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HEAT TRANSFER ENHANCEMENT WITH NANOFLUIDS – A REVIEW

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ABSTRACT

This paper presents a review of the studies undertaken on convection heat transfer with nanofluids. Initial studies were directed towards the determination of the properties of nanofluids, especially their thermal conductivity and viscosity. The studies indicate that thermal conductivity and viscosity increase with an increase in the concentration of the nanofluid. Experiments were conducted with different nanofluids, at various concentrations and temperature ranges, for the estimation of the heat transfer coefficient and friction factor for water-based nanofluids. All the studies confirmed enhancement of the heat transfer coefficient with an increase in concentration. The experimental ranges of temperature undertaken by the authors were different for different nanofluids. Certain studies with smaller particle sizes indicated an increase in heat transfer enhancements when compared with values obtained when using larger particle sizes. It is observed that the concentration of the nanofluid, the operating temperature, the particle size and shape, together with the material of the nanoparticle dispersed in the base liquid, have significant influence on the heat transfer coefficient. All the studies indicate a nominal increase in pressure drop.

Keywords: Convection heat transfer; thermal conductivity; viscosity; friction factor; nanofluid.

INTRODUCTION

A nanofluid is prepared by dispersing particles of metal or metal oxide with sizes of 100 nm or less, in a base liquid such as water. The purpose of using nanofluids is to achieve higher values of heat transfer coefficient compared with that of the base liquid. This is achieved by the dispersion of solid particles, which have higher thermal conductivity than the base liquid. There are many engineering applications that can benefit from the use of nanofluids, for example absorption refrigeration, micro electromechanical systems, lubrication of automotive systems, the manufacture of advanced miniature camera lenses, coolant in machining, automobile radiator cooling, personal computers, solar water heating, heat exchangers, several medical applications, nuclear reactors, and in several aerospace applications. Recent advances in material technology have made it possible to produce innovative heat transfer fluids by suspending nanometer-sized particles in base fluids, which could change the transport and thermal properties of the liquids. Nanofluids represent solid-liquid composite materials consisting of solid nanoparticles with sizes no larger than 100 nm suspended in liquid (Ferrouillat, 2011). This study presents the work undertaken by various investigators and the possible impact of nanofluids on the enhancement of heat transfer in the near future.

THERMAL PROPERTIES OF NANOFLUIDS

The thermal properties of nanofluids have received significant attention. Nanofluids are considered to offer important advantages over conventional heat transfer fluids. A number of experimental studies to investigate the transport properties of nanofluids have been carried out (Choi, 1995; Masuda, Ebata, Teramae, & Hishinuma, 1993; Eastman, Choi, Li, Thompson, & Lee, 1997; Lee, Choi, Li, & Eastman, 1999). Many researchers have used regression equations of density and specific heat capacity (Gianluca, 2012; Heris, Etemad, & Esfahany, 2006; Incropera & DeWitt, 1996; Kulkarni, Namburu, Ed Bargar, & Das, 2008; Lee et al, 1999; Li & Xuan, 2002; Ma, Wilson, Borgmeyer, Park, & Yu, 2006; Masuda et al, 1993; Namburu, Kulkarni, Misra, & Das, 2007) as:

$$\rho_{\mathrm{nf}} = \left(\frac{\phi}{100}\right)\rho_p + \left(1 - \frac{\phi}{100}\right)\rho_f \qquad (1)$$

$$C_{nf} = \frac{\frac{\phi}{100}(\rho C)_p + \left(1 - \frac{\phi}{100}\right)(\rho C)_f}{\rho_{nf}} \qquad (2)$$

Thermal Conductivity

The thermal conductivity of nanofluids was found to increase with concentration (Pak & Cho, 1998; Xuan and Li 2000; Xuan & Roetzel, 2000; Xue, Keblinski, Phillpot, Choi, & Eastman, 2004; Heris et al., 2006) following experiments undertaken with Cu, Al_2O_3, CuO, and TiO_2 nanoparticles in water, they also, observed heat transfer enhancement as high as 40% with Al_2O_3 particles. A simulation of the effective thermal conductivity of nanofluids through modeling has been undertaken by Bhattacharya, Saha, Yadav, Phelan, & Prasher (2004). Nanoparticles dispersed in ethylene glycol (EG), water (H_2O) and oil have shown an increase in the thermal conductivity ratio (k_{nf} / k_f) with a decrease in the thermal conductivity values of the base fluid (Lee et al., 1999). With regard to the volume concentrations and magnitude of particle-particle interaction that are affected by pH, surfactant additives, and particle size and shape, agglomeration equilibrium is established in nanoparticle suspension. It should be noted that two types of agglomeration are possible in nanofluids. The first type of agglomeration occurs when nanoparticles are agglomerated through the solid-solid interface, which can potentially provide increased thermal conductivity, as described by Prasher, Wang, and Phelan (2006). The SiC in water and EG/water mixed with volume concentration and pH was studied by Timofeeva, Yu, France, Singh, and Routbort (2010). It showed that the change in thermal conductivity ratio was 5% higher in EG/water than in water considering all other parameters. The base fluid effect, observed with different nanofluid systems, is most likely related to the lower value of the thermal resistance in the EG/water than in the water-based nanofluids. Both thermal conductivity and viscosity are related strongly to the nanofluid microstructure. The nanoparticles dispersed in a base fluid are in random motion under the influence of several forces such as Brownian motion, intermolecular van-der-Waals interaction (repulsion, polarization, and dispersion forces), and electrostatic interactions between ions and dipoles. Thermal conductivity measurement of pure fluids by the transient hot-wire method has been investigated (Wang, 2009). A transient hot wire is in contact with the liquid being studied (Pawan, Singh, & Anoop, 2010; Gianluca, 2012) and the effect of temperature on thermal conductivity is shown in Figure 1. The thermal conductivity of nanofluids has been found experimentally, and data of the thermal conductivity for metal and metal oxides, such as Al_2O_3, Fe_3O_4, TiO_2, ZnO, ZrO_2, and CuO nanofluids, available in the literature, have used in the development of the regression Eq. (3).

$$k_r = \frac{k_{nf}}{k_f} = \left\{ 0.8938 \left(1+\frac{\phi}{100}\right)^{1.37} \left(1+\frac{T_{nf}}{70}\right)^{0.2777} \left(1+\frac{d_p}{150}\right)^{-0.0336} \left(\frac{\alpha_p}{\alpha_f}\right)^{0.01737} \right. \qquad (3)$$

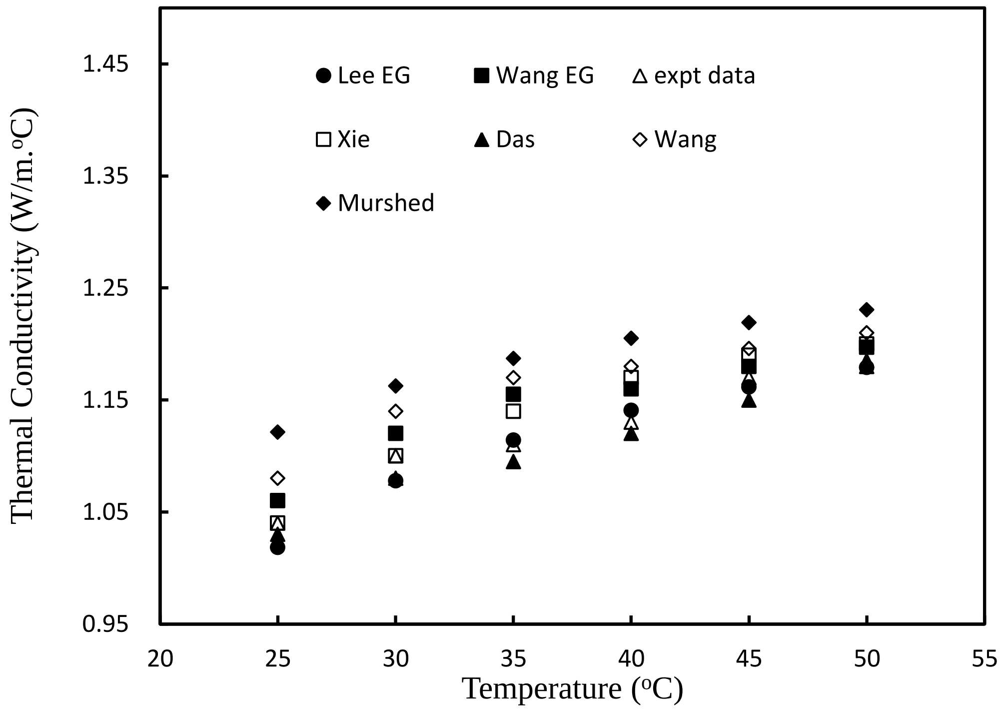


Figure 1. Effect of temperature on thermal conductivity.

Viscosity

Rheometers are used to measure the rheological properties of nanofluids (Ding et al., 2006; Prasher et al., 2006). The viscosity has been shown to decrease with an increase in the average diameter size in both EG/water and water-based suspensions. However, at the same volume concentration of nanoparticles, the relative viscosity increase was smaller in EG/water than in the water-based nanofluids, especially in suspensions of smaller nanoparticles (Timofeeva et al., 2010). According to the classic Einstein equation for hard non-interacting spheres (Vold, Kristiansen, & Christensen, 2007), the viscosity increase should be independent of the viscosity of the base fluid and only proportional to the volume concentration. Viscometers have been used by Nguyen, Desgranges, Galanis, Roy, Mare, Boucher, & Angue (2008), Namburu et al. (2007), and Pozhar (2000) who studied the effect of nanofluid concentration on viscosity (Figure 2). The experimental data of viscosity obtained at 4% volume fraction, consisting of many data points, was subjected to regression and the following correlation obtained.

$$\mu_r = \frac{\mu_{nf}}{\mu_f} = \left(1 + \frac{\phi}{100}\right)^{11.3} \left(1 + \frac{T_{nf}}{70}\right)^{-0.038} \left(1 + \frac{d_p}{170}\right)^{-0.061} \tag{4}$$

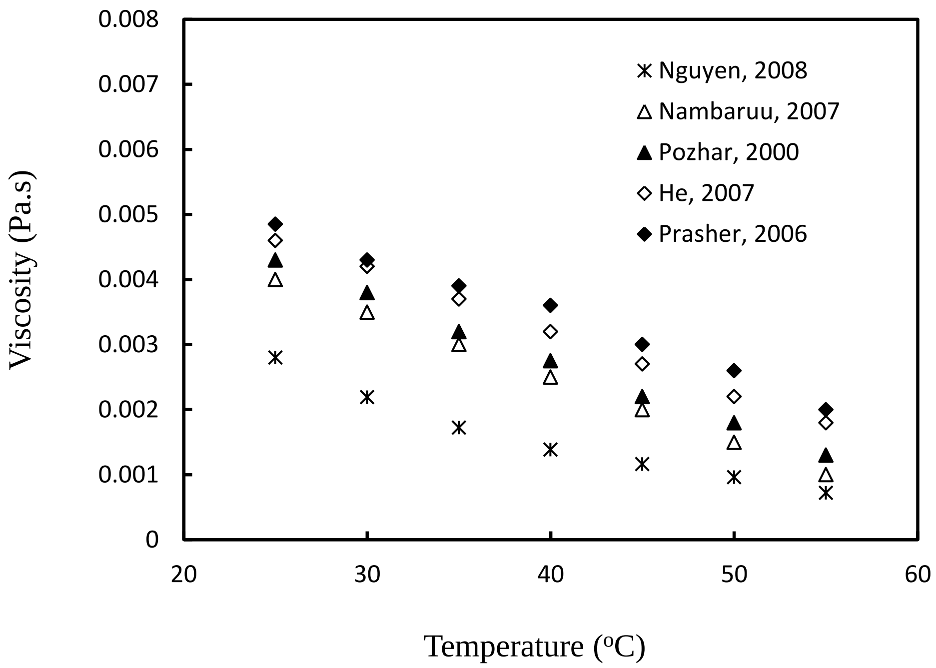


Figure 2. Effect of temperature on viscosity.

Friction Factor

Turbulent friction factors have been evaluated for the flow of nanofluids in a tube (Pak & Cho, 1998; Xuan & Li, 2002; Yang, Zhang, Grulke, Anderson, & Wu, 2005; Ding et al., 2006; Ma et al., 2006; Kulkarni et al., 2008). Some of the studies are in agreement with values estimated by using the Blasius equation:

$$f = \frac{0.316}{\mathrm{Re}^{0.25}} \qquad \text{For} \qquad \mathrm{Re} > 1 \times 10^4 \tag{5}$$

The calculated friction factors were then compared for validation (Incropera & DeWitt, 1996; Dong & Leyuan, 2010). Figure 3 shows the friction factors at different Reynolds numbers.

HEAT TRANSFER ENHANCEMENT

Numerical studies of steady-state turbulent convection of a water-Al_2O_3 nanofluid inside a circular tube, by means of the finite volume method, has been investigated by many researchers (Li, 2002; Bianco & Manca, 2011; Syam Sundar & Sharma, 2011a,b; Rao, Sharma, Chary, Bakar, Rahman, Kadirgama, & Noor, 2011), and the results showed that heat transfer is enhanced with particle volume concentration and Reynolds number. A study of forced convection heat transfer of nanofluids inside a horizontal circular tube, subjected to a constant and uniform heat flux at the wall, was carried out by Shuichi (2012), and the results showed that the heat transfer enhancement was caused by the suspended nanoparticles. A bibliography of works on experimental forced convective heat transfer with nanofluids with Al_2O_3 particles and TiO_2 particles at the same concentration levels is presented in Table 1. When taking into account thermal conductivity, it is not surprising that the Cu-water nanofluid shows the highest heat transfer enhancement. However, the thermal conductivity enhancements of Al_2O_3 and TiO_2 in water are similar, although the heat transfer enhancement of Al_2O_3 in water is higher than that of TiO_2 in water.

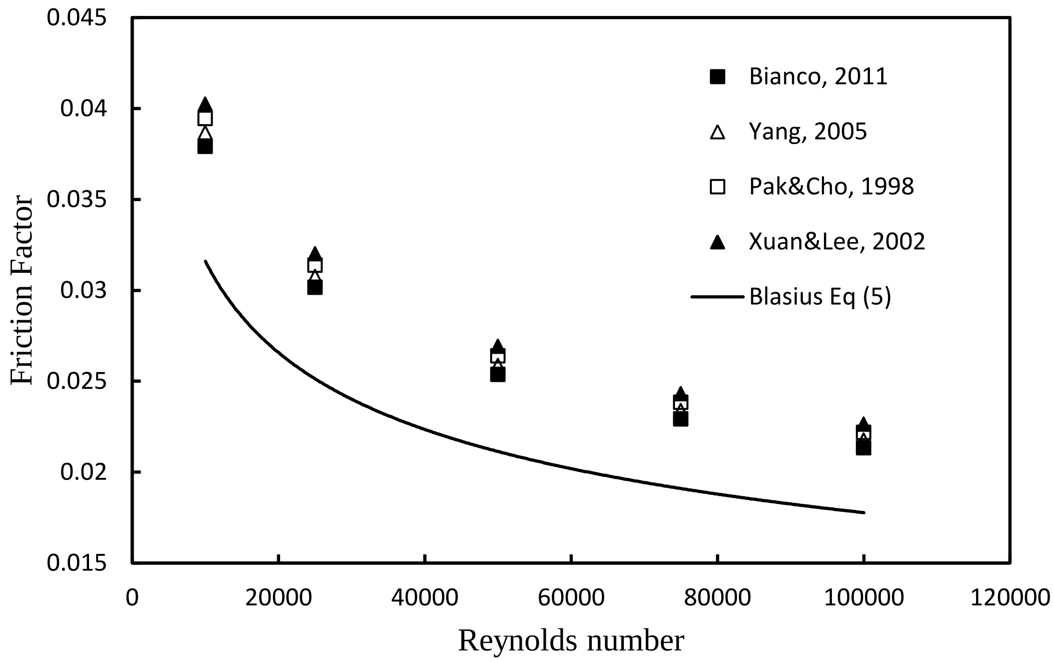


Figure 3. Friction factor at different Reynolds number.

However, two groups found that the heat transfer coefficient of nanofluids (Al_2O_3 and TiO_2 in water and SiC in water) was lower than for pure water for constant average velocity in a turbulent flow (Ferrouillat, 2011; Vijaya Lakshmi, Subrahmanyam, Dharma Rao, & Sharma, 2012). A hybrid nanofluid was used to enhance the heat transfer and pressure drop in a fully developed laminar flow through a uniformly heated circular tube (Suresh, Venkitaraj, Selvakumar, & Chandrasekar, 2012). Experimental results using Cu-Al_2O_3 in water synthesized with 0.1% volume concentration showed a maximum enhancement of 13.56% in the Nusselt number at a Reynolds number of 1730 when compared to the Nusselt number of water. Results also showed that 0.1% Cu-Al_2O_3/water nanofluids have a slightly higher friction factor when compared with 0.1% Al_2O_3-water nanofluid. Correlations of the Nusselt number and friction factor were found and there was good agreement with the experimental data of other researchers. A number of studies concluded that heat transfer enhancement depends on the Dittus-Boelter equation (Eq. (6)):

$$Nu = 0.032 \times \mathrm{Re}^{0.8} \times \mathrm{Pr}^{n} \tag{6}$$

where *n* is 0.4 at heating and 0.3 at cooling. The increase in heat transfer coefficient in the turbulent flow of a nanofluid in a tube was reported by Pak & Cho (1998). They showed the increase was 45% with 1.34% volume fraction of Al_2O_3 in water, and 75% with a nanoparticle loading of 2.78%. They predicted that the Nusselt number was a function of the Reynolds number and Prandtl number.

$$Nu = 0.021 \times \mathrm{Re}^{0.8} \times \mathrm{Pr}^{0.5} \tag{7}$$

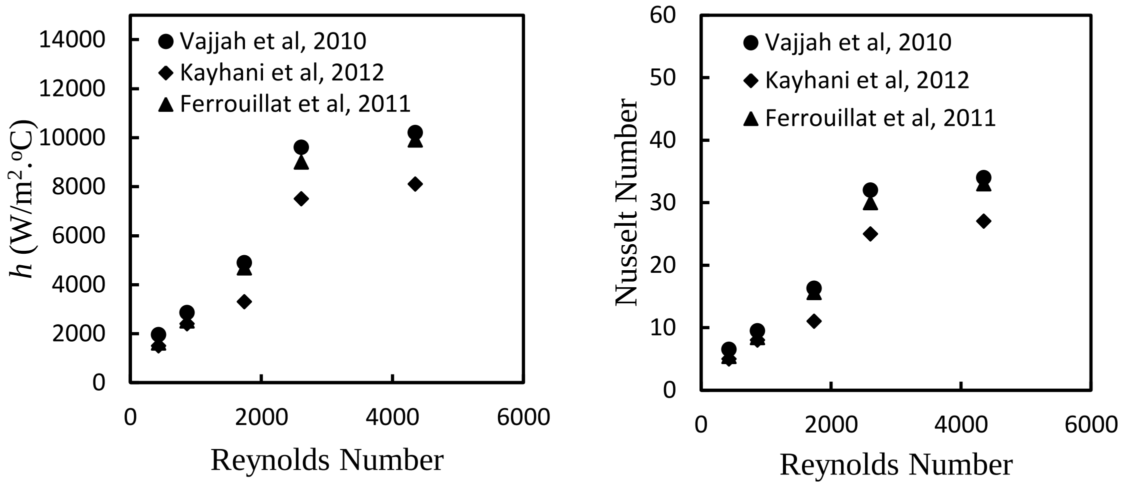
Xuan and Li (2002) followed Pak and Cho (1998) in showing that an increase of as much as 40% could be achieved in the heat transfer coefficient of a nanofluid. The correlation of the convection heat transfer coefficient of nanofluids in a horizontal tube was found to be:

$$Nu = 0.4328(1.0 + 11.285\phi^{0.75} Pe^{0.218})\,\mathrm{Re}^{0.333}\,\mathrm{Pr}^{0.4} \tag{8}$$

Table 1. Results of papers published on different types of nanofluid.

Ref.	Nanofluid	Re	Nu_{nf}/Nu_f
Lee & Choi (1996)	Metallic nanoparticle suspension	Laminar	+100%
Pak & Cho (1998)	Al_2O_3–water TiO_2–water 3 vol.%	Turbulent	3% to _12% for constant average velocity
Li & Xuan (2002)	Cu–water 2 vol.%	800–23,000	60%
Xuan & Li (2002)	Cu–water 0.3–2 vol.%	Laminar, turbulent	30%
Wen & Ding (2004)	Al_2O_3–water 0.2–1.6%	650–2050	Nu > Nu Shah especially near the entrance
Yang et al. (2005)	Graphite 2–2.5 wt.% 5	110	$N_{unf}/N_{uf} < k_{nf}/k_f$ (aspect ratio *L/d* = 0.02)
Ding et al. (2006)	CNT–water (aspect ratio > 100) 0.1–1 wt.%	800–1200	+350%
Zeinali, Esfahany & Etemad (2007)	Al_2O_3–water 0.2-2.5 vol.%	700–2050	Enhancement of with Pe increases with aspect ratio (nanoparticle shape)
Rea, McKrell, Hu & Buongiorno (2009)	Al_2O_3–water 0.6–6.0 vol.% ZrO_2–water 0.32–3.5 vol.%	Laminar	32%enhancement of with 1.8 vol.% without major friction loss
Jung, Natter, Hempelmann & Lach (2009)	Al_2O_3–water 0.6–1.8 vol.% Al_2O_3–water	5-300	+8% at 0.3 vol.%
Hwang, Lee, Park, Park, Jung, Lee & Song (2009)	0.01–0.3 vol.%	550–800	Heat transfer enhancement

An experimental study was carried out by Yang (2005) who reported that nanoparticle concentration, material, temperature, and base fluid all affected the heat transfer coefficient. Bianco and Manca (2011) showed the effect of concentration on the heat transfer coefficient and Nusselt number with Reynolds number, as shown in Figure 4**.**



(a) Heat transfer coefficient (b) Nusselt number

Figure 4. (a) heat transfer coefficient, (b) Nusselt number, at different Reynolds number.

CONCLUSION

The review of these studies shows that nanofluids are very important for many applications. Many studies showed good agreement between experimental and numerical studies. Some general conclusions are:

1. An increase in thermal conductivity occurred by adding nanoparticles to liquids.
2. Viscosity increased as the concentration of particles increased.
3. Friction factor increased with Reynolds number from experimental results and from the Blasius equation.
4. The convection heat transfer coefficient was shown to increase with Reynolds number and volume concentration by experimental results and the Dittus-Boelter equation.
5. There are many correlation equations among the input parameters (volume concentrations, Reynolds number, and temperature) and output parameters (friction factor and Nusselt number).

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NOMENCLATURE

C specific heat capacity ($J/kg.^{o}k$)
d diameter of tube (*m*)
f friction factor
h heat transfer coefficient ($W/m^2.^{o}C$)
k thermal conductivity ($W/m.^{o}C$)
Nu Nusselt number ($Nu = \frac{h \times d}{k}$)
Pe Peclet number (*Pe=RePr*)
Pr Prandtl number ($Pr = \frac{C \times \mu}{k}$)
Re Reynolds number ($Re = \frac{\rho \times v \times d}{\mu}$)
v velocity of fluid (*m/s*)
ϕ concentration of solid particles
μ Viscosity (*Pa.s*)
ρ Density (kg/m^3)

SUBSCRIPTS

av average value
f fluid
nf nanofluid
s solid