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RESEARCH ARTICLE

Numerical analysis of the thermal state of a cylindrical body cooled by an internal fluid flow

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ABSTRACT - Mechanical engineering has its own specifics when it comes to describing the thermal state of cylinders. The heating and cooling of bodies with a cylinder surface as their heat exchange area can be considered an important technical task that requires appropriate mathematical foundation. The purpose of this work is to construct a mathematical description of the thermal state of a thermally massive cylinder cooled by a liquid passing through a coaxially located channel. The study proposed a one-dimensional mathematical model for the numerical study of the thermal state of a thermally massive cylinder cooled by a liquid passing through a coaxially arranged channel inside the body under consideration. The mesoscopic modeling scale is the basis of the mathematical model, which employs the mathematical approach of Markov chains theory. The numerical evaluation of cooling scenarios in the flow and looping mode of the cooling fluid movement is carried out. The operability of the mathematical model was investigated by performing a series of numerical experiments. The numerical experiments with the model have shown the possibility of a qualitatively consistent analysis of possible cooling scenarios and their significant differences. The qualitative reliability of the results allows us to consider the proposed model as a reliable scientific basis for describing more complex cooling systems used, for example, in transport technologies and processes.

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1. INTRODUCTION

Heat and mass transfer phenomena are widespread in technology [1]. A widely employed heat exchange area in engineering practice is a cylindrical surface. In some cases, a liquid flow is organized around a cylindrical body cooling or heating it [2]. At the same time, a streamlined body does not necessarily represent a stable structure. For example, cylindrical particles of a particulate medium can dry, deform, and undergo chemical changes [3, 4]. Aslightly different scenario occurs when a cylindrical body is cooled or heated by an internal fluid flow passing through the body. A typical example of such task in industry is a tube heat exchanger. A characteristic feature of such tasks is that the cylindrical surface is in contact with liquids from two sides. Numerous calculation methods have been developed for the calculation and description of such systems [3, 5, 6].

In technology, there are a variety of tasks that do not coincide by their concept with those described above. In mechanical engineering, the tasks of describing the thermal state of cylinders have their own specifics [7, 8]. In other cases, for example, a significant object of research can be turbulent convective heat transfer in a long closed cylindrical tube filled with liquid metal [9]. In addition, a liquid does not always act as a cooling agent, for example, at high temperatures of the cooled body a heterogeneous cooling agent may form instead of water [10]. Thus, heating and cooling of bodies for which the heat exchange area is a cylinder surface can be considered as an important technical task, which is needed to appropriate mathematical basis. In the opinion of the authors, all mathematical approaches that are used to solve such problems can be divided into three main groups. According to the first concept a heated or cooled object can be considered as a whole with instant averaging of properties inside. This approach cannot be called informative, however, it often allows you to have sufficient accuracy for engineering tasks, especially when it comes to thermally thin bodies [11]. The opposite approach assumes the decomposition of the heat exchange region into conditionally infinitesimal representative volumes. This approach is highly informative, but it is also characterized by computational cumbersomeness and complexity of parametric identification of model solutions. To eliminate these shortcomings the so-called "coarse grain" simplifications are used [12, 13]. The introduction of such a domain as "coarse grain" actually means a transition to the mesoscale, and many authors point out the need to consider phenomena on such scale of modeling [14, 15]. However, the use of mesoscopic scale modeling involves a fairly wide range of mathematical approaches, among which one can distinguish the use of the mathematical apparatus of Markov chains [16], approximation using cellular automata [17, 18], discrete analogues of the Boltzmann equation [19] and other approaches.

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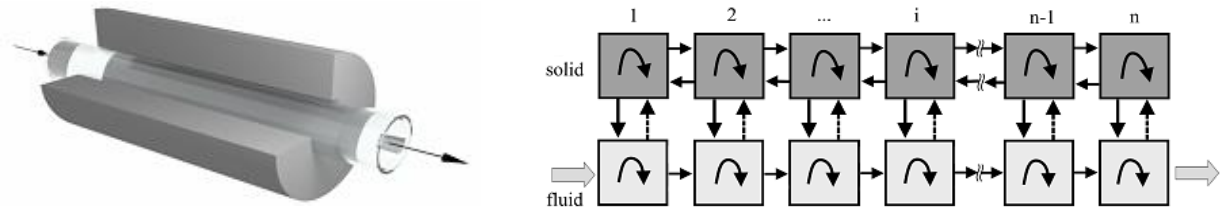
The mathematical simplicity and ease of implementation of Markov chains make them accessible for a wide range of applications in thermodynamics [20]. They are capable of effectively modeling complex systems where exact analytical solutions are hard to obtain. Examples include systems with numerous degrees of freedom or thermodynamics that are not in equilibrium [21, 22]. By changing transition probabilities, Markov chains can be modified to meet a variety of problems, resulting in customized simulations that fit specific needs [23]. And finally, accurate estimates of thermodynamic quantities are provided by Markov chain Monte Carlo methods when achieving thermodynamic equilibrium states in simulations [24]. For thermodynamic simulations, Markov chains can be scaled effectively. The law of large numbers and the central limit theorem ensure that averages computed from the Markov chain converge to true values as the system size increases [25, 26]. Nonetheless, they also pose challenges, particularly when it comes to convergence and autocorrelation. The effective application of thermodynamics requires balancing these factors [27]. There is a quite a few articles that describe the usage of this method. Dehling, Hoffmann, and Stuut [28], and some other authors successfully employed Markov chains in their thermodynamic process modeling. Faggionato, Gabrielli,. and Ribezzi Crivellari [29], as well as Bechhoefer [30] applied this method for non-equilibrium thermodynamics. The model by Khan and Elkamel [31]shows that the heat transfer coefficient depends on the void fraction and physical properties, consistent with experimental results. The referenced studies primarily focus on highly specialized applications. However, relatively simple models can be remarkably effective and sophisticated, particularly when they are designed to be universal and scalable. These models, despite their simplicity, can capture essential dynamics and provide significant insights across a wide range of systems. The universal nature of these models ensures their applicability to various contexts, while their scalability allows them to handle increasing system sizes efficiently. Consequently, simple yet robust models can offer substantial advantages, making them valuable tools in both theoretical research and practical applications.

The objective of this research is to develop a comprehensive mathematical framework that accurately describes the thermal state of a thermally massive cylinder. This cylinder is subjected to cooling by a liquid that flows through a channel positioned coaxially within the cylinder. By constructing this mathematical model, we aim to capture the intricate thermal dynamics and interactions between the cylinder and the cooling liquid, providing a detailed understanding of the system's thermal behavior under various conditions. This work not only seeks to enhance the theoretical understanding but also to offer practical insights that can be applied in engineering and industrial processes involving thermal management.

2. MATERIALS AND METHODS

In engineering practice, mathematical models are widely used as an essential tool to model heat and mass transfer phenomena in various systems. The model introduced here is a discrete one and is based on the stochastic concept of the Markov chain approach, but transfer probabilities will be related to physical parameters describing by the difference approximation of physical laws. The problem is solved in a one-dimensional formulation. The object of the study is a thermally massive cylinder cooled by a liquid passing through a coaxially arranged channel inside the body under consideration. The cylindrical shape of the body makes the task axisymmetric, which simplifies the construction of the mathematical model. It should be noted that any channel in the cooled body of arbitrary shape usually has a circular cross-section, therefore naturally forms a cylindrical area around itself. Therefore, the model proposed here is considered by us as an element for assembling models of cooling systems for larger systems.

According to it, the operating volume of the object is separated into n perfectly mixed cells of the length $\Delta x = L/n$ where L is the length of the cylinder. This decomposition of the object along the axis of the cylinder allows us to talk about the existence of two chains of cells, each of which is described from the standpoint of the mathematical apparatus of the theory of Markov chains. One chain of cells describes the thermal state of a solid (characterized by a vector Q_s, whose elements represent the heat in the cells). The second chain describes the thermal state of the fluid elements inside the cylinder (characterized by a vector, Q_f). Figure 1 schematically shows the (a) modeling object and (b) the calculated scheme of heat flows between the cells of the chains. Despite the fact that the cells in the diagram are rectangular, all surfaces and volumes are calculated as cylinder elements.



(a) (b)

Figure 1. (a) The conditional image of the basic modelling object and (b) the design scheme of the representation of the object in the mathematical model

The chain of cells belonging to the solid phase is shown by dark color; between the cells of this chain the arrows indicate the possible directions of heat transfer (conduction). The chain of cells shown in a lighter color refers to the flow of the cooling agent. It is assumed that it moves through the tube in the mode of plug flow, so the arrows point only to

the right (in the accepted direction of fluid movement). Arrows indicating heat transfer are provided between the corresponding cells of these two chains in the diagram. Heat transfer is also possible in two directions (from liquid to solid or vice versa). The diagram shows the case when a solid is cooled (the body gives off heat, so the arrows to the liquid are shown solid, and the transfer of heat in the opposite direction is possible, but not realized – the arrows are dotted). The transfer of heat from one chain of cells to another is described on the basis of a difference approximation of the heat transfer equation, and then the amount of heat exchanged by the corresponding cells per unit of time can be written by Newton's Law of Cooling [32] as:

$$Q_{sf}^{k} = -\alpha \cdot \left(T_s^k - T_f^k\right)(2\pi r \Delta x)\Delta t \tag{1}$$

where, Q_{sf} is the vector, each element of which determines the portion of heat transferred between the fluid and the solid in the corresponding cell, α is the heat transfer coefficient (it is the same for all cells since the steady motion of the fluid is considered), T_s and T_f are temperature vectors for solid and liquid phases, respectively, Δt is time duration, r is radius of the internal channel, k is time step number (integer analog of the process time).

The description of migrations of a key additive property (heat) along the modeling object is based on the mathematical basis of the theory of Markov chains [20]. The main balance equations in matrix form are written in as follows:

$$Q_f^{k+1} = P_f^k \cdot \left(Q_f^k + q_f - q_{out}^k\right) \tag{2}$$

$$Q_s^{k+1} = P_s \cdot Q_s^k \tag{3}$$

where, P_s and P_f are transition probability matrices for heat in solid and liquid phases, respectively, q_f and q_{out} are the vectors of heat source and heat sink for the liquid phase. The q_f contains one non-zero element that define the amount of heat that appears in the first cell of the chain due to the arrival of a liquid with the specified properties and the qout contains one non-zero element that define the amount of heat that disappears in the last cell of the chain due to the leaks of a liquid with the specified properties. The matrixes, P_s and P_f contain probabilities of heat transfer from all cells (all possible probabilities for i-th cells are contained in i-th column of the matrix).

Let us first consider the process of heat transfer along the chains of cells introduced to describe thermal conductivity. The structure of the transition matrix in this case corresponds to the structure of any transition matrices for processes of the "diffusion" type, namely, the matrix will be tridiagonal and symmetric:

$$P_s = \begin{bmatrix} 1-d & d & 0 & \cdots & 0 & 0 & 0 \\ d & 1-2\cdot d & d & \cdots & 0 & 0 & 0 \\ 0 & d & 1-2\cdot d & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1-2\cdot d & d & 0 \\ 0 & 0 & 0 & \cdots & d & 1-2\cdot d & d \\ 0 & 0 & 0 & \cdots & 0 & d & 1-d \end{bmatrix} \tag{4}$$

where, d is a dimensionless coefficient of thermal conductivity showing the fraction of heat (temperature) transferred due to thermal conductivity from a given cell to neighbouring cells in one transition Δt:

$$d = \left(\frac{\lambda_s}{c_s \cdot \rho_s}\right) \cdot \frac{\Delta t}{\Delta x^2} \tag{5}$$

where, the expression in parentheses on the right side of equality Eq. (5) is the dimensional coefficient of thermal conductivity (λ_s, c_s and ρ_s are the thermal conductivity, heat capacity and density of the material, respectively).

The structure of the transition matrix for a flow moving in the ideal plug mode has the following form:

$$P_s = \begin{bmatrix} 1-v & 0 & 0 & \cdots & 0 & 0 & 0 \\ v & 1-v & 0 & \cdots & 0 & 0 & 0 \\ 0 & v & 1-v & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1-v & 0 & 0 \\ 0 & 0 & 0 & \cdots & dv & 1-v & v \\ 0 & 0 & 0 & \cdots & 0 & v & 1-v \end{bmatrix} \tag{6}$$

where, v is the proportion of fluid displaced from the cell over time Δt depending on the average flow velocity V in the following form:

$$v = V \cdot \frac{\Delta t}{\Delta x} \tag{7}$$

where, V is liquid velocity in the internal channel.

In fact, Eqs. (1) - (7) describe the heat balance in an open system. The vectors, q_f and q_{out} describe the energy exchange of the system with the external environment, which supplies a liquid with a certain temperature, T_0 to the cylinder and

takes the same volume of liquid with temperature, T_n^k from the last cell with the number n of the chain. If the inner channel has a flow area, s then the vectors have the following form:

$$q_f = \begin{bmatrix} T_0 \cdot c_f \cdot \rho_f \cdot V \cdot s \cdot \Delta t \\ 0 \\ \cdots \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

$$q_{out}^k = \begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ T_n^k \cdot c_f \cdot \rho_f \cdot V \cdot s \cdot \Delta t \end{bmatrix} \tag{9}$$

Here, it is noted that the heat input vector, q_f for the flow mode of fluid motion does not depend on time (on the calculated step k), at the same time, the vector, q_{out} depends on time (at least during the transient process of interphase heat transfer). It is also noted, that the asymptote is quite obvious for the flow mode of fluid motion, since the cylinder eventually cools down to the temperature of the incoming fluid, T_0.

Next, let's turn to the description of a slightly more complex case when the amount of fluid is limited and it cools the cylinder in a circulating mode. In other words, the fluid that is removed from one side of the cylinder should get back into the cylinder from the other side after some delay time, θ. In fact only the balance ratio Eq. (2) needs to be corrected in the model, rewriting it to describe such a situation as follows:

$$Q_f^{k+1} = P_f^k \cdot \left(Q_f^k + q_{out}^{k-\tau} - q_{out}^k\right) \tag{10}$$

where, $\tau = [\theta/\Delta t]$ is the number of time steps Δt that fit into the delay interval θ (the symbol [ ] represents taking the integer part of the ratio).

3. RESULTS AND DISCUSSION

A series of numerical experiments were performed to investigate the operability of the mathematical model. A cylinder with 1 m length was represented by a chain of n = 100 cells of the same length Δx. The cylinder diameter was 0.5 m and along the axis of the cylinder there was a channel with a diameter of 0.1 m for the cooling fluid, which moved at a velocity of 0.01 $m \cdot s^{-1}$. The heat capacity of the fluid and the cylinder material were assumed to be 4000 $J \cdot kg^{-1} \cdot K^{-1}$ and 600 $J \cdot kg^{-1} \cdot K^{-1}$ respectively. The time step was assumed to be Δt=0.05 s for all numerical experiments. The heat transfer coefficient was assumed to be constant and equal to α = 10 $J \cdot s \cdot m^{-2} \cdot K^{-1}$. Numerical experiments were performed for the following two fundamentally different cooling modes: for the flow mode and for the circulation mode. The characteristics of the flow mode of cooling are shown in Figure 2.

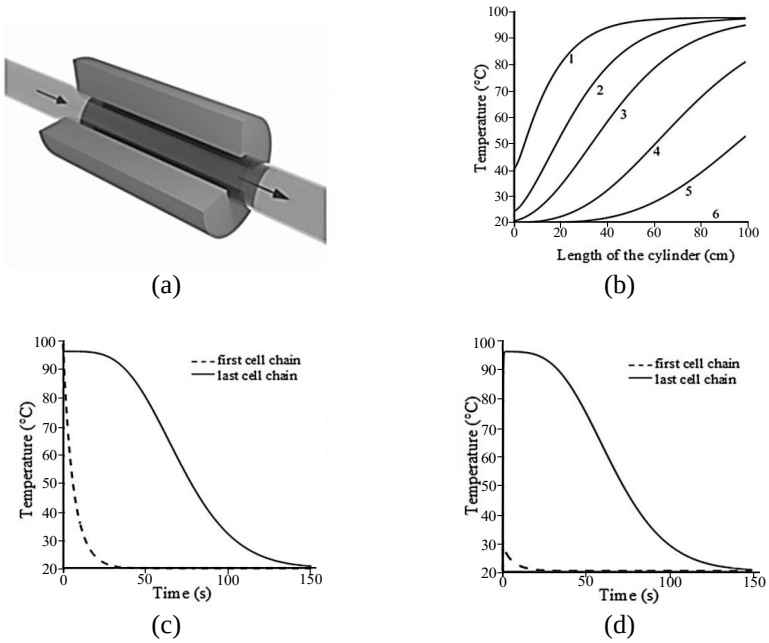


Figure 2. Results of numerical experiments for the flow mode of cooling: (a) conditional image of the process, (b) temperature distribution along the length of the cylinder at various points in time (1 – 8 min; 2 – 18 min; 3 – 30 min; 4 – 50 min; 5 – 75 min, 6 – 150 min), (c) temperature evolution of the cylinder in the first and last cells chain and (d) temperature evolution of the fluid in the first and last cells chain

The flow mode implies an essentially infinite supply of cooling agent with specific thermophysical properties, making the results somewhat predictable. It is evident that, over time, the entire cylinder will reach the temperature of the cooling agent. However, this mode allows for the observation of how different longitudinal sections of the cylinder are cooled by the fluid with varying intensities. This cooling process is schematically represented in Figure 2(a). Figure 2(b) depicts the temperature distribution along the cylinder's length at various observation time points, highlighting that sections of the cylinder closer to the cooling agent's supply point cool more rapidly and reach the agent's temperature faster. Furthermore, Figures 2(c-d) illustrates the differences in thermal state changes between the extreme cells of the chain –specifically, at the entry and exit points of the cylinder. From the solid's perspective, shown in Figure 2(c), the temperature gradient is more pronounced at the inlet compared to the outlet. In contrast, Figure 2(d) shows that the fluid's temperature consistently remains lower than that of the cylinder at all observation points until thermal equilibrium is achieved. This indicates that the cooling efficiency is highest near the inlet, diminishing progressively along the length of the cylinder. Figure 2(c-d) depicts the differences in temperature change between the extreme cells of the chain-specifically, at the entry and exit points of the cylinder. From the solid's perspective (see Figure 2(c)), the temperature gradient is more pronounced at the inlet compared to the outlet. From the fluid's perspective (see Figure 2(d)), the fluid temperature remains consistently lower than the cylinder temperature at all observation points until thermal equilibrium is achieved. The temperature distribution pattern observed in the flow mode is characterized by a progressive decrease from the inlet to the outlet. This pattern, which shows faster cooling near the inlet region, is evidenced by temperature profiles at different time intervals in Figure 2. This indicates that the cooling efficiency is highest near the inlet, diminishing progressively along the length of the cylinder. The temperature distribution in the flow mode indicates a gradual decline from the inlet to the outlet, with the inlet region cooling faster. This distribution can be seen in Figure 2 at different times.

It is also important with what time delay, the fluid element returns back to the cylindrical body. If this parameter is negligible (τ = 0), then the situation will be equivalent to cooling a cylinder immersed in a liquid that can average properties throughout its entire volume. In other words, the cylinder will cool evenly (this case is not interesting for analysis and we do not provide any graphs for it). The final temperature of the cylindrical body will be the same as if the cylinder was simply cooled in this volume of liquid, since the amount of circulating liquid in a closed circuit is limited by this volume and we did not introduce additional sources or discharges into the model. However at a significant value of this parameter (for example, case with τ = 15 min is illustrated in Figure 3), the cooling procedure can acquire an obviously oscillatory nature. The temperature profile along the cylinder in the circulating mode is highly influenced by the delay time and the initial temperature of the returning fluid. When the delay time is longer, the oscillatory nature of the temperature changes becomes particularly noticeable.

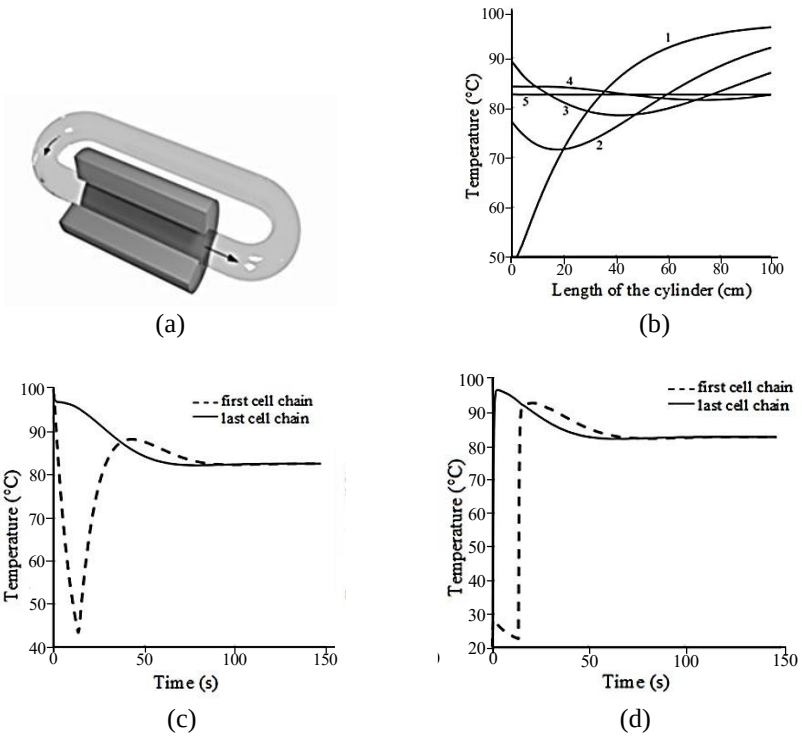


Figure 3. Results of numerical experiments for the flow mode of cooling: (a) conditional image of the process, (b) temperature distribution along the length of the cylinder at various points in time (1 – 8 min; 2 – 18 min; 3 – 30 min; 4 – 50 min; 5 – 75 min), (c) temperature evolution of the cylinder in the first (dotted) and last (solid line) cells chain and (d) temperature evolution of the fluid in the first and last cells chain

A general understanding of the cooling scheme is illustrated in Figure 3(a). Figure 3(b) shows the temperature distribution along the length of the cylinder for different observation time points. It can be seen (Figure 3(c)) that the temperature of the solid in the first cell (into which the cooling fluid falls) monotonously drops for the first 15 minutes of

the process (τ = 15 min). After that, the cylinder material in this cell begins to heat up, since the fluid that has completed a full revolution in the circulation system returns and has a higher temperature. The return of the liquid after the first cycle corresponds to a sharp jump in temperature, which is visible on the dotted graph in Figure 3(d). The temperature distribution of the material along the length of the cylinder is very diverse (see Figure 3(b)) and has a wavelike shape at some time intervals due to different cooling rates and fluid temperatures. The system's thermal behavior is critical due to the return of the heated fluid, which is highlighted by the sharp temperature jumps. The cooling characteristics of the flow mode and circulating mode are distinct. The flow mode results in a more uniform cooling along the length of the cylinder due to the continuous introduction of fresh cooling fluid. The thermal dynamics are more complex when using the circulating mode due to the reuse of the same fluid. The temperature profile in the circulation mode demonstrates how the cooling fluid's recirculation introduces periodic fluctuations in the thermal state of the cylinder. This is particularly evident when considering substantial delay times, which cause the temperature to oscillate before stabilizing. Such oscillatory patterns are critical for understanding the cooling dynamics in systems where fluid recirculation is involved. Based on the data obtained, it is possible to predict possible solutions to the problem of modeling and calculating heat transfer through a multilayer flat wall. For instance, the outer layers of the wall can be considered refractory and divided into several cells, and the inner layer, which occupies another amount cells, is fusible.

Analysis of the data obtained shows that, firstly, the temperature graph changes the angle of inclination when moving from cell to cell, and secondly, it will be possible to identify a phase-change zone when the temperature remains constant. The model can also display graphs of changes in the outer and inner wall surfaces, as well as the low-melting layer, and clearly highlight the melting period of the middle cells. The phase transition (melting) will begin from the outer boundary of the low-melting layer, spread deeper and gradually cover the entire layer. When the temperature of the heat source decreases, the reverse process will occur. The phase transition (solidification) will also begin from the outer boundary of the low-melting layer, since the temperature decreases faster on it, and spreads to the inner boundary, after which the entire wall will return to the solid state. The model is physically consistent and can work successfully not only at a constant temperature of the heat source, but also at an arbitrary temperature schedule, which makes it possible to calculate the kinetics of all thermophysical processes in the wall and can easily be generalized to the case of chemical reactions occurring in layers. Different scenarios can be explored through the adjustment of certain parameters. Although the model can be adapted to a new particulate system to explore the effects of varying delay times τ in the circulating cooling mode, the effect of varying the heat transfer coefficient, α, on the cooling process, and different velocities of the cooling fluid, it is not within the scope of the current study. However, it is obvious that the results will indicate that higher fluid velocities result in more rapid cooling, as the fluid can remove heat more efficiently. The temperature gradients will be steeper at higher velocities, especially near the inlet of the cooling fluid. A higher heat transfer coefficient will enhance the cooling rate. This is expected, as a larger α indicates more efficient heat transfer between the fluid and the solid, facilitating faster thermal equilibration. Also, longer delay times cause more pronounced temperature oscillations. This oscillatory behavior is due to the periodic return of the heated fluid, which temporarily increases the local temperature before subsequent cooling cycles. In practical cooling applications, optimizing fluid velocity, heat transfer coefficient, and delay times is highlighted by the extended analysis. The flow mode with a high fluid velocity and heat transfer coefficient is the preferred choice in applications that necessitate uniform cooling. However, the circulating mode could be more appropriate for systems where resource conservation is crucial, even though there is a possibility of temperature oscillations.

4. CONCLUSIONS

The study proposed a one-dimensional mathematical model for the numerical study of the thermal state of a cylindrical body, which is cooled by a fluid flow passing through a channel along its axis. The mathematical model is based on the mesoscopic scale modeling and uses the mathematical approach of the theory of Markov chains. Considering the object as a system with distributed spatial characteristics increases the reliability of the description of heat and mass transfer processes. At the same time, since the processes of heat and mass transfer themselves are actually described on the basis of difference approximations of conventional heat and mass transfer equations, therefore, the solutions obtained can be interpreted as a matrix formalization of the solution of systems of balance equations, and the obtained results have to be plausible. In this regard, in the opinion of the authors, the results of numerical experiments themselves deserve more attention. The numerical analysis of cooling scenarios in the flow mode and in the looping mode of the cooling fluid movement is carried out. The numerical experiments with the model have shown the possibility of a qualitatively consistent analysis of possible cooling scenarios and their significant differences. It is important that with sufficiently plausible physical constants of materials, the results of numerical experiments reveal the existence of a noticeable spatial heterogeneity in the distribution of the characteristics of these processes. This circumstance eloquently testifies that such systems need to be described using the software and algorithmic modeling tools, which should be simple and accessible in engineering practice, and on the other hand make it possible to track non-stationary local characteristics of heat and mass transfer processes. The qualitative reliability of the results allows us to consider the proposed model as a reliable scientific basis for describing more complex cooling systems used, for example, in transport technologies and processes.

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CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

AUTHORS CONTRIBUTION

V. S. Yessaulkov (conseptualization, methodology, data curation, investigation, original draft, review and editing)
A. V. Mitrofanov (methodology; validation; formal analysis, investigation, software, visualisation, original draft)
K. K. Abishev (validation, data curation; supervision).

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