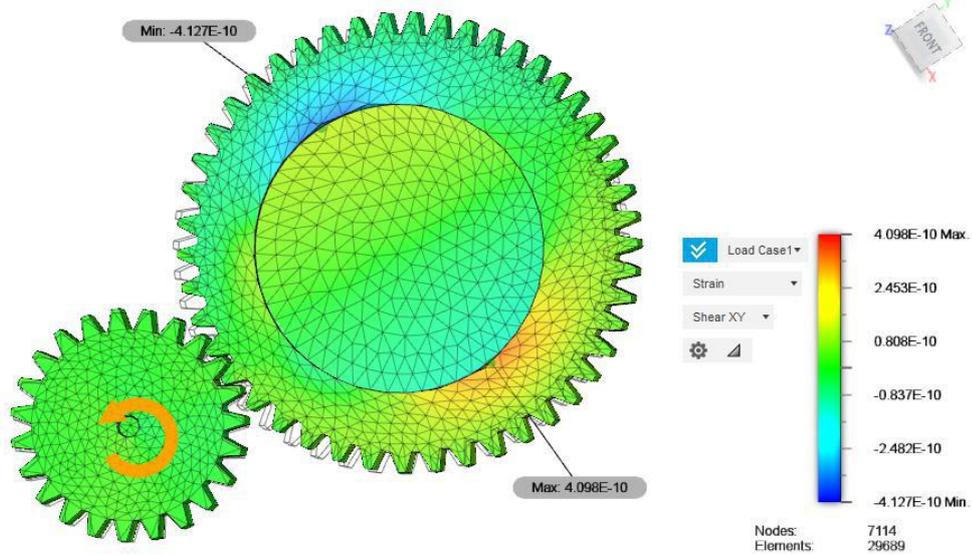


Fig. 12: Equivalent shear strain along XY plane.

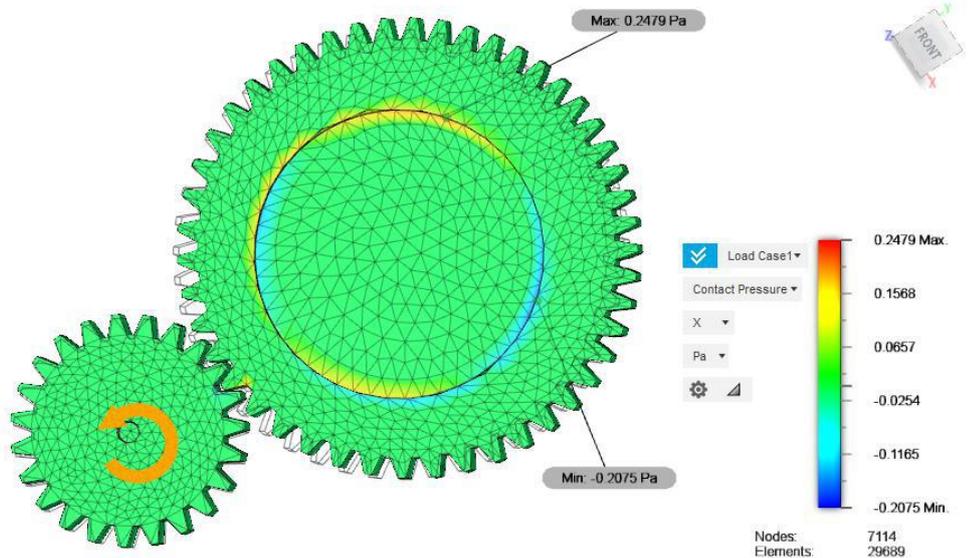


Fig. 13: Contact pressure along X- Axis in Pa.

## CONCLUSION

This gear arrangement has been designed for practical use in rotating mechanism where the angular displacement of the driven gear required smaller with respect to the driver gear. This simulation has been accomplished considering the load in real application. The results from the simulation can help to draw a decision that, the durability of the gear set will be very high even considering real conditions such as corrosion and erosion. The contact pressure and reaction force are so low in the working load that the wear and tear will be negligible. The vulnerable portions of the gears may be the contact zone where stress, strain and contact pressure etc. are higher among all. Proper lubrication can minimize that.

## REFERENCES

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