

RESEARCH ARTICLE

Optimization of insulation thickness of walls and roofs using energy, exergy, economic and environmental analyses

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ABSTRACT - Buildings play an important role in consumption of energy and carbon dioxide emissions all over the world. The optimum thickness of each insulation material of wall and roof of residential buildings depending on energy, environment, economy and exergy was determined in this study. For this purpose, an optimization model was established based on four different criteria: energy, environment, economics, and exergy. A function was defined containing these four criteria. It has been seen from the results that the optimum insulation thickness of the wall and roof depends on the weight coefficients of the energy, environment, economic and exergy parameters and insulation material types. The results of the economic analysis indicate that the optimum insulation thickness of wall varies from 1.01 to 7.7 cm and the optimum thickness of roof varies from 3.25 to 6.7 cm for XPS, EPS and GW insulation materials. According to the results of the environmental analysis, the optimum thicknesses of wall for different insulation materials are 6.5, 8.6, 9.4, and 9.55 cm and optimum insulation thicknesses of roof are 7.55, 8.1 and 8.2 cm, respectively. The effect of economic and energy parameters on the optimum thickness of the wall and roof for the three insulation materials was investigated using the sensitivity analysis method. It was seen from the results that impacts of interest rate, inflation rate, electricity cost, fuel cost, insulation material cost, heating and cooling degree-days on the optimum insulation thickness of wall and roof and 4E optimization function were different.

ARTICLE HISTORY
 Received : 16th June. 2023
 Revised : 19th Sept. 2023
 Accepted : 27th Dec. 2023
 Published : 04th Apr. 2024
KEYWORDS
*Optimum insulation
 Thickness
 4E analysis
 Optimization model
 CO₂ emissions
 Exergy*

1. INTRODUCTION

In recent years, energy consumption in buildings has increased significantly due to rapid population growth in the world, the increase in energy consumption of technological devices and global climate change [1]. As energy consumption in developing countries such as Turkey increase, environmental pollution and greenhouse gas emissions are increasing every year. Residential buildings are very important in reducing energy needs and greenhouse gas emissions. Approximately 31% of the energy requirement in Turkey is related to residential buildings. The thermal insulation of the external walls in building is very important to decrease the environmental effects and energy costs caused by energy consumption. The thickness of the insulation material in building should be selected taking into account the average ambient temperature of the region, the thermal conductivity of the insulation material and its price. Dombayci et al. [2] calculated the optimum insulation thickness of the external wall for Ankara, İzmir, Kars and Trabzon by using a thermo-economic method that takes into account the effect of inflation and interest rate. Özel et al. [3] are determined the optimum thickness of insulation material in buildings using environmental and life cycle cost analysis. They are calculated the CO₂ emissions and environmental impacts as a result of the fuel consumption of the system. Jie et al. [4] developed an optimization model covering primary energy consumption, global cost and pollutant emissions and they determined the optimum insulation thickness of wall and roof of existing buildings. There are many studies in which the optimum insulation thickness for exterior walls of buildings to save energy and reduce CO₂ emissions is calculated [5–8].

The Life Cycle Assessment (LCA) method is used to identify environmental impacts of produces along their life cycle. The parameters affected on the LCA of residential buildings: the climate, thicknesses of insulation materials, type of insulation material used, energy sources and the heating and cooling system. LCA is applied in many researches to evaluate the impact of different insulation materials [9-13]. Ashouri et al. [14] developed a new method. In this method, they combined exergy analysis with life cycle assessment. Rad and Fallahi [15] used an analysis based on three different criteria, including energy, economy and environment to find the optimum insulation material thickness. Tettey et al. [16] researched the impacts of different insulation materials on CO₂ emissions and primary energy for residential buildings. Their results showed that there was a decrease of about 6–7% in primary energy and 6–8 % in CO₂ emission in the optimum versions. Erdem et al. [17] calculated the values of heating degree days (HDD) and cooling degree days (CDD) for Turkey by using estimated temperature values. They applied an artificial neural network and an adaptive network-based fuzzy inference system for the estimation of temperature values. Gökhan [18] calculated the thermal performance and the amount of condensation on the walls of an iron profile building under steady-state conditions. In the study, the calculations in a real wall model and three different wall model scenarios are made.

In the present study, the optimum thickness of each insulation material of wall and roof of residential buildings depending on energy, environment, economy and exergy is determined. For this purpose, an optimization model is established based on four different criteria: energy, environment, economics, and exergy. The optimum thicknesses of the insulation material calculated from these four analyzes are different from each other. Therefore, it is necessary to consider these four criteria simultaneously for a more comprehensive analysis. For this purpose, a energy, environment, economics, and exergy function (4EF) was described as a function containing these four criteria. In the first step, the four parameters were calculated as total energy savings, total energy cost, total CO₂ emission and total exergy loss. In the second step, the 4EF function is calculated by combining these four parameters with equal weight coefficients.

2. MATERIALS AND METHODS

The gross area of studied building is about 140 m² per story, three stories, and two dwellings per story. Each dwelling unit has three bedrooms and a living room and a bathroom. Figure 1 presents the detailed floor plan of building in Elazığ. The annual heating and cooling degree-days of Elazığ is 2653 and 337. Thermal conductivities of the exterior wall structure components are given in Table 1. The exterior wall structure of the building consists of 20 cm thick brick, 2 cm thick interior plaster and exterior plaster. Roof of building is a compound structure consisting of chipping, felt, insulation material, concrete. In this study, expanded polystyrene (EPS), glasswool (GW) and extruded polystyrene (XPS) are selected as insulation materials. The physical properties of each material in the wall structures are given in Table 2.

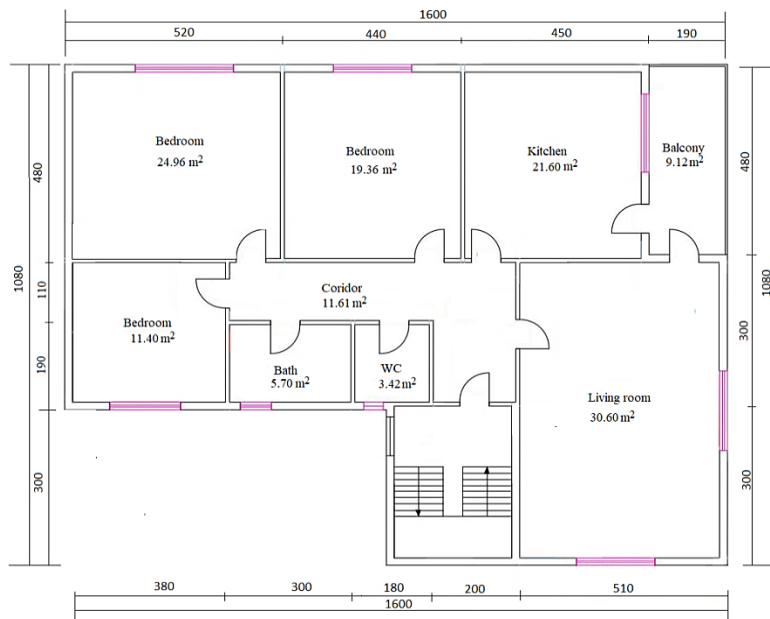


Figure 1. The floor plan of the studied building

Table 1. Layers and thermal conductivities (W/mK) of the building envelope components

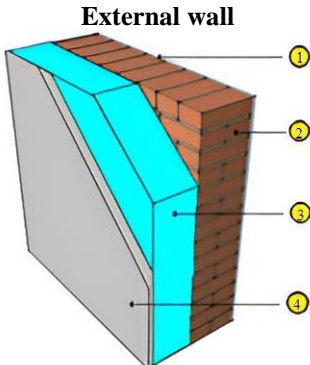
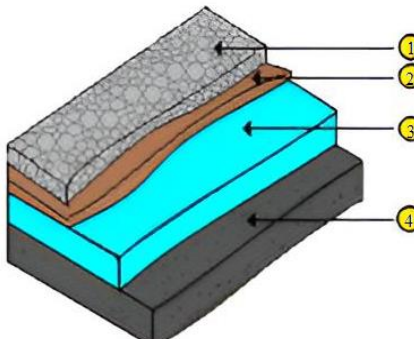
Items	Layers	Thickness (m)	Conductivity (W/mK)	Insulation cost (\$/m ³)
	1-Internal plaster	0.02	0.87	
	2-Brick	0.2	0.45	
	3-Insulation material	*	0.031-0.040	
	4-External plaster	0.03	0.87	

Table 1. (cont.)

Layers	Thickness (m)	Conductivity (W/mK)	Insulation cost (\$/m ³)
Roof			
	1-Chipping	0.012	1.436
	2-Felt	0.010	0.190
	3-Insulation material	*	0.031-0.040
	4-Concrete	0.1	0.173
Insulation Materials			
Extruded polystyrene	*	0.031	180
Expanded polystyrene	*	0.039	120
Glass wool	*	0.040	75

* The optimum thickness of insulation material which is found by the life cycle cost analysis

Table 2. The parameters used in calculations

Parameter	Value
Elazığ	
HDD	2653
CDD	337
Fuel Natural gas (Heating)	
C_f	0.332 \$/m ³
H_u	34.526 x10 ⁶ J/m ³
η_s	0.93
f_H	0.181 kgCO ₂ /kWh
Electricity (Cooling)	
C_e	0.475 \$/kWh
f_c	0.588 kgCO ₂ /kWh
COP	2.5
Insulation Material	
Extruded polystyrene, f_{ins}	4.42 kgCO ₂ /kg
Expanded polystyrene, f_{ins}	2.35 kgCO ₂ /kg
Glass wool, f_{ins}	1.16 kgCO ₂ /kg
Interest rate, i	24 %
Inflation rate, g	21 %
Lifetime, N	10

In the present work, the optimum thickness of each insulation material of wall and roof of residential buildings is established. For this purpose, an optimization model is established depending on energy, environment, economy and exergy.

2.1 Energy Analysis

Heat losses from the roof and exterior wall create most of the heat losses in buildings. The life cycle analysis method is one of the methods used to determine the annual heating and cooling energy demands of buildings. Heat transfer rate from a unit area of building wall and roof can be calculated as:

$$q_H = 86400 \times U \times HDD \tag{1}$$

$$q_C = 86400 \times U \times CDD \tag{2}$$

The total heat transfer coefficient for an insulated wall and roof is written by:

$$U = \frac{1}{1/h_i + R_w + R_{ins} + 1/h_o} \quad (3)$$

The thermal resistances of the insulation of wall and roof are calculated as:

$$R_{ins,w} = \frac{x_{ins}}{k_{ins}}, R_{ins,f} = \frac{y_{ins}}{k_{ins}} \quad (4)$$

Total thermal resistance of the non-insulated wall and roof $R_{w,t}$ is:

$$R_{w,t} = 1/h_i + R_w + 1/h_o \quad (5)$$

The pre-insulation annual required energy for heating and cooling are determined as:

$$E_{AH,pre-ins} = \frac{86400 HDD}{R_{w,t} H_u \eta_s} \quad (6)$$

$$E_{AC,post-ins} = \frac{86400 CDD}{R_{w,t} COP} \quad (7)$$

The post-insulation required energy of annual heating and cooling are determined [19]:

$$E_{AH,post-ins} = \frac{86400 HDD}{(R_{w,t} + R_{ins}) H_u \eta_s} \quad (8)$$

$$E_{AC,post-ins} = \frac{86400 CDD}{(R_{w,t} + R_{ins}) COP} \quad (9)$$

The TESR is a rate of total heating and cooling energy savings to the pre-insulation total heating and cooling energy requirement and stated as:

$$TESR = \frac{(E_{AH,pre-ins} + E_{AC,pre-ins}) - (E_{AH,post-ins} + E_{AC,post-ins})}{(E_{AH,pre-ins} + E_{AC,pre-ins})} = 1 - \frac{(E_{AH,post-ins} + E_{AC,post-ins})}{(E_{AH,pre-ins} + E_{AC,pre-ins})} \quad (10)$$

2.2 Economic Analysis

The post-insulation annual heating and cooling energy cost are:

$$C_{AH,post-ins} = \frac{86400 HDD c_f}{(R_{w,t} + R_{ins}) H_u \eta_s} \quad (11)$$

$$C_{AC,post-ins} = \frac{86400 CDD c_e}{(R_{w,t} + R_{ins}) COP} \quad (12)$$

The pre-insulation annual energy cost heating and cooling are determined as:

$$C_{AH,pre-ins} = \frac{86400 HDD c_f}{R_{w,t} H_u \eta_s} \quad (13)$$

$$C_{AC,pre-ins} = \frac{86400 CDD c_e}{R_{w,t} COP} \quad (14)$$

Total energy costs of heating and cooling are calculated as:

$$C_{H,post-ins} = C_{AH,post-ins} PWF + C_{ins} = C_{AH,post-ins} PWF + (c_{insw}x + c_{insf}y) \quad (15)$$

$$C_{C,post,ins} = C_{AC,post-ins} PWF + C_{ins} = C_{AC,post-ins} PWF + (c_{insw}x + c_{insf}y) \quad (16)$$

The pre-insulation total energy cost of heating and cooling can be determined as:

$$C_{H,pre-ins} = C_{AH,pre-ins} PWF \quad (17)$$

$$C_{C,pre-ins} = C_{AC,pre-ins} PWF \quad (18)$$

The present worth factor (PWF) is determined as:

$$r = \frac{i - g}{1 + g} \tag{19}$$

$$PWF = \frac{(1 + r)^N - 1}{r (1 + r)^N} \tag{20}$$

The ECSR is a rate of the total energy cost of heating and cooling to the pre-insulation total energy cost of heating and cooling and shown as follows:

$$ECSR = \frac{(C_{H,pre-ins} + C_{C,pre-ins}) - (C_{H,post-ins} + C_{C,post-ins})}{(C_{H,pre-ins} + C_{C,pre-ins})} = 1 - \frac{(C_{H,post-ins} + C_{C,post-ins})}{(C_{H,pre-ins} + C_{C,pre-ins})} \tag{21}$$

2.3 Environmental Analysis

The post-insulation annual heating and cooling CO₂ emissions can be determined by [6]:

$$EM_{CO_2,H,post-ins} = \frac{q_{H,post-ins} f_H}{\eta_s} = \frac{86400 HDD f_H}{(R_{w,t} + R_{ins}) \eta_s} \tag{22}$$

$$EM_{CO_2,C,post-ins} = \frac{q_{C,post-ins} f_C}{COP} = \frac{86400 CDD f_C}{(R_{w,t} + R_{ins}) COP} \tag{23}$$

The annual embodied CO₂ emissions of insulation material can be defined by:

$$EM_{ins} = \frac{\rho x_{ins} f_{ins}}{N} \tag{24}$$

The post-insulation total annual CO₂ emissions are calculated as:

$$EM_{tot,post-ins} = EM_{CO_2,H,post-ins} + EM_{CO_2,C,post-ins} + EM_{ins} \tag{25}$$

The annual embodied CO₂ emissions of heating and cooling can be defined by:

$$EM_{CO_2,H,pre-ins} = \frac{q_{H,pre-ins} f_H}{\eta_s} = \frac{86400 HDD f_H}{R_{w,t} \eta_s} \tag{26}$$

$$EM_{CO_2,C,pre-ins} = \frac{q_{C,pre-ins} f_C}{COP} = \frac{86400 CDD f_C}{R_{w,t} COP} \tag{27}$$

The pre-insulation total annual embodied CO₂ emissions are calculated as:

$$EM_{tot,pre-ins} = EM_{CO_2,H,pre-ins} + EM_{CO_2,C,pre-ins} \tag{28}$$

The CERR is described as the rate of the total annual embodied CO₂ emission declines to the pre-insulation total annual embodied CO₂ emissions and defined as follows:

$$\begin{aligned} CERR &= \frac{(EM_{tot,pre-ins} + EM_{tot,pre-ins}) - (EM_{tot,post-ins} + EM_{tot,post-ins})}{(EM_{tot,pre-ins} + EM_{tot,pre-ins})} \\ &= 1 - \frac{(EM_{tot,post-ins} + EM_{tot,post-ins})}{(EM_{tot,pre-ins} + EM_{tot,pre-ins})} \end{aligned} \tag{29}$$

2.3 Exergy Analysis

The exergy loss in buildings during the heating season is due to the heat transferred from the inside to the outside of the walls [20]. The post-insulation annual exergy losses due to heat transfer, $Ex_{loss,Q}$, are determined by [14]:

$$Ex_{loss,Q,post-ins} = \frac{86.4 HDD}{(R_{w,t} + R_{ins}) \eta_s} \left(1 - \frac{T_0}{T_{rt}}\right) \tag{30}$$

$$Ex_{loss,Q,post-ins} = \frac{86.4 CDD}{(R_{w,t} + R_{ins}) COP} \left(1 - \frac{T_0}{T_{rt}}\right) \tag{31}$$

The pre-insulation annual exergy losses, $Ex_{loss,Q}$, are determined as:

$$Ex_{loss,QH,pre-ins} = \frac{86.4 HDD}{R_{w,t} \eta_s} \left(1 - \frac{T_0}{T_{rt}}\right) \tag{32}$$

$$Ex_{loss,QC,pre-ins} = \frac{86.4 \text{ CDD}}{R_{w,t} \text{ COP}} \left(1 - \frac{T_0}{T_{rt}}\right) \quad (33)$$

The net savings in exergy loss depending on use of insulation material is:

$$SE = Ex_{loss,Q,pre-ins} - Ex_{loss,Q,post-ins} \quad (34)$$

The EXSR is defined as the ratio of the net saving in exergy loss to the pre-insulation annual exergy losses depending on heat transfer and expressed as follows:

$$EXSR = \frac{(Ex_{loss,QH,pre-ins} + Ex_{loss,QC,pre-ins}) - (Ex_{loss,QH,post-ins} + Ex_{loss,QC,post-ins})}{(Ex_{loss,QH,pre-ins} + Ex_{loss,QC,pre-ins})} \quad (35)$$

$$EXSR = 1 - \frac{(Ex_{loss,QH,post-ins} + Ex_{loss,QC,post-ins})}{(Ex_{loss,QH,pre-ins} + Ex_{loss,QC,pre-ins})} \quad (36)$$

2.4 Optimization Model

TESR, ECSR, CERR and EXSR which were presented in the previous sections are represented the amount of the energy, environment, economic and exergy of each material for wall and roof insulation. The optimum insulation thicknesses of roof and wall by using each separate analysis can be calculated. However, it is necessary to consider these four parameters simultaneously for a more comprehensive analysis. For this purpose, a 4EF function was defined as a function containing these four parameters and expressed as follows:

$$4EF = \lambda_1 \text{TESR} + \lambda_2 \text{ECSR} + \lambda_3 \text{CERR} + \lambda_4 \text{EXSR} \quad (37)$$

where, $0 \leq \lambda_1, \lambda_2, \lambda_3, \lambda_4 \leq 1$ and $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$.

4EF function are taken objective function and the optimum value of insulation thickness is defined by maximizing Eq. (37). In this study, MATLAB optimization toolbox was used for this optimization problem.

3. RESULTS AND DISCUSSION

The optimum thicknesses of insulation material for the external wall (ITW) and roof (ITR) are found by solving the above optimization model. Figure 2 shows the variations of annual total energy requirement, energy costs, CO₂ emissions and exergy losses according to different insulation thickness of wall and roof for three insulation materials. The heat loss from external wall and roof decreases with the insulation thickness increases. The total energy requirement decreases with increase of insulation thickness until it reaches a minimum point and it increases again after a minimum value. This minimum point shows optimum insulation thickness. It can be seen from the figures that there is an optimum point where the total energy requirement, energy costs, CO₂ emissions and exergy losses reach minimum values for each insulation type. The energy consumption in the building are decreased with the increase of the insulation thickness and therefore the CO₂ emissions as depending on the total energy consumption of the building are reduced. The insulation cost increases linearly with insulation thickness, while operating costs corresponding to heating and cooling decreases. The optimum insulation thickness for external wall is minimum value of total cost which equals the summation of the insulation cost and operating cost. The external wall insulated with GW at the optimum thickness has the least total energy cost among other insulation materials. It seen from the figures that the exergy losses reduce with the increase in the insulation thickness and it reaches the minimum value at the optimum point.

The variations of TESR according to the insulation thickness of wall and roof for three insulation materials are shown in Figure 3. Total heating and cooling energy savings increase with the increase of the insulation thickness. For the three selected insulation materials, TESR increases with insulation thickness at insulation thicknesses less than the optimum value. TESR reduces with rise in insulation thickness, with an insulation thickness greater than the optimum value. As can be seen in Figure 3(c), the optimum values of insulation thickness of wall and roof for GW insulation material are 71 mm and 58.50 mm, respectively. TESR reaches maximum value, when the insulation thickness of wall and roof are 65 mm and 55 mm for XPS insulation material, respectively. The variations of ECSR according to the insulation thickness of wall and roof for three different insulation materials are shown in Figure 4. According to the results of the economic analysis, the optimum thicknesses of wall for XPS, EPS and Glasswool insulation materials were 55.5, 79 and 10.15 mm, respectively. As is shown in Figure 4, the optimum thicknesses of roof for XPS, EPS and Glasswool insulation materials were 32.5, 48.5 and 67 mm, respectively.

The variations of CERR according to insulation thickness of walls and roofs for different three insulation materials is shown in Figure 5. The total CO₂ emissions decreases with increase of insulation thickness until it reaches a minimum point and it increases again after a minimum value. It is obtained that the total CO₂ emission of the building where exterior walls and roofs were insulated with GW were highest compared to the XPS and EPS insulation materials. The variations of EXSR according to insulation thickness of wall and roof for three insulation materials is shown in Figure 6. The optimum thicknesses of wall for different insulation materials, including XPS, EPS, and Glasswool are 6.5, 8.6, 9.4, and 9.55 cm, optimum insulation thicknesses of roof are 7.55, 8.1 and 8.2 cm, respectively.

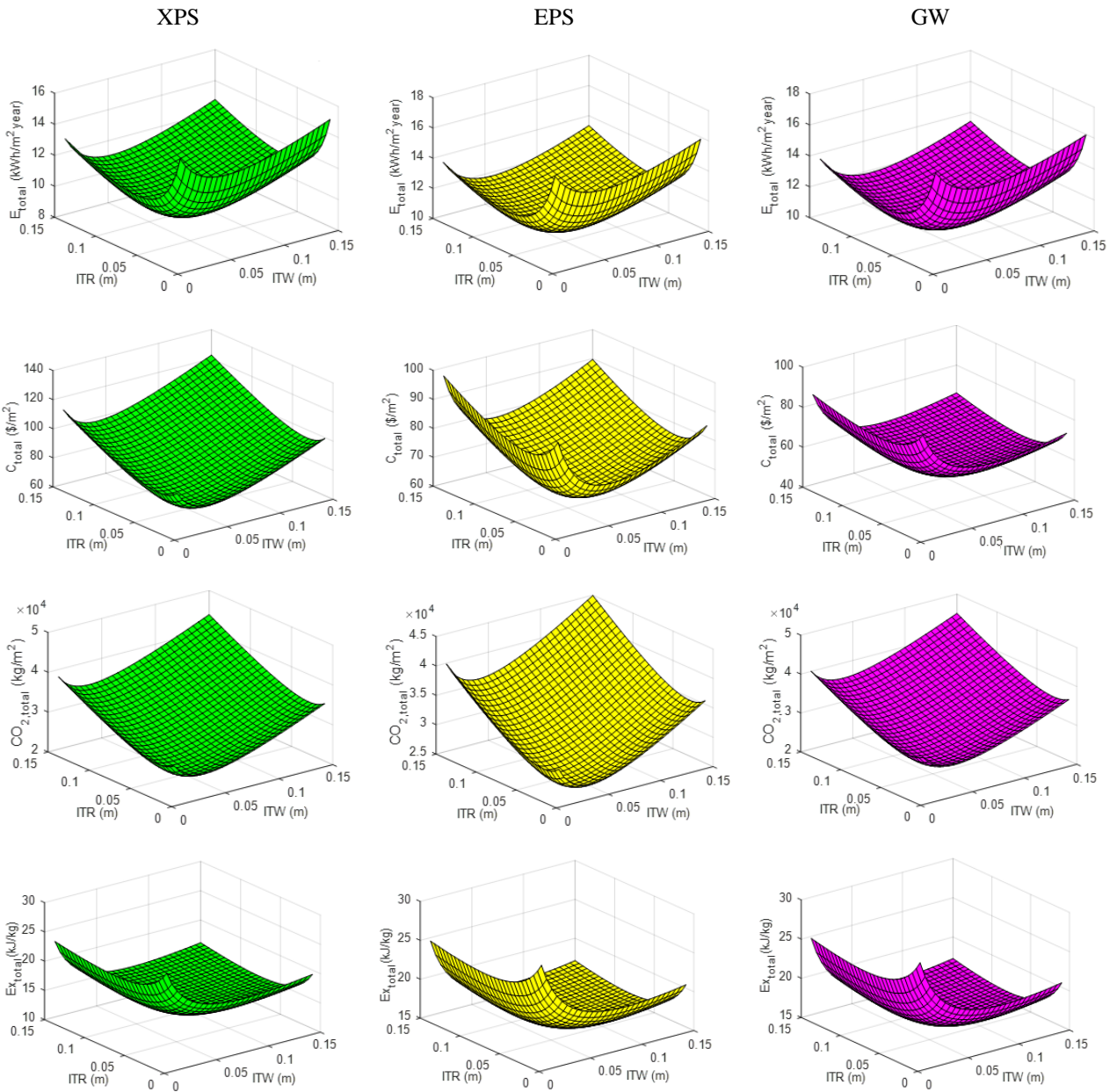


Figure 2. Variations of annual total energy requirement, energy costs, CO₂ emissions and exergy losses versus different insulation thickness of walls and roofs

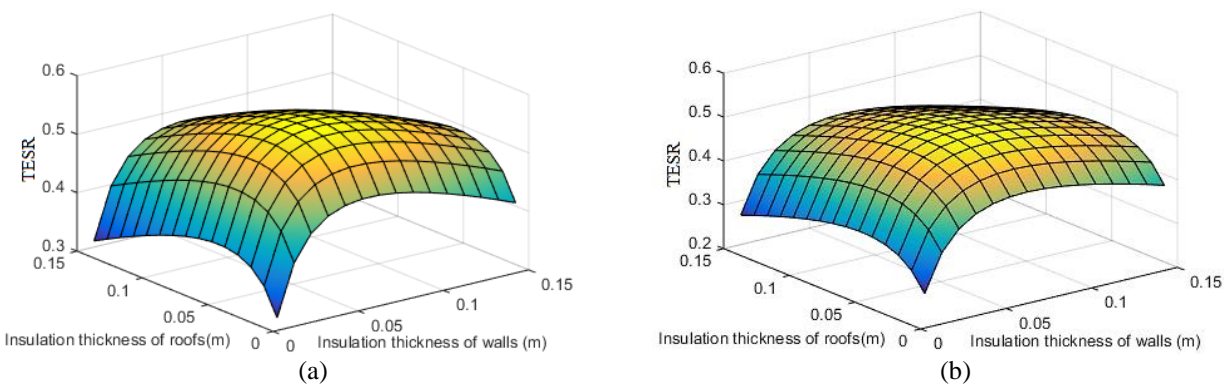


Figure 3. Variations of TESR versus different insulation thickness of walls and roofs for different insulation materials: (a) XPS, (b) EPS and (c) GW

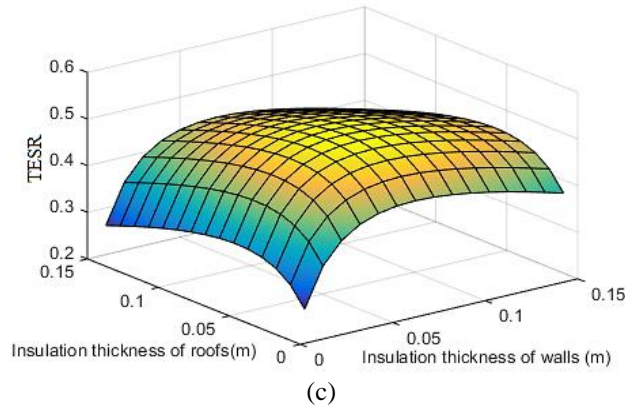


Figure 3. (cont.)

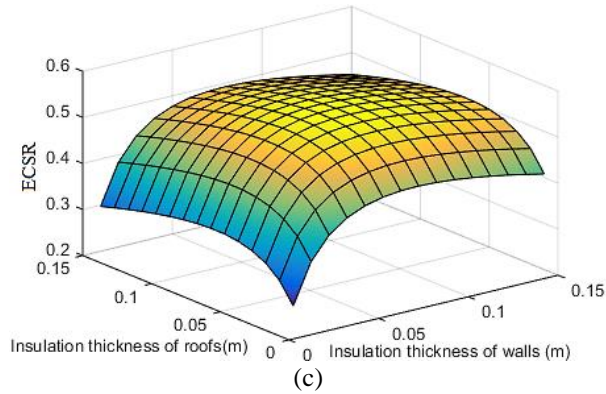
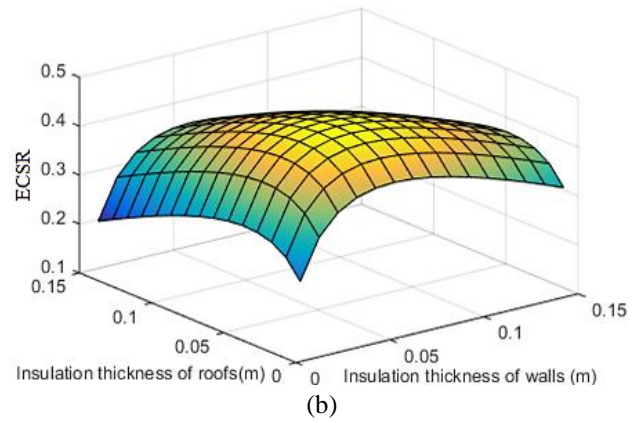
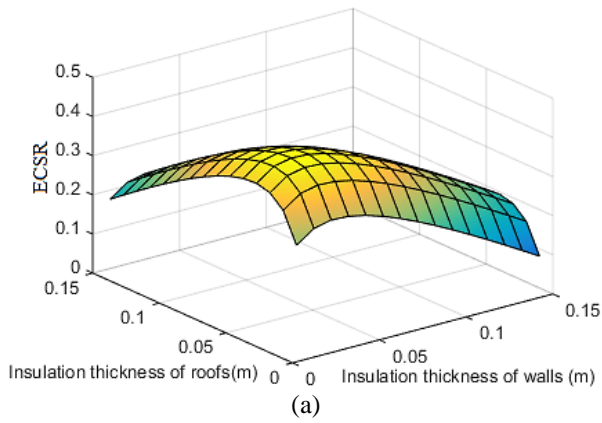


Figure 4. Variations of ECSR versus different insulation thickness of walls and roofs for different insulation materials: (a) XPS, (b) EPS and (c) GW

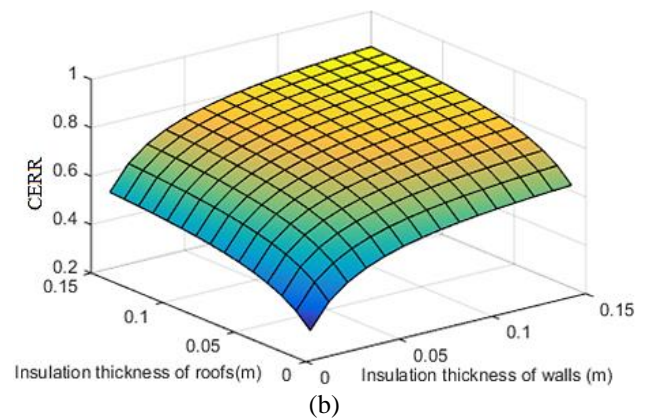
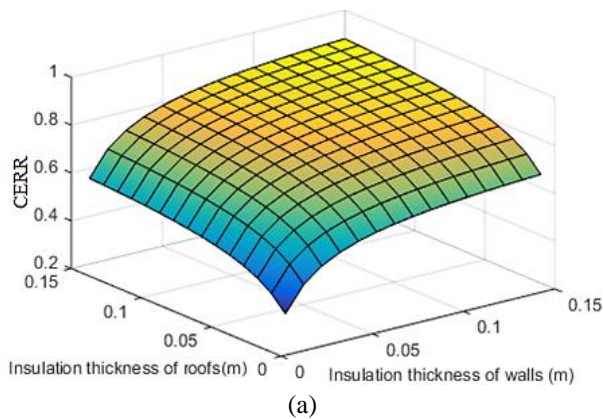


Figure 5. Variations of CERR versus different insulation thickness of walls and roofs for different insulation materials (a) XPS, (b) EPS and (c) GW

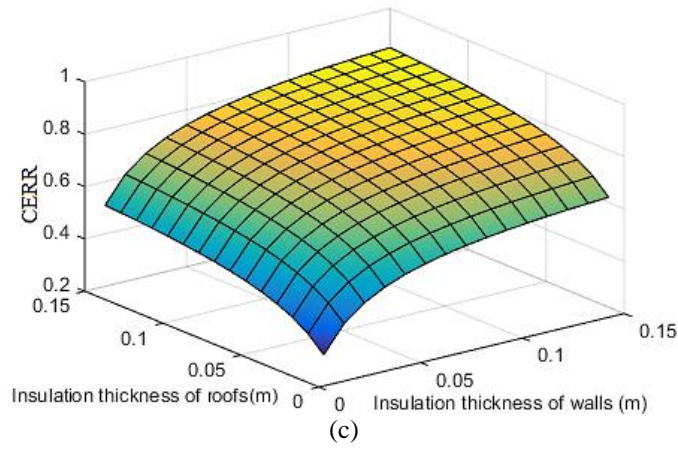


Figure 5. (cont.)

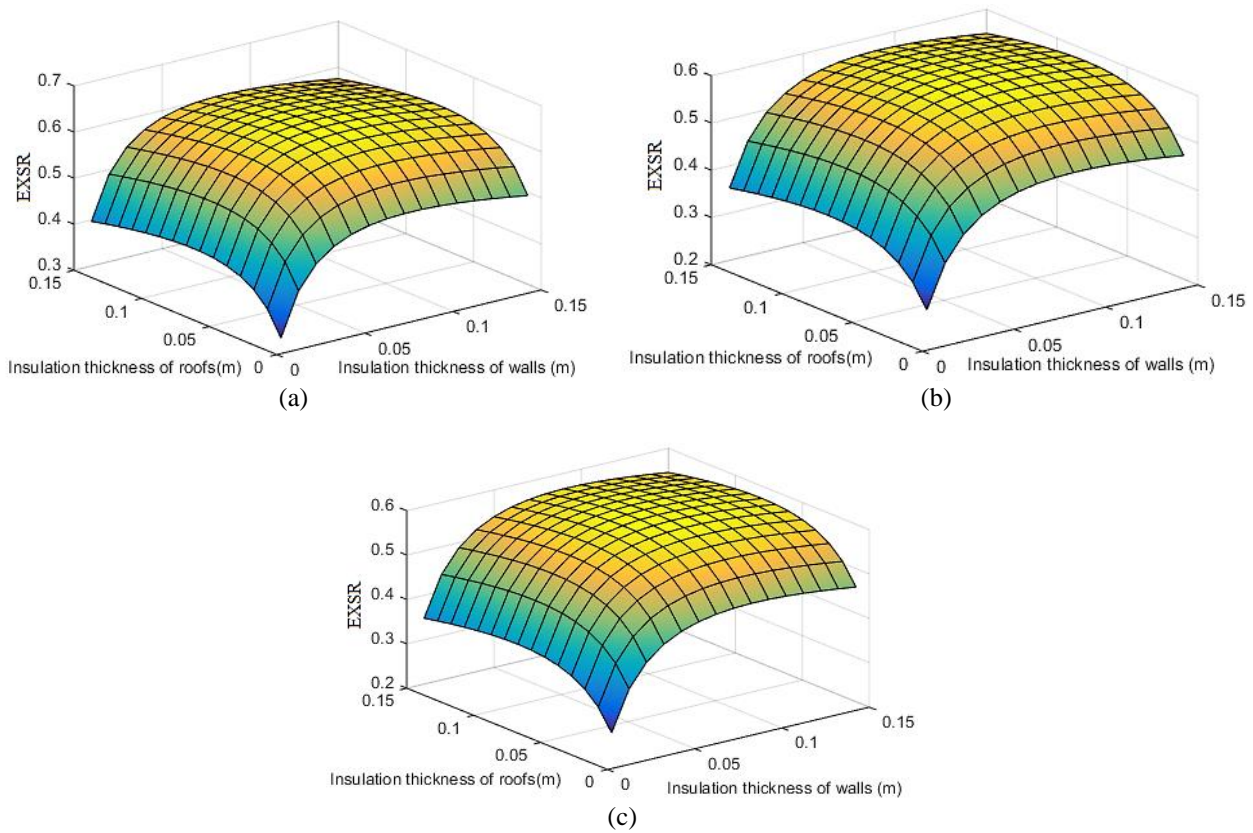


Figure 6. Variations of EXSR versus different insulation thickness of walls and roofs for different insulation materials: (a) XPS, (b) EPS and (c) GW

Figure 7 shows the variations of 4EF function according to insulation thickness of walls and roofs for different insulation materials, when $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are equal to $1/4$. As is shown in the three graphics, the maximum values of 4EF function for walls are obtained as 55, 64 and 68.5 cm for XPS, EPS and GW, respectively. When the insulation thicknesses are 42, 48.5 and 52 cm, the maximum values of this 4EF function for the roof are calculated for XPS, EPS and GW insulation materials, respectively.

The optimization results for selected three insulation materials are shown in Tables 3-5. When TESR, ECSR, CERR and EXSR are equally considered (i.e., $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1/4$), 4EF function is 0.6116, 0.6057 and 0.6348 for XPS, EPS and GW insulation materials, respectively. It is obtained that the optimum thicknesses of wall are 55, 62.5 and 67.5 cm, while the optimum thicknesses of roof are 42.5, 47.5 and 52.5 cm for XPS, EPS and GW insulation materials, respectively. When $\lambda_1=1$, the values of the 4EF function are obtained as maximum. 4EF functions are 0.7472, 0.7146 and 0.7095 for XPS, EPS and GW insulation materials, respectively. The optimum thicknesses of wall and roof are 65 and 55 cm for XPS insulation material, while the optimum thicknesses of wall and roof are 70 and 57.5 cm for EPS and GW insulation materials, respectively.

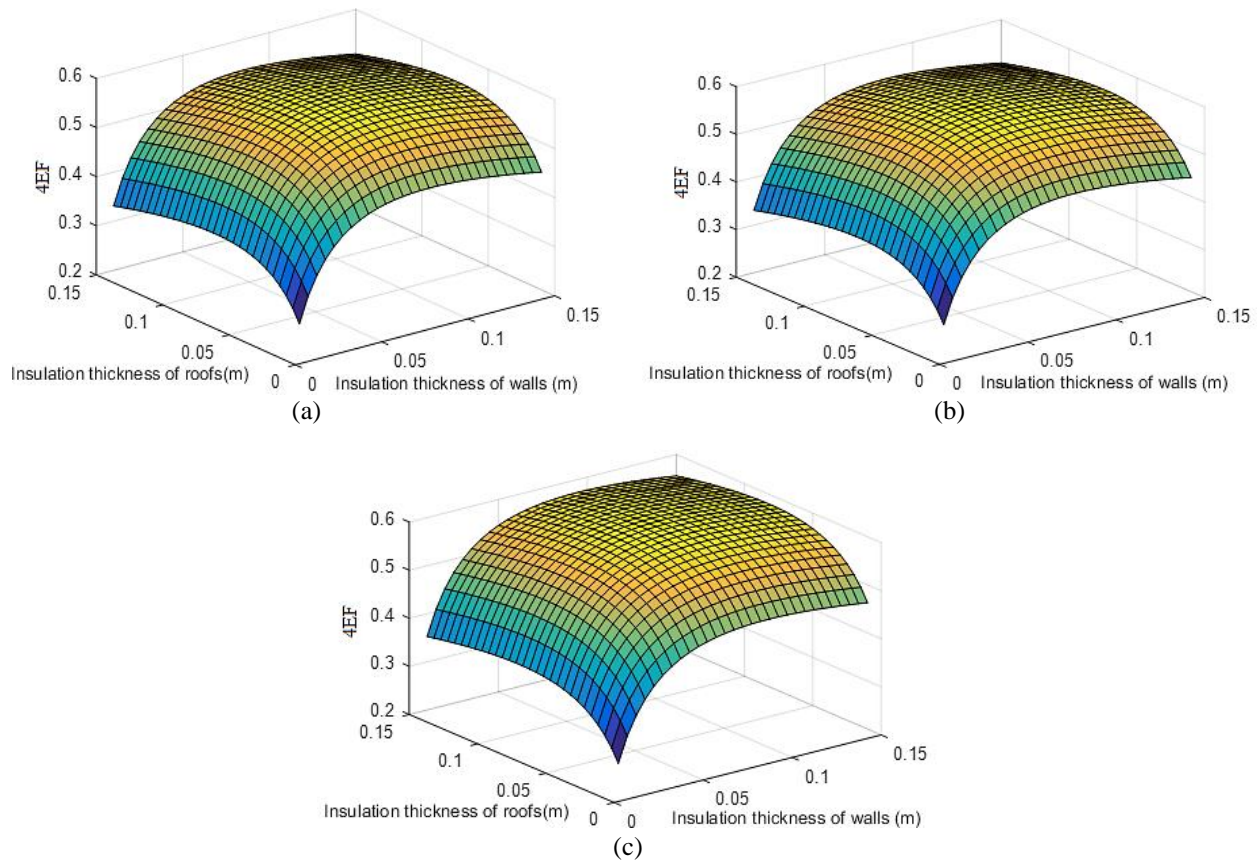


Figure 7. Variations of 4EF function versus different insulation thickness of walls and roofs for for different insulation materials: (a) XPS, (b) EPS and (c) GW

Table 3. Optimization results for XPS insulation material

λ_1	λ_2	λ_3	λ_4	$x_{opt,w}$	$x_{opt,f}$	4EF	TESR	ECSR	CERR	EXSR	E_{tot}	C_{tot}
1/4	1/4	1/4	1/4	0.0625	0.0475	0.6057	0.6842	0.4655	0.6840	0.5890	6.77	64.54
1	0	0	0	0.070	0.0575	0.7146	0.7146	0.4612	0.7145	0.6343	6.12	65.06
0	1	0	0	0.0775	0.0475	0.4592	0.7077	0.4592	0.7075	0.5890	6.27	65.30
0	0	1	0	0.040	0.030	0.5827	0.5828	0.4436	0.5827	0.4751	8.95	67.19
0	0	0	1	0.0925	0.080	0.7070	0.7712	0.4283	0.7710	0.7070	4.91	69.03
0.9	0.1	0	0	0.070	0.0575	0.6893	0.7146	0.4612	0.7145	0.6343	6.12	65.06
0.9	0	0.1	0	0.065	0.0525	0.6980	0.6980	0.4645	0.6979	0.6129	6.48	64.66
0.9	0	0	0.1	0.070	0.060	0.7110	0.7185	0.4601	0.7183	0.6441	6.04	65.20
0.7	0.3	0	0	0.0725	0.055	0.6383	0.7143	0.4609	0.7142	0.6239	6.13	65.09
0.7	0.2	0.1	0	0.0675	0.050	0.6510	0.6977	0.4642	0.6976	0.6013	6.48	64.70
0.7	0.1	0.1	0.1	0.0675	0.0525	0.6695	0.7023	0.4637	0.7021	0.6129	6.39	64.75
0.7	0.1	0.2	0	0.0625	0.0475	0.6623	0.6842	0.4655	0.6840	0.5890	6.77	64.54
0.7	0	0	0.3	0.0725	0.0625	0.7041	0.7259	0.4576	0.7258	0.6534	5.88	65.50
0.7	0	0.1	0.2	0.0675	0.055	0.6900	0.7066	0.4631	0.7064	0.6239	6.29	64.83
0.5	0.5	0	0	0.075	0.0525	0.5869	0.7136	0.4602	0.7135	0.6130	6.14	65.18
0.5	0	0	0.5	0.0775	0.0675	0.7051	0.7396	0.4514	0.7395	0.6706	5.58	66.24
0.5	0	0.1	0.4	0.0675	0.060	0.6863	0.7145	0.4610	0.7143	0.6441	6.12	65.08
0.5	0	0.5	0	0.0525	0.040	0.6466	0.6467	0.4628	0.6466	0.5468	7.58	64.87
0.5	0	0.4	0.1	0.055	0.0425	0.6486	0.6583	0.4645	0.6582	0.5618	7.33	64.66
0.3	0.7	0	0	0.075	0.050	0.5352	0.7091	0.4606	0.7090	0.6013	6.24	65.13
0.3	0.6	0.1	0	0.070	0.0475	0.5567	0.6969	0.4633	0.6968	0.5890	6.50	64.80
0.3	0.4	0.2	0.1	0.065	0.0475	0.5892	0.6886	0.4650	0.6885	0.5890	6.68	64.60
0.3	0	0.7	0	0.0475	0.035	0.6208	0.6209	0.4569	0.6208	0.5136	8.13	65.58

Table 3. (cont.)

λ_1	λ_2	λ_3	λ_4	$x_{opt,w}$	$x_{opt,f}$	4EF	TESR	ECSR	CERR	EXSR	E_{tot}	C_{tot}
0.3	0	0	0.7	0.080	0.0725	0.7050	0.7490	0.4459	0.7488	0.6862	5.38	66.91
0.3	0.1	0.1	0.5	0.070	0.060	0.6554	0.7185	0.4601	0.7183	0.6441	6.04	65.20
0.1	0.9	0	0	0.0775	0.0475	0.4841	0.7077	0.4592	0.7075	0.5890	6.27	65.30
0.1	0	0.9	0	0.0425	0.030	0.5910	0.5910	0.4469	0.5910	0.4751	8.77	66.79
0.1	0	0	0.9	0.0875	0.0775	0.7066	0.7634	0.4354	0.7632	0.7003	5.07	68.17
0.1	0.8	0.1	0	0.0725	0.045	0.5087	0.6955	0.4620	0.6954	0.5759	6.53	64.97
0.1	0.7	0.2	0	0.0675	0.0425	0.5292	0.6822	0.4636	0.6821	0.5619	6.82	64.77

4EF functions for all insulation materials increase with the decrease of λ_2 , while the optimum thicknesses of wall and roof decrease. When $\lambda_2 = 1$, the minimum values of the 4EF function are calculated, i.e. 0.4204, 0.4592 and 0.5365. It indicates these results that the total energy cost savings have less impact compared to the total energy savings, exergy loss savings and CO₂ emission reductions. The optimum thicknesses of both wall and roof for all insulation materials are reached the minimum values when $\lambda_3 = 1$. The optimum thicknesses of wall are 42, 42.5 and 72.5 cm, while the optimum thicknesses of roof are 29.5, 30 and 57.5 cm for XPS, EPS and GW insulation materials, respectively. The maximum values of optimum thicknesses of both wall and roof for all insulation materials are obtained when $\lambda_4 = 1$. It can also be seen that the greater the optimum insulation thickness of the wall and roof, the lower the total energy savings. Table 6 given results of sensitivity analysis for three insulation materials, when $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are equal to $\frac{1}{4}$.

Table 4. Optimization results for EPS insulation material

λ_1	λ_2	λ_3	λ_4	$x_{opt,w}$	$x_{opt,f}$	4EF	TESR	ECSR	CERR	EXSR	E_{tot}	C_{tot}
1/4	1/4	1/4	1/4	0.0675	0.0525	0.6348	0.6971	0.5381	0.6970	0.6069	6.50	55.78
1	0	0	0	0.070	0.0575	0.7095	0.7095	0.5406	0.7094	0.6284	6.23	55.47
0	1	0	0	0.1025	0.0675	0.5365	0.7617	0.5365	0.7616	0.6650	5.11	55.97
0	0	1	0	0.0425	0.030	0.5851	0.5851	0.4890	0.5851	0.4688	8.90	61.70
0	0	0	1	0.095	0.080	0.7017	0.7692	0.5374	0.7691	0.7017	4.95	55.86
0.9	0.1	0	0	0.0725	0.0575	0.6961	0.7134	0.5411	0.7133	0.6284	6.15	55.41
0.9	0	0.1	0	0.065	0.0525	0.6928	0.6928	0.5371	0.6927	0.6069	6.59	55.90
0.9	0	0	0.1	0.0725	0.060	0.7093	0.7172	0.5417	0.7117	0.6383	6.06	55.34
0.7	0.3	0	0	0.080	0.060	0.6720	0.7276	0.5421	0.7275	0.6383	5.84	55.29
0.7	0.2	0.1	0	0.070	0.055	0.6723	0.7054	0.5398	0.7053	0.6180	6.32	55.57
0.7	0.1	0.1	0.1	0.070	0.055	0.6801	0.7054	0.5398	0.7053	0.6180	6.32	55.57
0.7	0.1	0.2	0	0.065	0.050	0.6729	0.6882	0.5358	0.6881	0.5952	6.69	56.05
0.7	0	0	0.3	0.0725	0.0625	0.6989	0.7209	0.5420	0.7208	0.6476	5.99	55.30
0.7	0	0.1	0.2	0.0675	0.055	0.6847	0.7014	0.5391	0.7013	0.6179	6.40	55.66
0.5	0.5	0	0	0.085	0.0625	0.6397	0.7374	0.5420	0.7373	0.6477	5.63	55.31
0.5	0	0	0.5	0.0775	0.0675	0.6999	0.7348	0.5427	0.7347	0.6650	5.69	55.22
0.5	0	0.1	0.4	0.0675	0.060	0.6809	0.7093	0.5404	0.7092	0.6382	6.23	55.50
0.5	0	0.5	0	0.0525	0.040	0.6410	0.6410	0.5184	0.6410	0.5405	7.70	58.15
0.5	0	0.4	0.1	0.055	0.0425	0.6430	0.6527	0.5235	0.6526	0.5555	7.45	57.53
0.3	0.7	0	0	0.0925	0.0625	0.6018	0.7456	0.5402	0.7454	0.6477	5.46	55.52
0.3	0.6	0.1	0	0.0825	0.0575	0.6156	0.7269	0.5414	0.7268	0.6284	5.86	55.38
0.3	0.4	0.2	0.1	0.0725	0.0525	0.6289	0.7050	0.5393	0.7048	0.6069	6.33	55.63
0.3	0	0.7	0	0.0475	0.035	0.6151	0.6151	0.5058	0.6150	0.5073	8.26	59.68
0.3	0	0	0.7	0.0825	0.0725	0.7007	0.7474	0.5420	0.7472	0.6807	5.42	55.30
0.3	0.1	0.1	0.5	0.075	0.0625	0.6679	0.7246	0.5423	0.7244	0.6476	5.91	55.26
0.1	0.9	0	0	0.0975	0.065	0.5602	0.7539	0.5386	0.7538	0.6566	5.28	55.71
0.1	0	0.9	0	0.0725	0.0575	0.7133	0.7134	0.5411	0.7133	0.6284	6.15	55.41
0.1	0	0	0.9	0.090	0.0775	0.7017	0.7615	0.5396	0.7614	0.6950	5.11	55.59
0.1	0.8	0.1	0	0.0875	0.0575	0.5790	0.7327	0.5406	0.7326	0.6284	5.73	55.47
0.1	0.7	0.2	0	0.080	0.0525	0.5924	0.7154	0.5398	0.7152	0.6070	6.10	55.57

Sensitivity analysis method is used to research the effect of parameters such as interest rate, inflation rate, electricity cost, fuel cost, insulation material cost, heating and cooling degree-days on the optimum insulation thickness of wall and roof and 4EF optimization function. Figure 8 shows a sensitivity analysis results of optimum insulation thickness of wall and roof and 4EF function for XPS insulation material. It is seen that the sensitivity degrees of rise of the interest rate, discount rate and heating degree-days the impact on the optimum insulation thickness of wall and roof and 4EF function are greater than other parameters. The average variation of the optimum insulation thicknesses of wall and roof for three insulation materials is shown in Figure 9. It appears that the impact of each sensitivity factor on the optimum insulation thickness of wall and roof for the three insulation materials is different. It is seen these results that the average variation of the optimum insulation thicknesses of wall and roof for XPS insulation material are greater than other two insulation materials.

Table 5. Optimization results for GW insulation material

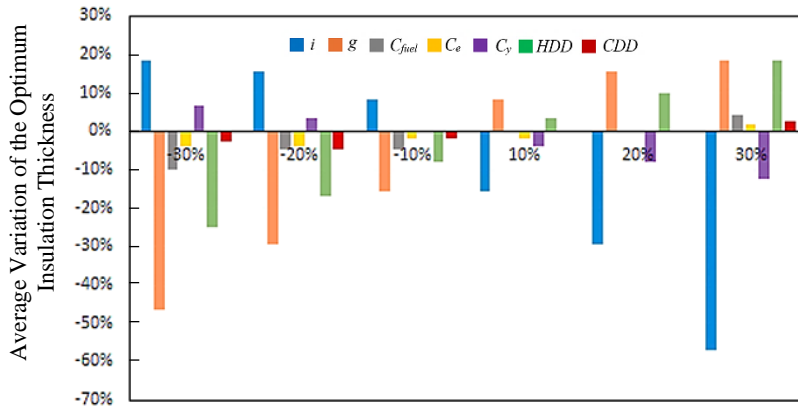
λ_1	λ_2	λ_3	λ_4	$x_{opt,w}$	$x_{opt,f}$	4EF	TESR	ECSR	CERR	EXSR	E_{tot}	C_{tot}
1/4	1/4	1/4	1/4	0.0675	0.0525	0.6348	0.6971	0.5381	0.6970	0.6069	6.50	55.78
1	0	0	0	0.070	0.0575	0.7095	0.7095	0.5406	0.7094	0.6284	6.23	55.47
0	1	0	0	0.1025	0.0675	0.5365	0.7617	0.5365	0.7616	0.6650	5.11	55.97
0	0	1	0	0.0425	0.030	0.5851	0.5851	0.4890	0.5851	0.4688	8.90	61.70
0	0	0	1	0.095	0.080	0.7017	0.7692	0.5374	0.7691	0.7017	4.95	55.86
0.9	0.1	0	0	0.0725	0.0575	0.6961	0.7134	0.5411	0.7133	0.6284	6.15	55.41
0.9	0	0.1	0	0.065	0.0525	0.6928	0.6928	0.5371	0.6927	0.6069	6.59	55.90
0.9	0	0	0.1	0.0725	0.060	0.7093	0.7172	0.5417	0.7117	0.6383	6.06	55.34
0.7	0.3	0	0	0.080	0.060	0.6720	0.7276	0.5421	0.7275	0.6383	5.84	55.29
0.7	0.2	0.1	0	0.070	0.055	0.6723	0.7054	0.5398	0.7053	0.6180	6.32	55.57
0.7	0.1	0.1	0.1	0.070	0.055	0.6801	0.7054	0.5398	0.7053	0.6180	6.32	55.57
0.7	0.1	0.2	0	0.065	0.050	0.6729	0.6882	0.5358	0.6881	0.5952	6.69	56.05
0.7	0	0	0.3	0.0725	0.0625	0.6989	0.7209	0.5420	0.7208	0.6476	5.99	55.30
0.7	0	0.1	0.2	0.0675	0.055	0.6847	0.7014	0.5391	0.7013	0.6179	6.40	55.66
0.5	0.5	0	0	0.085	0.0625	0.6397	0.7374	0.5420	0.7373	0.6477	5.63	55.31
0.5	0	0	0.5	0.0775	0.0675	0.6999	0.7348	0.5427	0.7347	0.6650	5.69	55.22
0.5	0	0.1	0.4	0.0675	0.060	0.6809	0.7093	0.5404	0.7092	0.6382	6.23	55.50
0.5	0	0.5	0	0.0525	0.040	0.6410	0.6410	0.5184	0.6410	0.5405	7.70	58.15
0.5	0	0.4	0.1	0.055	0.0425	0.6430	0.6527	0.5235	0.6526	0.5555	7.45	57.53
0.3	0.7	0	0	0.0925	0.0625	0.6018	0.7456	0.5402	0.7454	0.6477	5.46	55.52
0.3	0.6	0.1	0	0.0825	0.0575	0.6156	0.7269	0.5414	0.7268	0.6284	5.86	55.38
0.3	0.4	0.2	0.1	0.0725	0.0525	0.6289	0.7050	0.5393	0.7048	0.6069	6.33	55.63
0.3	0	0.7	0	0.0475	0.035	0.6151	0.6151	0.5058	0.6150	0.5073	8.26	59.68
0.3	0	0	0.7	0.0825	0.0725	0.7007	0.7474	0.5420	0.7472	0.6807	5.42	55.30
0.3	0.1	0.1	0.5	0.075	0.0625	0.6679	0.7246	0.5423	0.7244	0.6476	5.91	55.26
0.1	0.9	0	0	0.0975	0.065	0.5602	0.7539	0.5386	0.7538	0.6566	5.28	55.71
0.1	0	0.9	0	0.0725	0.0575	0.7133	0.7134	0.5411	0.7133	0.6284	6.15	55.41
0.1	0	0	0.9	0.090	0.0775	0.7017	0.7615	0.5396	0.7614	0.6950	5.11	55.59
0.1	0.8	0.1	0	0.0875	0.0575	0.5790	0.7327	0.5406	0.7326	0.6284	5.73	55.47
0.1	0.7	0.2	0	0.080	0.0525	0.5924	0.7154	0.5398	0.7152	0.6070	6.10	55.57

Table 6. A sensitivity analysis results for three insulation material

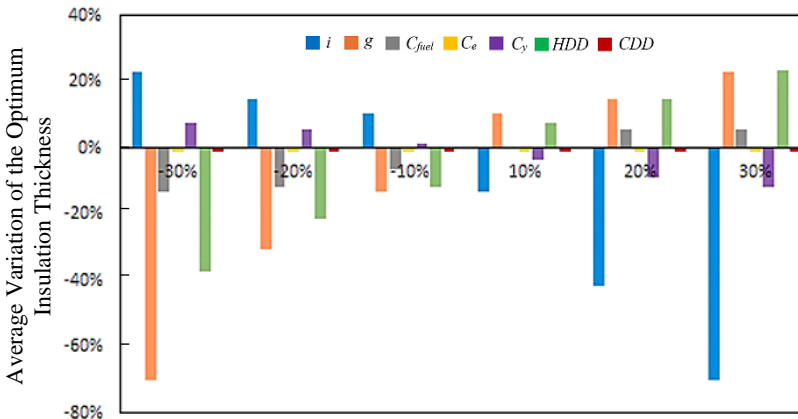
	XPS			EPS			GW			
	%	$x_{opt,w}$	$x_{opt,f}$	Z_{max}	$x_{opt,w}$	$x_{opt,f}$	Z_{max}	$x_{opt,w}$	$x_{opt,f}$	Z_{max}
<i>i</i>	-30	0.0675	0.0550	0.7195	0.0760	0.0590	0.6950	0.0775	0.0595	0.6955
	-20	0.0650	0.0500	0.6903	0.0730	0.0565	0.6750	0.0765	0.0575	0.6821
	-10	0.0600	0.0475	0.6563	0.0690	0.0530	0.6459	0.0740	0.0555	0.6630
	10	0.0475	0.0375	0.5594	0.0580	0.0435	0.5656	0.0640	0.0480	0.6020
	20	0.0425	0.0300	0.6433	0.0530	0.0380	0.5145	0.0590	0.0435	0.5620

Table 6. (cont.)

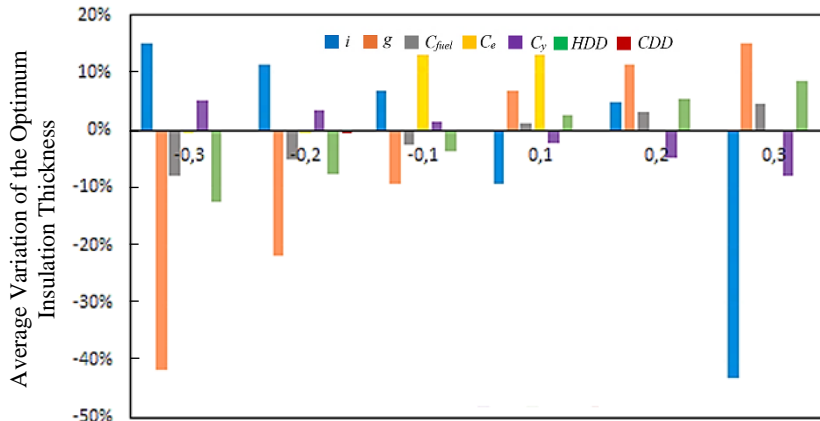
	%	XPS			EPS			GW		
		$x_{opt,w}$	$x_{opt,f}$	Z_{max}	$x_{opt,w}$	$x_{opt,f}$	Z_{max}	$x_{opt,w}$	$x_{opt,f}$	Z_{max}
<i>g</i>	30	0.0350	0.0250	0.4278	0.0465	0.0320	0.4540	0.0535	0.0385	0.5143
	-30	0.0375	0.0250	0.4315	0.0465	0.0325	0.4577	0.0540	0.0390	0.5176
	-20	0.0425	0.0325	0.5026	0.0525	0.0325	0.5001	0.0595	0.0440	0.5643
	-10	0.0475	0.0375	0.5598	0.0590	0.0430	0.5655	0.0640	0.0485	0.6033
	10	0.0600	0.0475	0.6563	0.0690	0.0525	0.6448	0.0720	0.0555	0.6612
	20	0.0650	0.0500	0.6904	0.0730	0.0565	0.6751	0.0755	0.0585	0.6834
	30	0.0675	0.0550	0.7198	0.0760	0.0595	0.6963	0.0760	0.0600	0.6952
C_{fuel}	-30	0.0500	0.0375	0.5668	0.0615	0.0440	0.5739	0.0670	0.0490	0.6100
	-20	0.0525	0.0380	0.5816	0.0620	0.0460	0.5883	0.0670	0.0505	0.6202
	-10	0.0525	0.0400	0.5960	0.0635	0.0470	0.6005	0.0680	0.0510	0.6276
	10	0.0550	0.0425	0.6189	0.0640	0.0495	0.6170	0.0690	0.0525	0.6401
	20	0.0550	0.0450	0.6319	0.0645	0.0505	0.6253	0.0695	0.0540	0.6479
	30	0.0575	0.0450	0.6417	0.0660	0.0510	0.6336	0.0695	0.0550	0.6541
	-30	0.0530	0.0420	0.6089	0.0620	0.0485	0.6074	0.0665	0.0520	0.6327
C_e	-20	0.0530	0.0420	0.6089	0.0625	0.0485	0.6079	0.0675	0.0520	0.6337
	-10	0.0540	0.0420	0.7036	0.0635	0.0485	0.6088	0.0685	0.0520	0.6346
	10	0.0540	0.0420	0.7034	0.0645	0.0485	0.6096	0.0685	0.0520	0.6346
	20	0.0550	0.0420	0.6105	0.0650	0.0485	0.6100	0.0695	0.0520	0.6355
	30	0.0560	0.0420	0.6112	0.0665	0.0475	0.6090	0.0710	0.0520	0.6367
	-30	0.0590	0.0460	0.6452	0.0675	0.0510	0.6349	0.0715	0.0540	0.6538
	-20	0.0570	0.0450	0.6337	0.0665	0.0505	0.6271	0.0710	0.0540	0.6492
C_{ins}	-10	0.0550	0.0430	0.6200	0.0665	0.0495	0.6191	0.0705	0.0525	0.6415
	10	0.0530	0.0410	0.5987	0.0620	0.0470	0.5981	0.0670	0.0510	0.6267
	20	0.0510	0.0390	0.5830	0.0605	0.0460	0.5871	0.0665	0.0505	0.6197
	30	0.0490	0.0380	0.5673	0.0595	0.0435	0.5712	0.0650	0.0490	0.6083
	-30	0.0440	0.0310	0.5433	0.0515	0.0345	0.5388	0.0555	0.0375	0.5684
	-20	0.0470	0.0350	0.5690	0.0555	0.0395	0.5661	0.0605	0.0425	0.5944
	-10	0.0510	0.0380	0.5897	0.0600	0.0440	0.5892	0.0640	0.0475	0.6158
<i>HDD</i>	10	0.0570	0.0460	0.6282	0.0680	0.0525	0.6266	0.0725	0.0565	0.6514
	20	0.0610	0.0500	0.6468	0.0730	0.0645	0.6460	0.0780	0.0625	0.6712
	30	0.0675	0.0555	0.6680	0.0785	0.0575	0.6681	0.0845	0.0695	0.6917
	-30	0.0535	0.0420	0.6093	0.0620	0.0485	0.6074	0.0660	0.0520	0.6323
	-20	0.0525	0.0420	0.6084	0.0630	0.0485	0.6083	0.0675	0.0520	0.6337
	-10	0.0540	0.0420	0.6097	0.0635	0.0485	0.6088	0.0680	0.0520	0.6341
	10	0.0550	0.0420	0.6105	0.0640	0.0475	0.6070	0.0690	0.0520	0.6350
<i>CDD</i>	20	0.0550	0.0420	0.6105	0.0650	0.0480	0.6089	0.0695	0.0525	0.6365
	30	0.0565	0.0420	0.6116	0.0660	0.0480	0.6097	0.0705	0.0525	0.6374



Parameter
(a)



Parameter
(b)



Parameter
(c)

Figure 8. A sensitivity analysis results of optimum insulation thickness of: (a) wall and (b) roof and (c) 4EF function for XPS insulation material

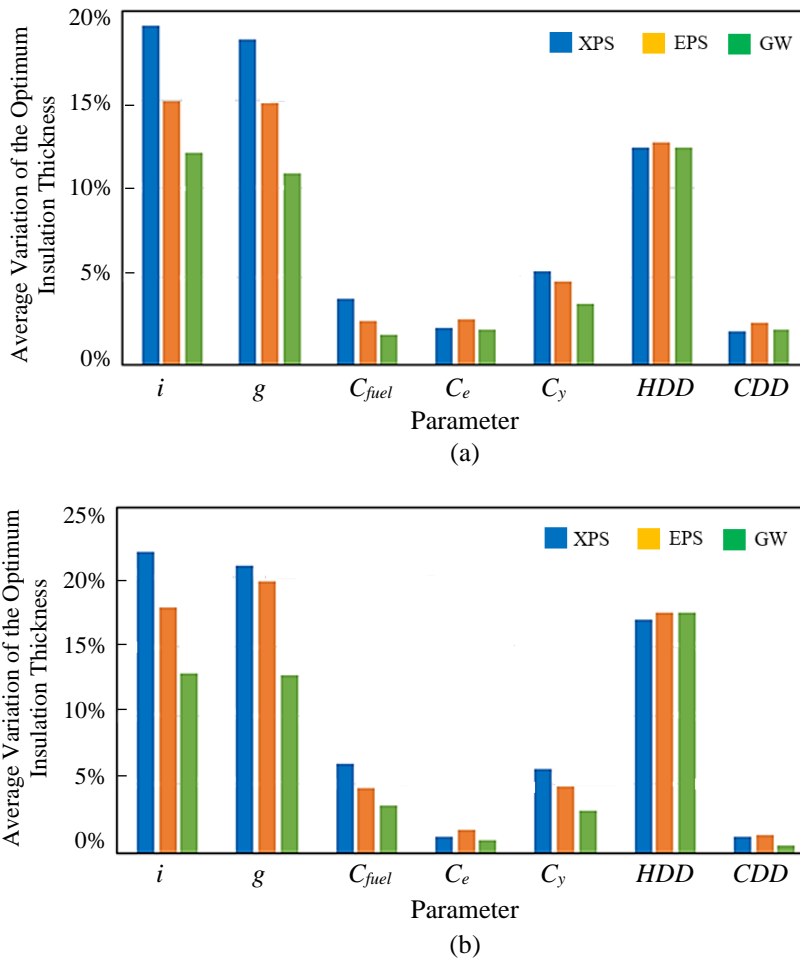


Figure 9. Average variation of the optimum insulation thicknesses of: (a) wall and (b) roof for three insulation materials

4. CONCLUSIONS

The optimum thickness of each insulation material of wall and roof of residential buildings depending on energy, environment, economy and exergy is determined for Elazığ. Firstly, the total energy requirements, energy costs, CO₂ emissions and exergy losses were determined in sections 1 to 4 by energy, environmental, economic and exergy analyses separately. In the fifth section, a 4EF function was defined as a function containing the four parameters, including energy, environment, economics and exergy. The results of this study can be summarized as follows:

- i) It is obtained that the total CO₂ emission of the building where exterior walls and roofs were insulated with GW were highest compared to the XPS and EPS insulation materials.
- ii) According to the economic analysis, The external wall insulated with GW at the optimum thickness has the least total energy cost among other insulation materials.
- iii) Based on the 4E analysis, the total energy cost savings have less impact compared to the total energy savings, exergy loss savings and CO₂ emission reductions.
- iv) The maximum values of optimum thicknesses of both wall and roof for all insulation materials were obtained when $\lambda_4 = 1$ according to the 4E function.
- v) When $\lambda_3 = 1$, the optimum thicknesses of wall and roof for all insulation materials were reached the minimum values.
- vi) According to sensitivity analysis, the sensitivity degrees of rise in the interest rate, discount rate and heating degree-days the impact on the optimum thickness of insulation for wall and roof and 4EF function are greater than other parameters.

5. ACKNOWLEDGEMENTS

The authors would like to acknowledge the Department of Mechanical Engineering, Firat University for providing the necessary facilities to conduct the present research.

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7. NOMENCLATURE

Symbol

C_A	yearly energy cost (\$/m ² year)
C_{ins}	cost of insulation material (\$)
c_e	electricity price (\$/kWh)
c_f	cost of fuel (\$/kg)

Symbol

<i>CDD</i>	cooling degree days (°C-days)
<i>d</i>	inflation rate
<i>E_A</i>	annual energy need (J/m ² year)
<i>Ex_{loss,Q}</i>	annual exergy losses depending on heat transfer (kJ/m ²)
<i>Hu</i>	heating value of the fuel (J/kg)
<i>f_H</i>	CO ₂ emission factor for thermal energy production from fuel (kgCO ₂ /kWh)
<i>f_C</i>	CO ₂ emission factor resulting from the electricity (kgCO ₂ /kWh)
<i>f_{ins}</i>	CO ₂ emission factor of insulation material (kgCO ₂ /kg)
<i>HDD</i>	heating degree days (°C-days)
<i>i</i>	interest rate
<i>k</i>	heat conduction coefficient of material (W/m K)
<i>N</i>	lifetime (years)
<i>T₀</i>	environmental reference temperature
<i>x_{ins}</i>	insulation material thickness of walls (m)
<i>y_{ins}</i>	insulation material thickness of roofs (m)
<i>η_s</i>	efficiency of fuel

Abbreviations

CERR	Rate of Total Annual Embodied CO ₂ Emission
COP	Coefficient of Performance
ECSR	Rate of Total Energy Cost of Heating and Cooling
EM	Annual Embodied CO ₂ Emission
EPS	Expanded Polistiren
EXSR	Rate of Total Savings in Exergy loss
GW	Glasswool
PWF	Present Worth Factor
SE	Savings in Exergy Loss
TESR	Rate of Total Heating and Cooling Energy Savings
XPS	Extrude Polistiren