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RESEARCH ARTICLE

Establishment of theoretical models for beam structures with particle impact damper

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ABSTRACT - Over the past few decades, research on impact dampers has grown significantly, leading to numerous analytical and experimental investigations in that field. Vibrations can harm industrial equipment and cause process errors. Particle impact damper has a significant impact on lowering vibration and has been broadly implemented in various engineering applications. This work deals with the theoretical modeling for cantilever beam and fixed-fixed beam attached with a cylindrical enclosure filled with particles. Transient and forced excitations are considered to trace the motion of the primary mass with respect to time. Also, theoretical modeling of particle arrangement in enclosure, condition for particles to detach from base or ceiling of enclosure, motion of particle after detachment from enclosure, and mechanism of collision and friction of particles are discussed and presented in detail. Theoretical models proposed in this study can be used to generate theoretical readings for practical engineering applications.

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1.0 INTRODUCTION

Particle impact damper (PID) consists of rigid containers containing one or more spherical balls that can move freely but are confined by the container's borders [1–5]. When a primary system is excited, damper balls and rigid boundaries exchange kinetic energy, and the damper dissipates it in different forms [4–8]. In the aerospace and civil engineering fields, nonlinear PIDs were developed where broadband vibration reduction was needed. The fundamental idea, historical evolution, and current research status of PID technology were discussed [9]. With a large reduction frequency band, long-term durability, insensitivity to higher temperatures, and great ruggedness, PIDs have been capable of attaining desirable performance to conventional tuned mass dampers in real-world applications [1, 10–13]. In recent years, PIDs have been extensively used to minimize excessive vibration of multi-story buildings, spacecrafts, robots, gas turbines and boring tools [14–18].

Researchers have so far made significant contributions to the design criteria for particle dampers and impact dampers [19–21]. In contrast to tuned mass dampers, the impact damper's nonlinear behaviour allows it to be effective over a broad frequency range, making it potentially appealing for the defence of civil structures from natural disturbances like wind and earthquakes [22]. Several researchers have also explored the performance of PIDs in forced vibration [1]. In addition to computational and experimental studies, several real-world uses of impact dampers have been reported in the literature to decrease extreme vibrations of mechanical systems, including high-speed railway bridges, printed circuit boards, light poles and turbine blades [23–26]. The first major part of this paper, thus, studies the theoretical modeling PID subjected to transient and forced excitations considerations. Another major part of this paper is dedicated to the theoretical modeling of particle arrangement in enclosure.

The paper begins by briefly introducing the mathematical model for the cantilever beam and fixed-fixed beam attached with a cylindrical enclosure filled with particles subjected to transient and forced excitations to predict the damping behavior of PID. Subsequently, theoretical modeling of particle arrangement in enclosure, condition for particles to detach from base or ceiling of enclosure, motion of particle after detachment from enclosure, and mechanism of collision and friction of particles were discussed and presented in detail.

2.0 STEPS IN THEORETICAL MODELING

The behavior of the PID attached with both the beams (cantilever and fixed-fixed beams) under transient and force excitations was predicted with the help of the following steps:

1) Well-known governing equations were used to predict the behavior of cantilever and fixed-fixed beam.
2) The arrangement of particles in the enclosure was predicted with the help of geometry and mathematics.
3) The instant when the particles will detach the base and will detach the ceiling was decided in terms of gravity conditions.
4) The path of particle in the cavity, from base to ceiling and ceiling to base was decided.

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5) The mechanism of collision and friction was formulated, and the loss of kinetic energy in every impact and friction was expressed in terms of displacement loss.
6) Then, with the reduced displacement, the motion of the beams was traced and the above procedure is repeated till the impacts of particles stop in case of transient vibration and till the beam vibrates with resonant frequency in case of forced vibration.

3.0 THEORETICAL MODELING OF BEAM WITH ENCLOSURE

To do the theoretical study, it was necessary to model both the beams with enclosure attached as a single degree of freedom (s.d.o.f) system. By finding the effective mass of the beam with Rayleigh's Energy method, damping coefficient with half power bandwidth method and stiffness with standard formula, both the beams were modeled as s.d.o.f system having *M*, *K* and *C*.

3.1 Theoretical Modeling of Cantilever Beam with PID

In this section, elementary model of cantilever beam with PID attached to its free end is discussed. A schematic of the cantilever beam and the enclosure containing the particles is shown in Figure 1(a). The beam modeled as a standard Euler-Bernoulli beam is shown in Figure 1(b). Mathematical modeling of the cantilever beam with enclosure and particles in it as s.d.o.f. system is shown in Figure 1(c). This theoretical modeling is used for machines having one part as cantilever such as arm of radial drilling machine, ram of shaping machine, arm of robot, crane, cantilever RCC beam, chuck of lathe machine, flywheel of engine etc. [27–29].

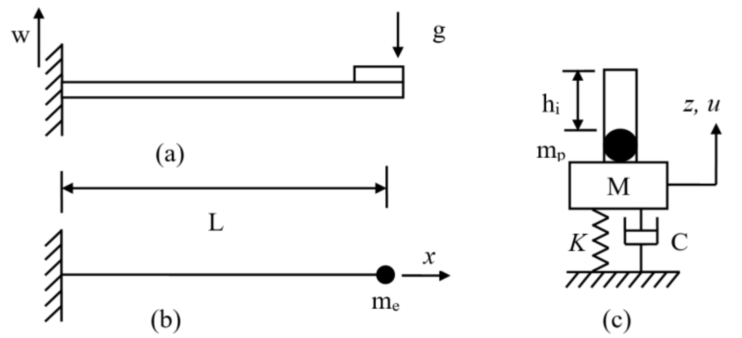

Figure 1. Theoretical modeling of cantilever beam with PID: (a) A schematic diagram of cantilever beam and enclosure, (b) A model of the beam with end mass, and (c) The equivalent single degree of freedom system

The enclosure along with the accelerometer is idealized as a point mass attached to the tip of the beam. Here the end mass, m_e is the summation of mass of the enclosure (m_c) and mass of accelerometer (m_a), mathematically it is given as,

$$m_e = m_c + m_a \tag{1}$$

The total consolidated mass (M) of beam at its free end is a function of end mass (m_e) and effective mass of beam (mb_{eff}), which is given as,

$$M = m_e + mb_{eff} \tag{2}$$

where, mb_{eff} is a function of its length and is expressed with the help of Rayleigh's Energy method. Also, the end mass (m_e) will have two different values as per the position of particles in enclosure. When the particle mass (m_p) is moving in contact with the enclosure base then,

$$m_e = m_c + m_a + m_p \tag{3}$$

and when the particle mass (m_p) is moving separately from the enclosure,

$$m_e = m_c + m_a \tag{4}$$

As any mechanical system can be modeled as s.d.o.f. system, similarly, the cantilever beam is also modeled as s.d.o.f system with *M* as mass, *K* as stiffness and *C* as damping coefficient and with particle mass, m_p which is free to move in the enclosure. The beam length,*L* is measured in *x* -direction and the length of beam is a function of *x*. All the particles are assumed as accumulated single mass, m_p. The enclosure is a small casket in which particles can be filled and has a provision with which the clearance h_i can be changed. The beam has constrained motion along the direction z and with the instantaneous displacement, *u*. The origin of co-ordinate – z is located at the equilibrium position of the mass *M*, and also, the displacement *u* of the mass *M* is measured from the same equilibrium position with the particle mass, m_p resting at the bottom of the enclosure.

In most vibration problems, the end mass of the beam remains constant. Therefore, the static deflection due to its own weight also remains constant and can be neglected. But for the case under study, the particles sometimes move in contact with the enclosure base and at other times move separately from the enclosure [30]. This makes the end mass, m_e a two-valued function. Therefore, $\Phi(x)$, M, K, ω, and u also become two valued functions. Moreover, the static deflection due to gravity is no longer a constant. Therefore, the static deflection must be taken into account in the analysis of the problem [31]. In this way, the analogy between the continuous beam, Figure 1(b) and its equivalent s.d.o.f. system, Figure 1(c) is appropriate.

3.1.1 Transient excitation

Let Figure 1(c) represent the primary mass's static equilibrium position, including the particles. The equilibrium position is considered as the origin of co-ordinate – z and for the measurement of displacement of the primary mass [31]. The well-known fundamental differential equation of second order, first degree describing the motion for s.d.o.f system under transient excitation is used to trace the motion of primary mass with respect to time,

$$M\frac{d^2u}{dt^2}+c\frac{du}{dt}+Ku=F(t) \tag{5}$$

$$\frac{d^2u}{dt^2}+2\omega\zeta\frac{du}{dt}+\omega^2u=\frac{F(t)}{M} \tag{6}$$

where $F(t)$ is a two-valued function of time, $F(t) = 0$, when the particles move separately from the beam and $F(t) = -m_p.g$, when the particles move with the beam. Let us consider these two cases separately.

Case 1: When the particles move in contact with the beam

All the quantities under this case are denoted by subscript $(\ )_1$. Let the m_c be the mass of the enclosure. The end mass m_{e1} is given as $m_{e1}= (m_c + m_a + m_p)$. Let Φ_1, M_1, K_1, ω_1 and $F_1 = (- m_p.g)$ are the corresponding quantities. Let u_0, v_0 and a_0 be, respectively, the displacement, velocity, and acceleration of primary mass, M_1 at any time taken to be zero (t = 0). After solving the above Eq. (6), the instantaneous displacement, u_1 at any time, t where, t > 0 is given as,

$$u_1(t)=\frac{U_0}{2\sqrt{\zeta^2-1}}\left[\left(\zeta+\sqrt{\zeta^2-1}\right)e^{(-\zeta+\sqrt{\zeta^2-1})\omega_n t}+\left(-\zeta+\sqrt{\zeta^2-1}\right)e^{(-\zeta+\sqrt{\zeta^2-1})\omega_n t}\right] \tag{7}$$

where, U_0 be the initial displacement given to the free end of the beam when the mass of particles (m_p) is in contact with the base of enclosure.

Case 2: When the particles move separately from the beam

All the quantities under this case are denoted by subscript $(\ )_2$. The end mass m_{e2} is given as $m_{e2} = (m_c+m_a)$. Let Φ_2, M_2, K_2, ω_2 and $F_2 = 0$ are the corresponding quantities. Let u_0, v_0 and a_0 be, respectively, the displacement, velocity, and acceleration of primary mass, M_2 at any time taken to be zero (t = 0). After solving the above Eq. (6), the instantaneous displacement, u_2 at any time, t where, t > 0 is given as,

$$u_2(t)=\frac{(U_0-U_{st})}{2\sqrt{\zeta^2-1}}\left[\left(\zeta+\sqrt{\zeta^2-1}\right)e^{(-\zeta+\sqrt{\zeta^2-1})\omega_n t}+\left(-\zeta+\sqrt{\zeta^2-1}\right)e^{(-\zeta+\sqrt{\zeta^2-1})\omega_n t}\right] \tag{8}$$

where, U_{st} is static deflection of beam under the load due to particle mass, m_p and is given as,

$$U_{st}=-(m_p.g/K_2) \tag{9}$$

(−) sign is used for U_{st} as displacement due to m_p is downward direction.

3.1.2 Forced excitation

The well-known fundamental differential equation of second order, first degree describing the motion for s.d.o.f system under forced excitation is used to trace the motion of primary mass with respect to time,

$$M\frac{d^2u}{dt^2}+c\frac{du}{dt}+Ku=F(t) \tag{10}$$

$$\frac{d^2u}{dt^2}+2\omega\zeta\frac{du}{dt}+\omega^2u=\frac{F(t)}{M} \tag{11}$$

where $F(t)$ is a two-valued function of time, $F(t) = F_0 \sin(\omega t)$, when the particles are moving separately from the beam and $F(t) = [F_0 \sin(\omega t) - m_p.g]$, when the particles are in contact with beam. F_0 = the excitation force applied by the exciter on the beam.

Eq. (11) is a linear, non-homogenous, second order, first degree differential equation. The solution of this equation consists of two parts viz. complementary function and particular integral. But in a practical system subjected to harmonic excitations, the transient vibrations die out within a short time, leaving only the steady-state vibrations. Therefore, it is important to study the steady-state behavior of the system under different excitation frequencies. Therefore, only particular part from the total solution is selected and is treated as solution to the Eq. (11) and is,

$$u = \frac{U_0 \sin(\omega t - \phi)}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2}} \tag{12}$$

where Φ is the angle by which the displacement vector lags force vector and U_0 is zero frequency deflection of beam under steady exciter force F_0.

$$\phi = \tan^{-1}\left[\frac{2\zeta(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right] \tag{13}$$

and,

$$U_0 = \left(F_0 / K\right) \tag{14}$$

as $F(t)$ is two-valued function of time, considering both cases separately.

Case 1: When the particles move in contact with the beam

All the quantities under this case are denoted by subscript $(\ )_1$. The end mass m_{e1} is given as $m_{e1} = (m_c + m_a + m_p)$. Let Φ_1, M_1, K_1, ω_1 and $F_1 = [F_0 \sin(\omega t) - m_p.g]$ be the corresponding quantities. Let u_0, v_0 and a_0 be, respectively, the displacement, velocity, and acceleration of primary mass, M_1 at any time taken to be zero (t = 0). Using Eq. (12) the instantaneous displacement, u_1 at any time, t for t > 0 is given as,

$$u_1(t) = \frac{U_0 \sin(\omega t - \phi)}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2}} \tag{15}$$

Case 2: When the particles move separately from the beam

All the quantities under this case are denoted by subscript $(\ )_2$. The end mass m_{e2} is given as $m_{e1} = (m_c + m_a)$. Let Φ_2, M_2, K_2, ω_2 and $F_2 = F_0 \sin(\omega t)$ be the corresponding quantities. Let u_0, v_0 and a_0 be, respectively, the displacement, velocity, and acceleration of primary mass, M_2 at any time taken to be zero (t =0). Using Eq. (12) the instantaneous displacement, u_2 at any time, t for t > 0 is given as,

$$u_2(t) = \frac{(U_0 - U_{st}) \sin(\omega t - \phi)}{\sqrt{[1-(\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2}} \tag{16}$$

where, U_{st} is static deflection of beam under the load due to particle mass, m_p and is given as,

$$U_{st} = -(m_p.g / K_2) \tag{17}$$

$(-)$ sign is used for U_{st} as displacement due to m_p is downward direction.

3.2 Theoretical Modeling of Fixed-Fixed Beam with PID

The developed elementary model of fixed-fixed beam with PID attached to it at the center of the beam is shown. A schematic of the fixed-fixed beam with the enclosure containing the particles is as shown in Figure 2(a). The beam modeled as a standard Euler-Bernoulli beam is as in Figure 2(b). The enclosure along with accelerometer is idealized as a point mass attached at the center of the beam. Mathematical modeling of fixed-fixed beam with enclosure and particles in it as s.d.o.f. system is shown in Figure 2(c). This model is the same as that of the cantilever beam model in Figure 1(c). Therefore, Eqs. (1) to (17) and explanation is as it is applicable with corresponding values of masses, stiffness, damping factor, excitation circular frequency, natural circular frequency, initial displacement, excitation force, zero frequency deflection etc. for fixed-fixed beam. So, with all the above procedures the modeling of fixed-fixed beam and tracking its motion at both conditions i.e. when particles move in contact or move in separate with enclosure and also under transient and forced excitation is done.

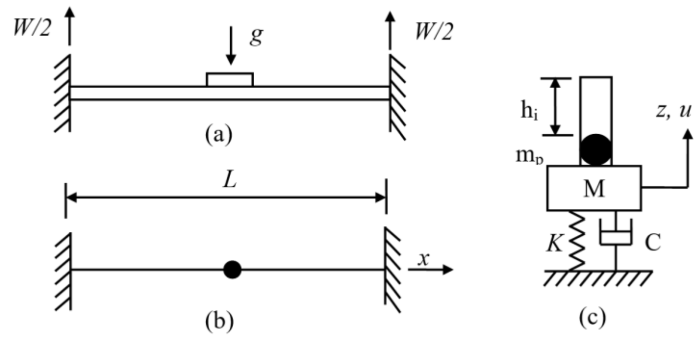

Figure 2. Theoretical modeling of fixed-fixed beam with PID: (a) A schematic of the fixed-fixed beam and enclosure, (b) A model of fixed-fixed beam with end mass and (c) The equivalent single degree of freedom system

The developed elementary model of fixed-fixed beam with PID attached to it at the center of the beam is shown in Figure 2. A schematic of the fixed-fixed beam with the enclosure containing the particles is shown in Figure 2(a). The beam modeled as a standard Euler-Bernoulli beam is shown in Figure 2(b). The enclosure along with accelerometer is idealized as a point mass attached at the center of the beam. Mathematical modeling of fixed-fixed beam with enclosure and particles in it as s.d.o.f system is shown in Figure 2(c). This theoretical modeling is used for machines having one part as fixed – fixed beam such as arbor of milling machine, cylindrical grinding machine, RCC bridges, turbines, rotors etc [32, 33]. This model is the same as that of the cantilever beam model in Figure 1(c). Therefore, all the Eqs. (1) to (17) and explanation is as it is applicable with corresponding values of masses, stiffness, damping factor, excitation circular frequency, natural circular frequency, initial displacement, excitation force, zero frequency deflection etc. for fixed-fixed beam. So, with all the above procedures the modeling of fixed-fixed beam and tracking its motion at both the conditions i.e. when particles move in contact or move in separate with enclosure and also under transient and forced excitation is done.

4.0 THEORETICAL MODELING OF PARTICLE ARRANGEMENT IN ENCLOSURE

When certain mass of particles is added in enclosure, it is straightforward to measure the bed height of particles. But it is practically difficult to predict the same height without direct measurement. To predict the same certain assumptions are accepted which are as follows:

1) It is assumed that every particle under the same material and same size category possesses identical mass and identical diameter.
2) While filling the enclosure with particles, particles will rest on the circular base of enclosure in the form of concentric circles, touching each other and progressing towards the axis of enclosure.
3) After forming the first layer, particles will form the second layer by occupying the gap among the particles of the first layer in the same way as above.
4) This process continues till the last particle will occupy the proper location in enclosure.
5) The particles in contact with the enclosure contribute to the phenomenon of friction, and the intra-particle friction is neglected.
6) It is assumed that all the particles move as a lumped mass. The relative motion between the particles is neglected.

4.1 Theoretical Modeling of Particle Arrangement in One Layer

As per the assumption, all particles of same material and of same size possess identical diameters and will occupy the places in one horizontal layer of enclosure as shown in Figure 3(a). Let the inside diameter of enclosure is D1 and *D2*, *D3*, *D4*,…., *Dn* are the diameters of concentric circles passing through the centers of particles. The diameter of particle is taken to be ‘d’.

A small sector of this particle arrangement with enlarged view is shown in Figure 3(b) to know the geometry of particle arrangement. From the figure, it is clear that every particle on the second concentric circle is finding its place in the gap of two particles of first concentric circle. Here the diameter of the smallest enclosure is 34 mm and the largest particle size is 3 mm, this wide difference in diameters further encourages for simplifying the geometry as straight parallel lines in place of concentric circles, Figure 4. Let the perpendicular distance between the two consecutive center lines is ‘h’ and lines joining the centers of particles forms equilateral triangles of side equal to d. Therefore, it can be shown that $h = \frac{\sqrt{3}}{2}d$. All the consecutive concentric circle diameters will differ by distance $2h = \sqrt{3}d$ from each other except the first one, it will differ by a distance, d from the inside enclosure diameter.

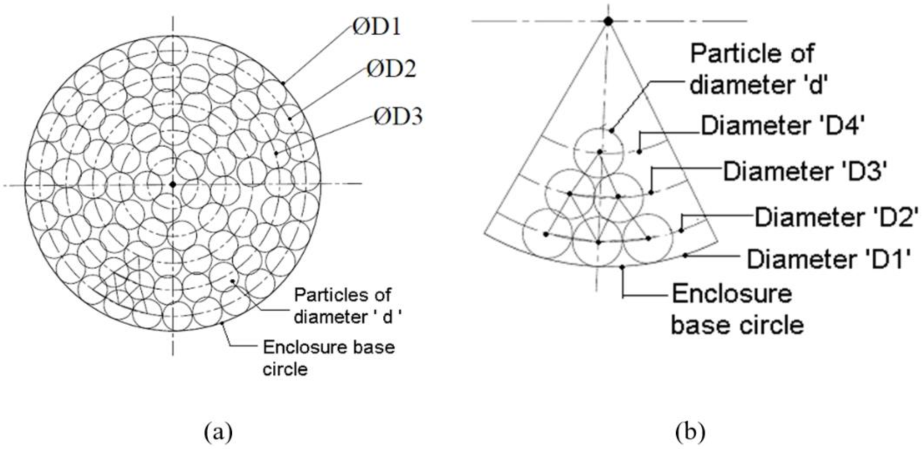

Figure 3. Arrangement of particles in enclosure: (a) In one layer and (b) Small sector of particle arrangement

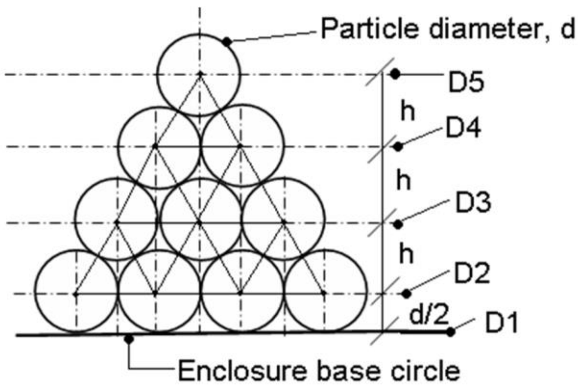

Figure 4. Particle arrangement in enclosure with simplified geometry

4.2 Theoretical Modeling of Particle Layers in Enclosure Space

The arrangement of particles in a horizontal layer parallel to the base of enclosure is explained above. Now, the possible ways the different horizontal layers will arrange themselves in enclosure is shown in Figure 5.

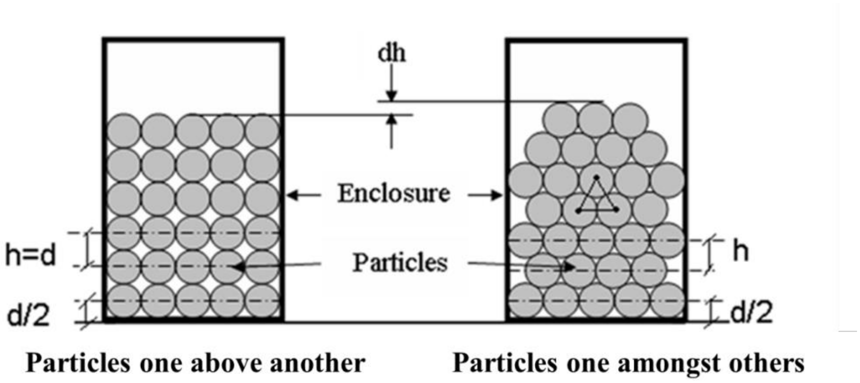

Figure 5. Particle arrangement in different layers in the enclosure

Either the particles of next layer will take place one above the earlier layer or they will adjust themselves amongst the particles of earlier layer. In the above figure, the fifteen particles arranged themselves in both ways and it clear that there is not much difference in the total height of particle bed so formed by two ways. So, both the arrangements are acceptable, but the particles' tendency to get arranged by second way, i.e. one amongst the others, is more due to its stable nature. Therefore, for modeling the particle layers, this method was selected. The distance between the center lines of successive layers is h and that of first layer from enclosure base is $d/2$. This h distance is d for one above another pattern and is $(\sqrt{3}/2)d$ for one among other patterns. In the Figure 5, 'dh' is the difference between the height of heap of one above another pattern and one among other patterns. This 'dh' is very small and can be neglected. With the help of modeling of particles in one layer and in different layers the bed height of particles for given mass, belonging to particular material and size can be predicted.

5.0 CONDITION FOR PARTICLES TO DETACH FROM BASE OR CEILING OF ENCLOSURE

Consider the cantilever beam under transient excitation. The free end of beam to which a particle-filled enclosure is attached, is excited with an initial transient displacement of 15 mm. The displacement, velocity, and acceleration curves for transient vibrations with respect to time are shown in Figure 6.

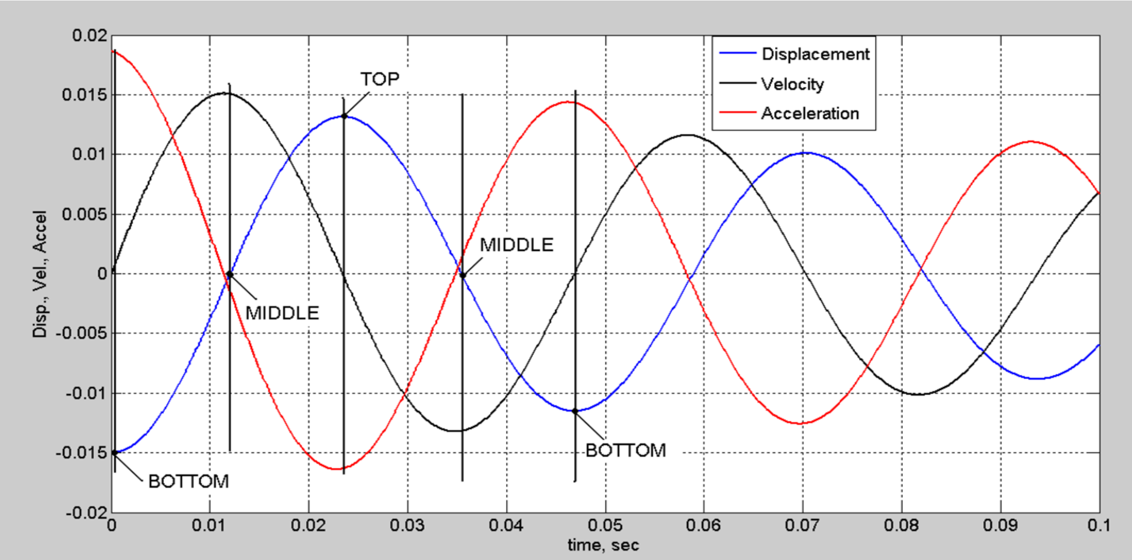

Figure 6. Displacement, velocity, and acceleration curves under transient excitation

Free end of the beam with enclosure moves in a sequence of its positions as bottom – middle – top – middle – bottom and this sequence keeps on repeating until transient vibration die out. Now consider the cantilever beam under forced excitation. The free end of beam to which a particle-filled enclosure is attached, is excited with forced excitation with excitation amplitude of 4 mm. The displacement, velocity, and acceleration curves for forced vibrations along with the locations of enclosure with respect to time are shown in Figure 7.

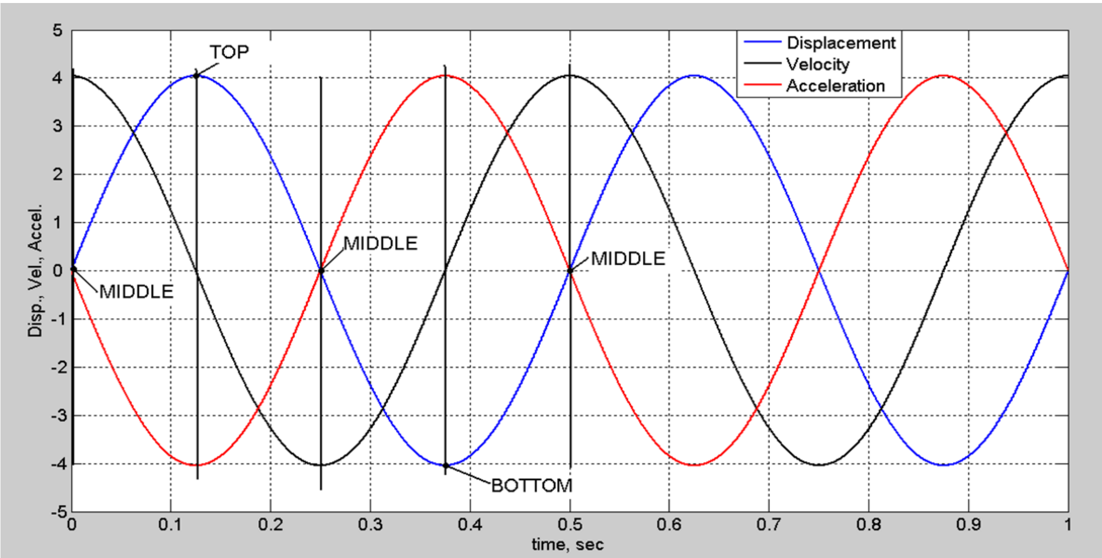

Figure 7. Displacement, velocity, and acceleration curves under forced excitation

The enclosure moves in a sequence of its positions as middle – top – middle – bottom – middle and this sequence keeps on repeating till forced vibrations exist. The beam is vibrating under simple harmonic motion (SHM) and the values of displacements, velocities, and accelerations of beam at different angular positions can be found from above graphs. The particle will be in contact with enclosure base as long as the enclosure acceleration is greater than –9.81 m/s^2 and particle acceleration is –9.81 m/s^2. And when enclosure acceleration becomes equal to or less than –9.81 m/s^2, particle will feel weightlessness and will get detached from base. After getting detached from base particle will hit to ceiling and will be in contact with ceiling as long as the enclosure acceleration is less than –9.81 m/s^2 and when it becomes equal to or greater than –9.81 m/s^2, particle will get detached from ceiling. With reference to the above graphs, Table 1 is formed which gives clear idea about angular displacement, displacement, velocity and acceleration of beam and the condition of detachment of particle form floor or ceiling of enclosure.

Table 1. Conditions of detachment of particle from enclosure

Sr. No.	Encl. Location	Ang. disp. of beam	Disp. of beam	Velocity of beam	Accel. of beam	Particle detachment
For transient excitation						
1	Bottom	0^0	– ve max.	0	+ ve max.	
2	Middle	90^0	0	+ ve max.	0	From floor at a_{abs} <= – g
3	Top	180^0	+ ve max.	0	– ve max.	To ceiling
4	Middle	270^0	0	– ve max.	0	From ceiling at (a_{abs}) >= – g
5	Bottom	360^0	– ve max.	0	+ ve max.	
For forced excitation						
1	Middle	0^0	0	+ ve max.	0	From floor at a_{abs} <= – g
2	Top	90^0	+ ve max.	0	– ve max.	To ceiling
3	Middle	180^0	0	– ve max.	0	From ceiling at (a_{abs}) >= – g
4	Bottom	270^0	– ve max.	0	+ ve max.	
5	Middle	360^0	0	+ ve max.	0	

6.0 MOTION OF PARTICLE AFTER DETACHMENT FROM ENCLOSURE

Now with a simple method the motion of particles and the primary mass can be tracked for all time, $t > 0$. At time $t = 0$, let the particles be in contact with the primary mass (case-1). Let M be released from rest with initial velocity, $v_0 = 0$ with an initial displacement, $u_0 = -U_0$ for transient vibration and that of for forced vibration will be $v_0 = +V_0$ and $u_0 = 0$. The motion of primary mass, M is described by Eqs. (7) and (15) for transient and forced vibration, respectively. The moment of particle detachment from floor and ceiling can be calculated using the conditions in Table 1.

Let v_s be the launch velocity of the particle at the time of separation, t_s. After separation the particle will travel under the influence of gravity and their motion is given by equation [31],

$$u_p(t) = u_s + v_s(t - t_s) - \frac{1}{2}g(t - t_s)^2, \quad \text{where, } t_s < t < t_1 \tag{18}$$

where t_1 is the instant at which the particles come across their first impact. After separation, the motion of M is given by Eqs. (8) and (16) for transient vibration and forced respectively. There are two different scenarios for the first impact to occur [30]:

1) If the clearance, $(H_i - h_i)$, is sufficiently small such that the "ceiling impact criterion", $u_p = u_2 + (H_i - h_i)$, is satisfied then the first impact of the particle will be with the ceiling; otherwise
2) The first impact will occur with the floor when "floor impact criterion", $u_p = u_2$, is satisfied.

7.0 MECHANISM OF COLLISION AND FRICTION OF PARTICLES

The number of particles in a multi-particle impact damper (MPID) is more. Every particle will receive vibration energy due to excitation and can move independently with an independent initial velocity. This means if total particles are 'n' then those particles will travel with 'n' different velocities and in 'n' different directions. All these particles with independent velocities and directions will collide either with each other or with enclosure wall and enclosure ceiling. After impact every particle will possess new velocity & direction and enclosure will have reduced velocity with is same up and down direction. Handling all these data of displacement, velocity, acceleration, direction of all particles and updating this data after every impact for the time until excitation dies out in case of transient vibrations and until excitation stops in case of forced vibrations. In this way, this problem comes under a theory of particle dynamics and real-time collision detection. The codes written earlier are based on certain assumptions and require high computational facility, still the time required to take a single reading will be in hours. Also, the accuracy of results through these codes is based on the closeness of the initial assumptions made towards the facts. The additional difficulty is that a new friction model is to be formed to take care of friction amongst the particles and between particles and enclosure.

7.1 Mechanism of Collision of Particles

Due to all the above difficulties, multi particles are assumed as a single particle moving in z-direction only so that there will not be question of particle-to-particle impacts and impact of particle with enclosure wall. This assumption simplifies the collision mechanism and reduces it to collision between two masses viz. system mass, M and particle mass, m_p, as shown in Figure 8.

The mechanism of energy dissipation is inelastic collisions amongst the particles and between particles and the enclosure walls. As the amount of energy dissipated by each mechanism cannot be estimated, all of these mechanisms are considered into an effective coefficient of restitution, R. Let v_1^-, v_2^- & v_1^+, v_2^+ be the before impact & after impact velocities of enclosure mass and particle mass respectively. Then R can be defined as,

$$R = -\left(\frac{v_2^+ - v_1^+}{v_2^- - v_1^-}\right), \tag{19}$$

where $0 \le R \le 1$ as per the law of conservation of linear momentum,

$$Mv_1^- + m_p v_2^- = Mv_1^+ + m_p v_2^+ \tag{20}$$

solving Eqs. (18) and (19), the velocities after impact can be expressed as,

$$v_1^+ = \frac{(1 - Rm_r)v_1^- + m_r(1+R)v_2^-}{(1+m_r)}, \quad v_2^+ = \frac{(1+R)v_1^- + (m_r - R)v_2^-}{(1+m_r)} \tag{21}$$

Now taking help of law of conservation kinetic energy, the kinetic energy converted into heat and noise can be written as,

Loss of KE = (KE of both masses before impact) – (KE of both masses after impact) (22)

$$\Delta KE = \left[\frac{1}{2}M\left(v_1^-\right)^2 + \frac{1}{2}m_p\left(v_2^-\right)^2\right] - \left[\frac{1}{2}M\left(v_1^+\right)^2 + \frac{1}{2}m_p\left(v_2^+\right)^2\right] \tag{23}$$

From Eqs. (21) and (23) it can be written,

$$\Delta KE = \frac{1}{2}\left(1 - R^2\right)\frac{m_p}{1+m_r}\left(v_2^- - v_1^-\right)^2 \tag{24}$$

This much energy is lost from the beam's total excitation energy.

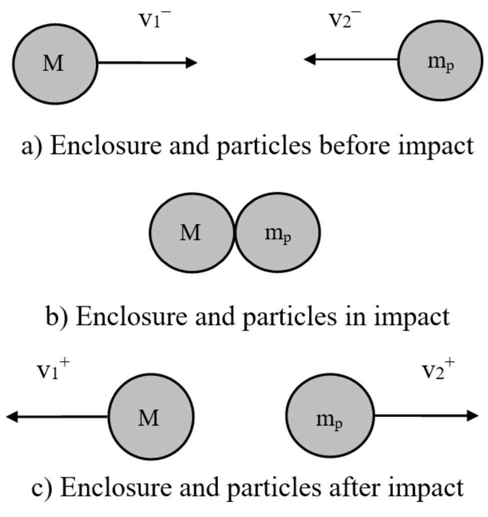

Figure 8. Collision mechanism of enclosure mass and particle mass: (a) before impact, (b) during impact and (c) after impact

The energy is nothing but the ability to do work, therefore this ΔKE is expressed as the work loss of beam,

$$\Delta KE = \Delta W \tag{25}$$

$$\Delta W = F.\Delta U_C \tag{26}$$

$$\Delta W = U_0.K.\Delta U_C \tag{27}$$

$$\Delta U_C = \frac{\Delta KE}{U_0.K.} \tag{28}$$

where, ΔW, F, ΔU_C, U_0, and K are work loss of beam, force,displacement loss, initial displacement, and stiffness of beam, respectively.

7.2 Mechanism of Friction of Particles

At resonance, the collision phenomenon dominates the friction phenomenon, but for more accuracy, the friction of particles with enclosure wall is considered. By knowing the arrangement of particles in the enclosure, it is easy to find the number of particles in contact with the enclosure wall. Let mass of those particles is $(m_p)_s$ and the normal reaction exerted by those particles on enclosure wall is R_n and which can be given as,

$$R_n = (m_p)_s.g \tag{29}$$

Then the force of friction F_f can be calculated as,

$$F_f = \mu.R_n \tag{30}$$

In this mechanism, friction of particles amongst themselves is neglected. To compensate for that frictional energy, normal reaction of particles in contact with enclosure wall is taken equal to their normal reaction when they are in contact with floor. Let h_s be the to and fro distance traveled by those particles by keeping the contact with enclosure wall. Then the energy lost in friction, ΔFE can be given as,

$$\Delta FE = F_f.h_s \tag{31}$$

and the displacement loss due to this friction is given as,

$$\Delta U_f = (\Delta FE/U_0.K) \tag{32}$$

The total displacement loss due to collision and friction is given as

$$\Delta U_t = \Delta U_c + \Delta U_f \tag{33}$$

The magnitude of this total displacement loss is subtracted from the initial displacement of primary mass, M and with new reduced beam displacement after first impact all the motion equations for primary mass M and for particle mass, m_p are traced out to find second impact. This procedure is repeated until the beam energy reduces to that extent where by particle-enclosure impacts ceases in case of transient vibrations and in case of forced vibrations till the exciter stops.

8.0 CONCLUSION

In this paper, a theoretical model for both cantilever and fixed-fixed beam with particle impact damper has been proposed. Transient and forced excitations are considered to trace the motion of the primary mass with respect to time. A method is also developed to find the effective size of particles and the packing ratio for a particle impact damper. Finally, the condition for particles to detach from base or ceiling of enclosure, motion of particle after detachment from enclosure, and the mechanism of collision and friction of particles are presented. The proposed theoretical model enables design and implementation of PID for numerous engineering applications subjected to various external loads.

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