

Second order sliding mode control for direct drive positioning system

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ABSTRACT

Second order sliding mode control is known for the ability to suppress chattering effect, often being associated with the implementation of a traditional sliding mode controller. The purpose of this paper is to demonstrate and compare the ability of super twisting sliding mode control to suppress chattering as well as to improve tracking performances against the traditional sliding mode controller. Both controllers were designed, numerically analysed and experimentally validated on a second order single-input-single-output system direct drive single axis positioning table. A continuous Kalman-Bucy filter was applied to estimate the velocity signal to further improve the overall tracking performance. Results showed that super twisting sliding mode controller was able to successfully suppress chattering effect by smoothening the control input through integration action to form a continuous function, thus dampening the effect of high frequency switching. The effectiveness of this control algorithm would promote its application in real-time application as it provides better control performance as compared to the standard sliding mode controller.

Keywords: accuracy; chattering; sliding mode control; super twisting; machine tools.

INTRODUCTION

Precision control is a popular focus of many researchers due to constant demands for precision and accuracy in most product development processes and technologies. In manufacturing, precision control is highly relevant, especially with respect to machine tools related applications. In literature, various control algorithms were proposed and demonstrated to replace the traditional control approaches to further improve accuracy and precision of machine tools. A popular nonlinear control algorithm is the sliding mode control (SMC). SMC provides robustness property on linear or nonlinear system with uncertainties or disturbances input [1]. However, the control algorithm has a main drawback in the form of chattering due to the high frequency switching behaviour of the control signal, specifically the after-effect of applying the signum function [2]. Various control approaches and mechanisms were proposed and presented to solve chattering problem of SMC and one of the good records can be found in the work of [3]. A typical solution is to use a smooth signum function through a continuous approximation method. A sigmoid-like function normally replaces the signum function and induces a pseudo-sliding instead of ideal sliding. Although this approach manages to reduce the chattering effect, the accuracy and robustness of the system is partially lost [1]. The decision on this

trade-off also becomes a challenge to control engineers as high chattering is not practical while low accuracy is not appropriate to be applied on the physical system.

Another novel approach of suppressing the chattering effect is through the use of high order sliding mode control (HOSMC). This approach was proposed by [4] and many findings of the HOSMC concepts could be found in the works of [1, 5-14]. Applications of HOSMC, specifically the second order sliding mode control (SOSMC), could be found in vibration suppression for diesel engines, as demonstrated by [15, 16] and application in DC motor by [17, 18]. Works in [11, 19] also demonstrated that the application of HOSMC as an observer for mechanical system. The SOSMC approach is able to maintain the original advantage of SMC and almost eliminate the chattering concurrently [1, 4, 20]. SOSMC applies sliding mode on higher order time derivatives of the sliding variable instead of first time derivatives normally found in standard SMC. Besides application in vibration suppression for diesel engines and DC motor, SOSMC is applied in marine engineering field as implemented by [21, 22] as well as aerial vehicles as demonstrated by [23, 24] due to its high robustness and disturbance rejection properties. In addition, some works on SOSMC are mainly focused on the finite time convergence and stability analysis by using time domain analysis [25], Lyapunov function [26, 27], and point-to-point method [10] for state trajectories, rather than the real-time applications of SOSMC. In this paper, a super twisting sliding mode (second order) control algorithm (ST-SMC) was applied on a single axis positioning system to improve tracking accuracy of the considered system through the reduction in degree of chattering observed. In addition, a Kalman-Bucy filter (KBF) was applied as an estimator for velocity of the system to further enhance tracking accuracy. In order to show the superiority of ST-SMC in chattering reduction, tracking results were compared with the standard SMC. Section 2 describes the system as well as the experimental setup. Section 3 discusses the design of the control algorithms as well as the estimator. Results and discussion are presented in Section 4 and lastly, Section 5 presents the conclusion and future recommendations.

MATERIALS AND METHODS

Experimental Setup

The considered setup is a direct drive single axis positioning system driven by an ironless flat linear motor equipped with a 4 μm -resolution linear encoder. Figure 1 shows the schematic diagram of the experimental setup. A host computer equipped with ControlDesk and MATLAB software was linked to the data acquisition unit, that is, dSPACE DS1104 Digital Signal Processor (DSP) controller board. This controller board was connected to the servo amplifier and lastly the single axis positioning system was connected to the servo amplifier.

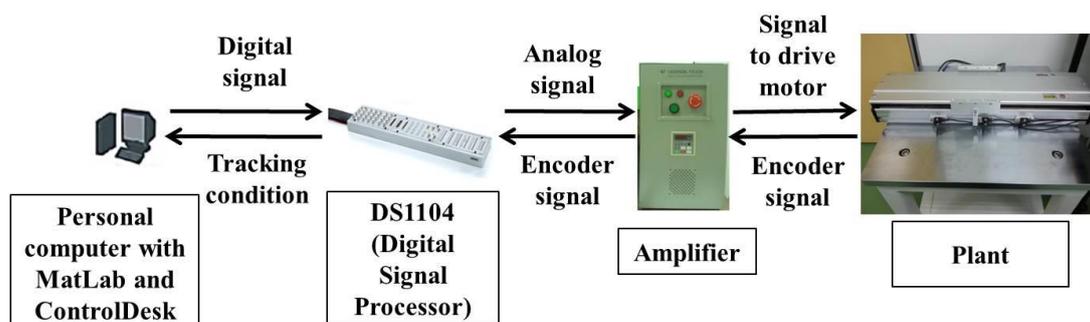


Figure 1. Configuration for experimental setup.

The considered system dynamics was described by using a single-input-single-output (SISO) model and estimated through frequency domain identification method. The SISO frequency response function (FRF) of the system was approximated by using H1 estimator, based on the measured input voltage and output position signals by applying the band-limited white noise excitation signal [28]. Sampling frequency used was 5000 Hz. Parametric model was then fitted on the measured FRF by using a nonlinear frequency domain identification method [28] and a second order transfer function with time delay of 0.00045 seconds, as shown in Equation (1) and can be represented in Equation (2).

$$\frac{Y(s)}{U(s)} = \frac{A}{s(s+B)} e^{-sT_d}, \quad (1)$$

$$\ddot{y}(t) = -B\dot{y}(t) + Au(t), \quad (2)$$

with $A = 7.5e8 \mu\text{m}/\text{V}\cdot\text{s}^2$, $B = 3622 \text{ s}^{-1}$, and $T_d = 0.00045 \text{ s}$. $Y(s)$ is the actual output position in micrometer while $U(s)$ or $u(t)$ is the control input voltage to the drive in volt, V, respectively. Both $\dot{y}(t)$ and $\ddot{y}(t)$ are the first and second time derivatives of actual output position, respectively.

Design of Controllers and Estimator

Two controllers were designed namely; SMC and ST-SMC. In addition, a continuous estimator known as Kalman-Bucy filter (KBF) was designed to estimate velocity of the system. Figure 2 shows the general block diagram of the designed control algorithm.

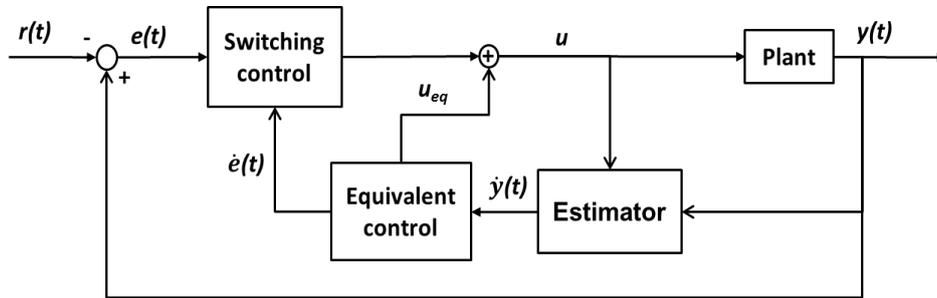


Figure 2. General block diagram of designed control algorithm.

Sliding Mode Control

Two main components considered in design of SMC are the switching function and the control laws [29-31]. Switching function is a function with respect to sliding surface, $s(t)$ and it is a function of the tracking error, $e(t)$ and its time derivative, $\dot{e}(t)$ as shown in Equation (3) and Equation (4).

$$s(t) = \left(\lambda + \frac{d}{dt} \right)^{n-1} e(t), \quad (3)$$

$$e(t) = y(t) - r(t), \quad (4)$$

where n is the order of uncontrolled system, λ is a positive constant while $r(t)$ and $y(t)$ both represent the desired position and actual output positions, respectively. Equations Equation (5) and Equation (6) show the first derivative of the sliding surface, $\dot{s}(t)$ and

second derivative of the tracking error, $\ddot{e}(t)$, respectively. Equation (7) is formed by substituting Equation (2) and Equation (6) into Equation (5).

$$\dot{s}(t) = \lambda \dot{e}(t) + \ddot{e}(t), \tag{5}$$

$$\ddot{e}(t) = \ddot{y}(t) + \ddot{r}(t), \tag{6}$$

$$\dot{s}(t) = \lambda \dot{e}(t) + Au(t) - B\dot{y}(t) - \ddot{r}(t). \tag{7}$$

Control law consists of a signum function with a positive constant M , and the equivalent control, $u_{eq}(t)$ that includes velocity and acceleration feedforward as shown in Equation (8) and Equation (9). Equivalent control is then obtained when $\dot{s}(t) = 0$ from Equation (7) and it is represented by Equation (10).

$$u(t) = u_{eq}(t) - M \cdot \text{sign}(s), \tag{8}$$

$$\text{sign}(s) = \begin{cases} 1, & s > 0 \\ 0, & s = 0 \\ -1, & s < 0 \end{cases}, \tag{9}$$

$$u_{eq}(t) = \frac{1}{A} (\ddot{r}(t) + B\dot{y}(t) - \lambda \dot{e}(t)). \tag{10}$$

λ and M are identified using heuristic method with values equal 800 and 0.004 respectively. M is tuned to small value to reduce the chattering effect. The stability of the system is ensured based on Lyapunov stability criterion. For practical issue (chattering), a sigmoid-like function, $s/(s+\delta)$ is used to replace the signum function during experiments [32-35]. The degree of continuous approximation, δ is selected as 400.

Super Twisting Sliding Mode Control (ST-SMC)

The main aim of second order sliding mode control is to steer the sliding surface as well as its first order time derivative to zero [1, 4], $s = \dot{s} = 0$. One of the popular second order sliding mode controls is ST-SMC. Unlike other second order SMC, such as twisting algorithm, ST-SMC does not require the information of \dot{s} in its formulation and application which is simpler and preferable [4]. The ST-SMC utilised the similar design steps as standard SMC. The same sliding surface as in Equation (3) is applied and the control laws are stated in Equation (11) and Equation (12).

$$u(t) = -L|s(t)|^{0.5} \text{sign}(s) + u_1(t), \tag{11}$$

$$\dot{u}_1(t) = -W \cdot \text{sign}(s), \tag{12}$$

where both L and W are positive constants.

Both parameters L and W are tuned by using heuristic approach in such that the chattering effect is minimised without affecting the accuracy of system and their values are 0.0001 and 0.02, respectively. The stability is ensured by using Lyapunov stability criterion, which was effectively demonstrated in the works of [36, 37].

Kalman-Bucy Filter

Kalman-Bucy Filter (KBF) is an estimator appropriate for continuous system [38]. In this paper, KBF is designed to estimate the velocity, v of the system instead of applying

numerical differentiation of measured position signal. Through the estimation process by using KBF, the amplification of noises due to numerical differentiation could be prevented and hence, further improving the performance of SMC and ST-SMC as the equivalent function, $u_{eq}(t)$ in both algorithms are using velocity signals. Equation (13) and Equation (14) consider the system in state-space form:

$$\dot{x}(t) = F(x(t), u(t), t) + w(t), \quad (13)$$

$$y(t) = H(x, t) + v(t), \quad (14)$$

where $y(t)$ is the measurement (output position) while F and H are the system and measurement matrices. The system and measurement noise are represented by the random variable $w(t)$ and $v(t)$, respectively. They are assumed to be uncorrelated, zero mean and white Gaussian noise with $w \sim N(0, Q)$ and $v \sim N(0, R)$.

KBF basically estimates the states based on Equation (15), Equation (16), and Equation (17) that denote the state estimation $\hat{x}(t)$, covariance estimation $\hat{P}(t)$, and filter gain $K(t)$ computation respectively [38-40]. Equations (18), Equation (19) and (20) show the matrices used in this paper.

$$\dot{\hat{x}}(t) = F(x(t), u(t), t) + K(t)[y(t) - H(\hat{x}(t), t)], \quad (15)$$

$$\dot{\hat{P}}(t) = F\hat{P}(t) + \hat{P}(t)F^T + Q(t) - \hat{P}(t)H^T K(t), \quad (16)$$

$$K(t) = \hat{P}(t)H^T R^{-1}(t), \quad (17)$$

$$F = \begin{bmatrix} 0 & 1 \\ 0 & -3622 \end{bmatrix} x + \begin{bmatrix} 0 \\ 7.5 \times 10^8 \end{bmatrix} u, \quad (18)$$

$$H = [1 \quad 0] x, \quad (19)$$

$$P = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}; \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad R = 1. \quad (20)$$

RESULTS AND DISCUSSION

Experimental results were obtained based on the control system block diagram, as shown in Figure 2. A sinusoidal reference (desired) input with amplitude 20, 000 μm and frequency of 0.2 Hz was used to excite the system to generate the continuous motion for tracking control. A second set of experiment was performed by using identical sinusoidal reference input signal but at a frequency of 0.5 Hz. This is to test the consistency of proposed control algorithm in chattering suppression. The actual position, estimated velocity, control (voltage) input signal and tracking errors were measured and captured. Figure 3 and Figure 4 were compared with the control inputs between SMC and ST-SMC by using sinusoidal reference inputs of 0.2 Hz and 0.5 Hz, respectively. Based on results shown in Figure 3(a) and Figure 4(a), it was observed that the control input for SMC was “dirty” (especially during the maximum and minimum region), indicating the presence of chattering expected from application of SMC due to the high frequency oscillation [33]. This phenomenon can be mathematically explained as there was discontinuity in signum function, as shown in Equation (8). Although the continuous approximation approach was applied in the algorithm, the chattering suppression effect was still under par as compared to the ST-SMC. In contrast, the control inputs for ST-SMC were clean and smooth (Figure 3(b) and Figure 4(b)), as discontinuity was removed and the continuous function was formed through integration action, as shown in Equation (11) and Equation (12).

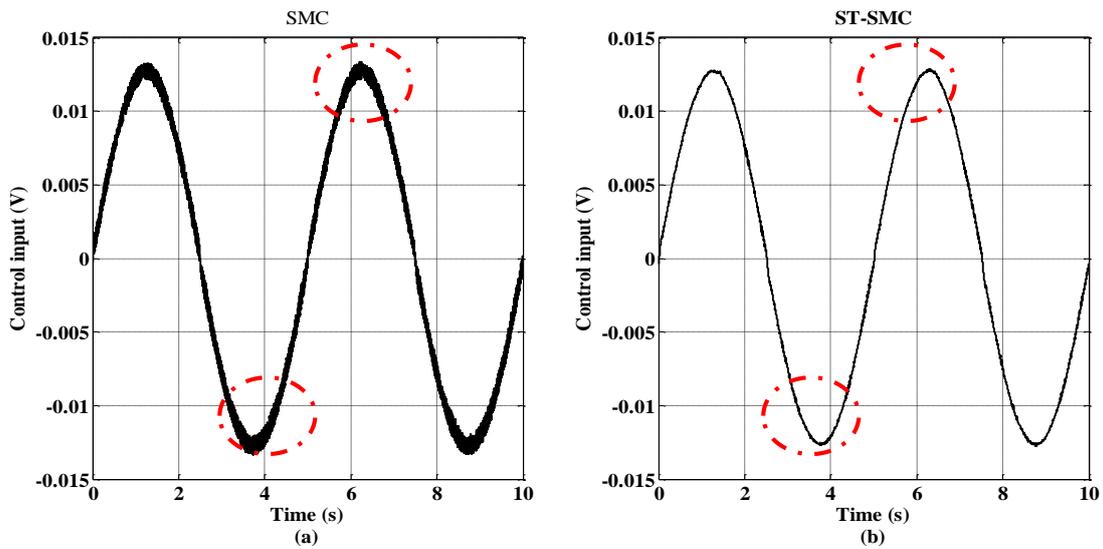


Figure 3. Comparison of control (voltage) input signals between (a) SMC and (b) ST-SMC for sinusoidal reference input with amplitude 20 000 μm and frequency of 0.2 Hz.

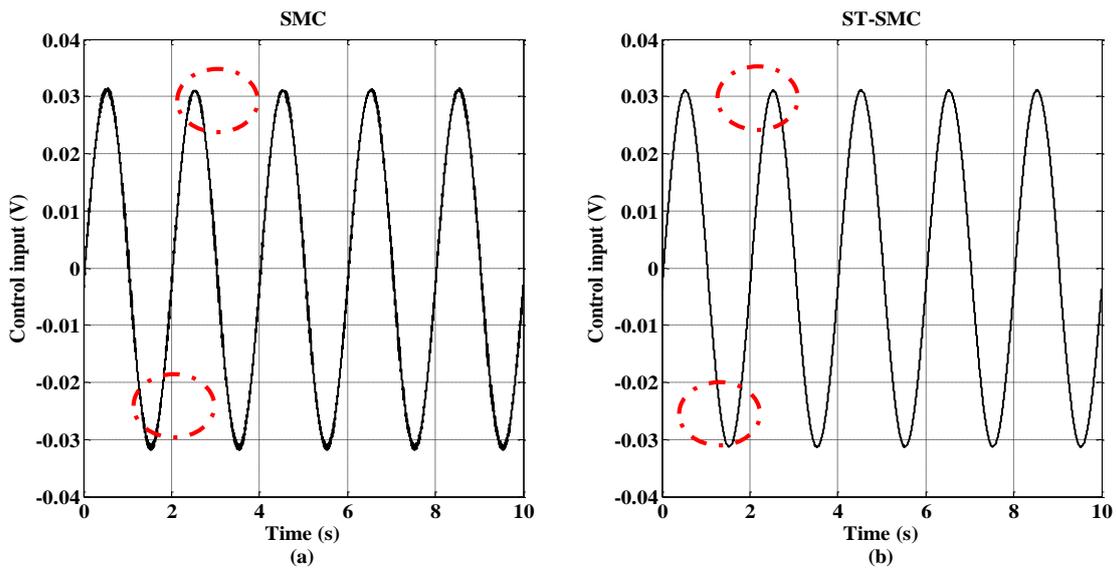


Figure 4. Comparison of control (voltage) input signal between (a) SMC and (b) ST-SMC for sinusoidal reference input with amplitude of 20 000 μm and frequency of 0.5 Hz.

In addition, Figure 5 and Figure 6 compare results of tracking errors between similar controllers and input references. The root mean square of the tracking error (RMSE) is tabulated in Table 1. According to the third row results from Figure 5(a) (left column) and Figure 6(a) (left column), the tracking errors of SMC was influenced by noises due to the chattering effect (Figure 3(a) and Figure 4(a)). In addition, it is clearly shown that the ST-SMC was able to produce smoothen signals (third row of Figure 5(b) (right column) and Figure 6(b) (right column)) as chattering effect was reduced (Figure 3(b) and Figure 4(b)). Improvements of 24% and 34% in tracking error were recorded for application of ST-SMC against SMC. It is also noted that there were spikes at near zero velocity region of ST-SMC. These spikes could be formed due to the unmodelled

dynamics [25] as well as friction forces existed in the system, which was not considered in this paper. However, this issue is a concern that needs to be attenuated and it is included in the future recommendation section. The RMSE of ST-SMC was greatly affected by the spikes formed, especially for the case of ST-SMC with input of 0.5 Hz. Overall, ST-SMC was able to reduce chattering on the two types of reference input signals. The suppression of chattering is a need, especially for large quantity production as chattering greatly affects the lifespan of machine tools as well as the output products.

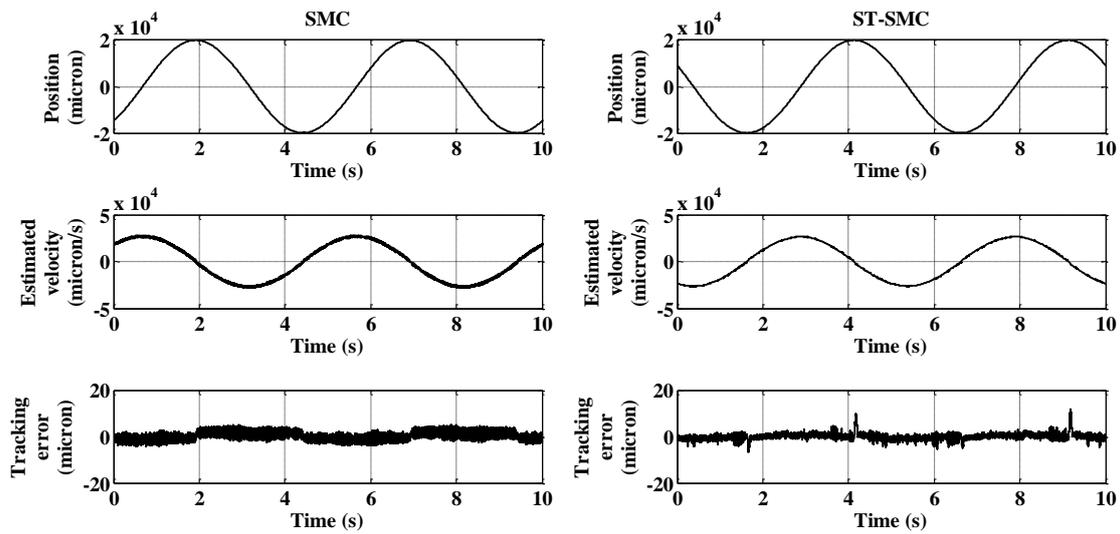


Figure 5. Results for (a) SMC and (b) ST-SMC for sinusoidal reference input of 20 000 μm in amplitude and 0.2 Hz in frequency.

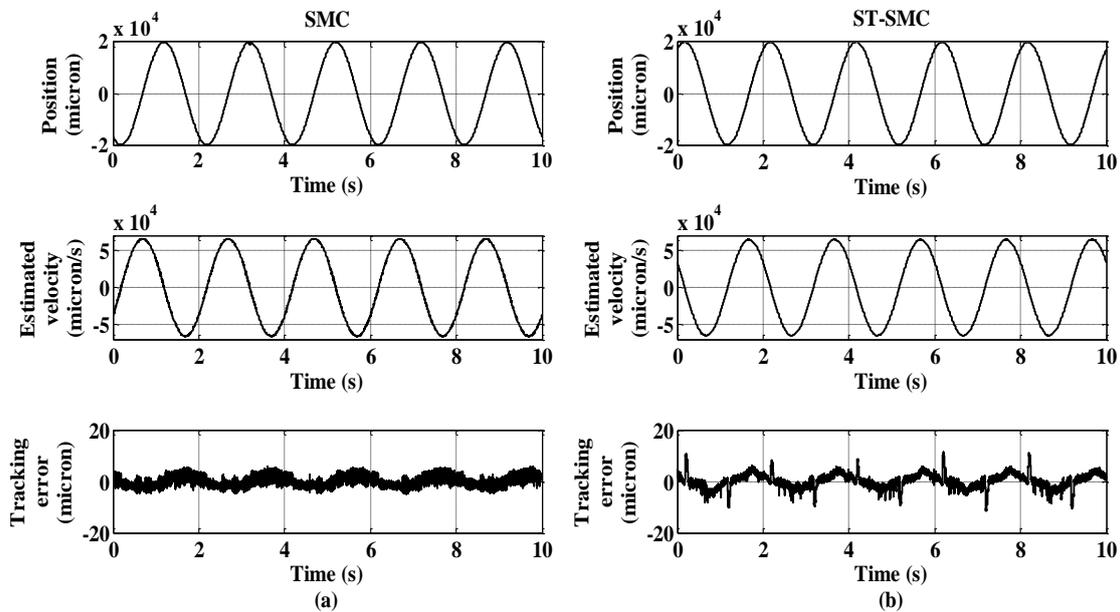


Figure 6. Results for (a) SMC and (b) ST-SMC for sinusoidal reference input of 20 000 μm in amplitude and 0.5 Hz in frequency.

Table 1. Root mean square error (RMSE) for designed control algorithms with reference input of varying frequencies.

Frequency (Hz)	RMSE for SMC (μm)	RMSE for ST-SMC (μm)
0.2	1.7808	1.3541
0.5	3.0779	2.0322

Furthermore, utilisation of KBF has contributed to the improvement in accuracy as the estimation of velocity was smoother as compared to velocity obtained from numerical differentiation of measured position. This is shown in Figure 7. Results obtained were consistent with literature as reduction of noises in velocity reduced burden of SMC or ST-SMC, specifically in the equivalent control section. The utilisation of KBF prevents the amplification of noises that are usually caused by the traditional direct differentiation of signals collected from encoder, thus; a better accuracy can be achieved.

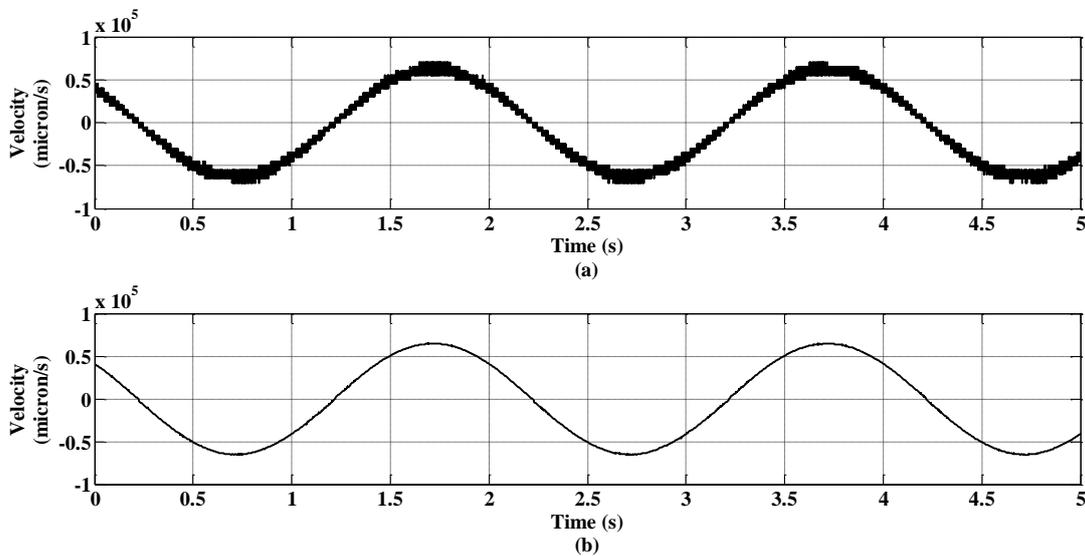


Figure 7. Velocity signals obtained from (a) direct differentiation of measured position signal, and (b) estimation from KBF.

CONCLUSIONS

This paper demonstrates the application of super twisting sliding mode controller applied on a direct drive single axis positioning table of a machine tool. Results showed that ST-SMC is effective in terms of chattering suppression. The smoothing or chattering suppression ability of ST-SMC proved its novelty against the common continuous approximation method. However, the formation of spikes at near zero velocity regions is an issue that needs to be solved. The estimation of velocity signal by using the Kalman-Bucy filter instead of numerical differentiation of measured position had also contributed in overall improvement of the tracking performance. For future works, friction forces compensation is to be considered and friction model feedforward is to be added into the control design scheme to attenuate the spikes that are formed during motion reversal. Generalised Maxwell-slip (GMS) model is an attractive friction model that effectively characterises friction behaviour in both pre-sliding and sliding regimes. This model can be used for feedforward application that would further enhance the tracking performances of the machine tools.

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