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ORIGINAL ARTICLE

Modeling of a non-linear multi-agent distributed control system

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ABSTRACT – The present study aims to develop a mathematical model for consensus control system based on Lyapunov Theory and nonlinear dynamics functional equations. This paper describes a new solution that deals with the general-consensus problem and the leader-following consensus problem of non-linear multi-agent system in which the parameters of all follower agents can be different, and with an unforced agent as the leader in the multi-agent system. Different control rules were constructed for each different follower agent based on its own state variables and its communication with adjacent agents. Numerical simulations are provided to demonstrate the feasibility of the developed mathematical model. The results have demonstrated the designed distributed control system satisfy the Lyapunov Theory since all the agents have converged to its steady state after a period of time.

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INTRODUCTION

The centralized control composes of several agents and one controller. When information is transmitted, large amount of information is transmitted to the controller. It makes the centralized controller heavy and computationally complex. When there is a large amount of information between the agents and the controller, it is easy to cause information blockage and errors in the transmission process [1]. The decentralized controller composes of multiple agents and multiple controllers. A controller controls an agent, so there is no information transfer between agents. Therefore, when the agent moves, it may cause a collision [2].

The distributed control system is a computer control system for centralized management and decentralization of the production process [3]. It is a more advanced control system developed to meet the increasingly complex process control requirements with the continuous development of modern large-scale industrial automation and production. It is a new type of control system that integrates risk control and optimization control which combines of computer, communication network and automatic control technology. The system adopts the idea of decentralized control and centralized management, and the design principles of separation, autonomy, and comprehensive coordination have a hierarchical structure. Compared with traditional methods, the introduction of multi-agent system (MAS) has many advantages in the process of solving practical problems. Firstly, it improves the efficiency of the system due to the coordination between individual agents. Secondly, the system has stronger robustness and reliability [2]. Regarding the research of MAS, one of the essential and important topics is none other than consensus control [4]. The key of achieving consensus is to design and construct an appropriate control protocol or algorithm so that all agents in the network can reach a consensus. The protocol or algorithm is mainly based on coordination with one's own neighbors. Many researchers have recently studied consensus issues from different perspectives. This is mainly due to their broad application in many fields, including robots [5-9], formations [10-13], unmanned aerial vehicles [14-18], unmanned surface vehicles [15, 19-21], unmanned underwater vehicles [21, 22], sensor networks [23, 24], etc.

The current consensus control mainly focuses on two control problems which are leaderless consensus and leader-following consensus [25]. Leaderless consensus requires the convergence of each agent to a certain agreement state based on the initial conditions. The introduction of a leader is a breakthrough in designing a distributed tracking controller for MASs which enables all agents to track the leader node's trajectory. Generally, Lyapunov theory, matrix theory and graph theory are applied to obtain consensus conditions [26]. The leader-following consensus has received the attention of many researchers. The semi-global leader-following consensus was studied for MASs with input saturation and low gain feedback on switching networks [27], discrete-time MASs with input saturation and external disturbances [28], discrete-time linear MASs for input saturation [29], imperfect linear actuator systems [30], etc. Global leader-following consensus problem was investigated for group of agents for linear systems under actuator saturation [31], bounded controls of a group of discrete-time linear systems [32], MASs with intermittent directed communication under actuator saturation [33], etc.

Many research projects have focused on linear MASs in recent years. Nevertheless, most of the engineering problems in the real world involving complex nonlinear systems [34]. Therefore, the nonlinearity in dynamics has attracted the attention of many researchers. The leader following consensus problem of second-order MASs under switching communication topologies with time-varying delays was studied by Zhu et al. [35]. Wen et al. also studied the similar

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problem of second order non-linear multi-agent system (NMAS) under fixed undirected and directed communication topologies respectively [36]. In addition, Huang et al. studied the similar problem with periodically intermittent communication [37]. The leader–follower fixed-time consensus problem of high-order MASs with external disturbances was studied by Tian et al. [38]. The tracking consensus problem for NMAS with under a reference leader by was studied by Zhao et al. [39]. Unique protocols were proposed based on the relative information between the neighbouring agents. The distributed leader-following consensus of a class of NMAS with switching topology and unreliable communication was studied by Chui et al. [40]. Each possible topology may contain a leader which was rooted by a directed spanning tree. An appropriate distributed controller was designed to make the asymptotically synchronization between all follower nodes and the leader node. A similar problem for a MAS with an affine nonlinear term was studied by Shi et al. [41]. An undirected connected-graph where the leader sends the information to one or multiple followers was adopted in the communication topology. A protocol was proposed to make the asymptotically synchronization between each follower with the leader.

The goal of the research aims is to develop a mathematical model for consensus control system based on Lyapunov Theory and nonlinear dynamics functional equations. A new solution that deals with the general-consensus problem and the leader-following consensus problem of NMAS is proposed in which the parameters of all follower agents can be different, and with an unforced agent as the leader in the multi-agent system. Different control rules were constructed for each different follower agent based on its communication with adjacent agents and its own state variables. If the non-linear multi-agent system satisfies the condition where the topological graph of communication among follower agents was undirected and connected, a leader-following consensus will be achieved. The overall structure of the paper is organized as follows. Firstly, it describes the methodology employed in this study. The mathematical model for consensus control system based on Lyapunov Theory and nonlinear dynamics functional equations is elaborated in detail. Then, some simulation results are presented to demonstrate the feasibility of the developed mathematical model for consensus control system. Finally, the research conclusions are summarized.

RESEARCH METHODOLODY

First, Algebraic Graph Theory was applied to solve graph problems, particularly on the transformation of distributed system's graphs into mathematical expressions. The second step was to solve the stability of the controller through coupling control. The third step was to solve the gain problem of the system according to Lyapunov theory. The fourth step was to use MATLAB simulation to prove our design theory.

The assumption made:

- The communication between an agent with an adjacent agent is stable.
- The adjacency matrix is constructed based on the relationship between adjacent agents.
- The designed controller can be implemented as expected.
- The state of the controller can be monitored.
- The states of all the agents in the network can be measured.

Distributed Non-Linear Multi-Agent System

As shown in Figure 1, at *t* moment, there is only one autonomous agent (V_0), in the multi-agent systems while there is at least a non-autonomous agent (V_1, V_2, V_3, V_4, V_5), obtaining information from the autonomous agent system. Thus, this autonomous agent(V_0) is the leader of the multi-agent system while the other non-autonomous agents (V_1, V_2, V_3, V_4, V_5) are the followers of the multi-agent system. Hence, under the designed control law, all the follower agents (V_1, V_2, V_3, V_4, V_5) will follow the leader (V_0) and eventually reach the same outputs or the same states. Figure 1 shows the communication topology of leader follower consensus of multi-agent systems. The leader has a directed path to the follower node while the communication between the follower agents was undirected.

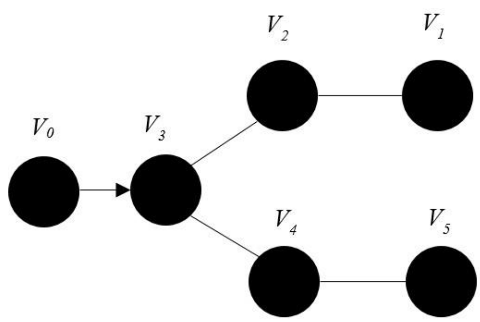

Figure 1. Communication topology of multi-agent systems

Notation

Note that the positive integers such as m and n, R_n stands for n dimensional Euclidean space, and $R_{m\times n}$, it represents a set of $m \times n$ matrices. This is the Euclidean norm of a vector. $Diag\{.\}$ used to represent a matrix, its diagonals are values of the corresponding parameter. All other entries are known as zero. 1_q denotes the column vector in R_q where all the entries are equal to one while I_q represents the $q \times q$ identity matrix. Furthermore, $\boldsymbol{A^T}$ represents the transpose of a matrix $\boldsymbol{A}$ while the sign $\otimes$ represents the Kronecker product has different properties, for example $(\boldsymbol{A} \otimes \boldsymbol{B})(\boldsymbol{C} \otimes \boldsymbol{D}) = \boldsymbol{AC} \otimes \boldsymbol{BD}$ *and* $(\boldsymbol{A} \otimes \boldsymbol{B})^T = \boldsymbol{A^T} \otimes \boldsymbol{B^T}$. Given a square matrix $\boldsymbol{Q}$, $Diag$ [5]>0 or $Diag$[5]<0 means it will be either positive definite or negative definite. Let's $\boldsymbol{A}$ to be an $m \times n$ matrix, let's $\boldsymbol{B}$ to be a $p \times q$ matrix, then the Kronecker's product of $\boldsymbol{A} \otimes \boldsymbol{B}$ will be the $mp \times nq$ block matrix as shown in Eqs. (1) and (2).

$$\boldsymbol{A} \otimes \boldsymbol{B} = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{bmatrix} \tag{1}$$

Namely,

$$\boldsymbol{A} \otimes \boldsymbol{B} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & \cdots & a_{11}b_{1q} & \cdots & \cdots & a_{1n}b_{11} & a_{1n}b_{12} & \cdots & a_{1n}b_{1q} \\ a_{11}b_{21} & a_{11}b_{22} & \cdots & a_{11}b_{2q} & \cdots & \cdots & a_{1n}b_{21} & a_{1n}b_{22} & \cdots & a_{1n}b_{2q} \\ \vdots & \vdots & \ddots & \vdots & & & \vdots & \vdots & \ddots & \vdots \\ a_{11}b_{p1} & a_{11}b_{p2} & \cdots & a_{11}b_{pq} & \cdots & \cdots & a_{1n}b_{p1} & a_{1n}b_{p2} & \cdots & a_{1n}b_{pq} \\ \vdots & \vdots & & \vdots & \ddots & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots & & \ddots & \vdots & \vdots & & \vdots \\ a_{m1}b_{11} & a_{m1}b_{12} & \cdots & a_{m1}b_{1q} & \cdots & \cdots & a_{mn}b_{11} & a_{mn}b_{12} & \cdots & a_{mn}b_{1q} \\ a_{m1}b_{21} & a_{m1}b_{22} & \cdots & a_{m1}b_{2q} & \cdots & \cdots & a_{mn}b_{21} & a_{mn}b_{22} & \cdots & a_{mn}b_{2q} \\ \vdots & \vdots & \ddots & \vdots & & & \vdots & \vdots & \ddots & \vdots \\ a_{m1}b_{p1} & a_{m1}b_{p2} & \cdots & a_{m1}b_{pq} & \cdots & \cdots & a_{mn}b_{p1} & a_{mn}b_{p2} & \cdots & a_{mn}b_{pq} \end{bmatrix} \tag{2}$$

The inverse of a Kronecker's product $(\boldsymbol{A} \otimes \boldsymbol{B})$ will be invertible if both A and B are invertible as discussed in Eq. (3).

$$(\boldsymbol{A} \otimes \boldsymbol{B})^{-1} = \boldsymbol{A}^{-1} \otimes \boldsymbol{B}^{-1} \tag{3}$$

The number of agents is assumed to be six. However, when calculating the agent control method, it is necessary to assume that the number of agents is q. It is commonly used to describe the communication among the different agents [23]. For example, a graph $W = \{\boldsymbol{v}, \boldsymbol{\varepsilon}, \boldsymbol{G}\}$ with q nodes. q nodes (the following agents) are represented by $v = \{v_1, v_2, \ldots, v_q\}$ where v_0 is the leader. All edges are represented as $\varepsilon \subseteq v \times v$. The pair $(v_j, v_i) \in \varepsilon$ indicates that there will be an edge which connects the agents j and i. Agent j will be a neighbor of agent i when $(v_j, v_i) \in \varepsilon$. The adjacency matrix $G = (g_{ij})$ can be applied to describe W's structure if agent i able to get information from agent j. Therefore, $g_{ij} > 0$, or else g_{ij} will be 0. If given $g_{ij} = 0$ for all $i = 1, 2, .., q$. The in-degree of agent i is represented as $d_i = \sum_{j=1}^{q} aij$ $(j = 1, \ldots, q)$, and the in-degree matrix $\boldsymbol{D}$ will be a diagonal matrix with $\boldsymbol{D} = Diag\{d_i\} \in R_{q\times q}$. Then, we let Laplacian matrix to be $\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{G}$.

If the information can be sent by the leader to its agent i, $a_{i0} > 0$ for $i = 1, 2, \ldots, q$, otherwise $a_{i0} = 0$. If $g_{ij} = g_{ji}$, it indicates that agent i can exchange information with agent j or vice versa. An important case is that for any two vertices, there is an undirected path, and then the undirected graph is connected. Then, $\boldsymbol{M} = \boldsymbol{L} + Diag\{g_{10}, g_{10}, \ldots, g_{q0}\}$. The dimensions of each matrix should be suitable.

Non-Linear Multi Agent Systems

Let's consider a multi-agent system,

$$\dot{x}_i = \boldsymbol{M}x_i + \boldsymbol{N}f(x_i) + \boldsymbol{F}(x_i)u_i \tag{4}$$

where $\dot{x}_i$ is dynamic variable quantity of follower; x_i is dynamic vector of follower, $x_i \in R_n$, the agent i's state vector with $i = 1,2, \ldots . q$; $f(x_i)$ is dynamic matrix of follower, $f(x_i(t)) \in R_r$; $F(x_i)$ *is* coefficient matrix of the i^{th} agent, $F(x_i) = (F_1(x_i), F_2(x_i), \ldots\ldots, F_m(x_i)) \in R_{n\times m}$; $\boldsymbol{M}$ and $\boldsymbol{N}$ are coefficient matrix, with $\boldsymbol{M} \in R_{n\times n}$ and $\boldsymbol{N} \in R_{n\times r}$; u_i is input. Eq. (5) describes the leader's dynamics in a non-linear form:

$$\dot{x}_0 = \boldsymbol{M}x_0 + \boldsymbol{N}f(x_0) \tag{5}$$

where $\dot{x}_0$is dynamic variable quantity of leader, $\boldsymbol{M}$ and $\boldsymbol{N}$ are coefficient matrix, $x_0 \in R_n$ denotes the states of the leader agent. It is important to show that the states of the leader just evolve in its own way. It is not affected by any of its followers. More importantly, the leader can provide information of its trajectory for the other agents to follow.

Definition 1 for initial condition x_i (0) of the q agents, i = 1, 2…N, the leader-following consensus of the system is achieved if the following condition is satisfied by applying the input u_i.

$$\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0 \tag{6}$$

In this paper, it is assumed that the communication topology and the undirected topology $\boldsymbol{G}$ is connected, then the leader sends its information to one follower. Assume that the non-linear term in the system satisfies the Lipschitz condition which there will be a constant $\theta > 0$, such that

$$\begin{gathered} \|f(a) - f(b)\| \le \theta\|a - b\| \\ \forall a, b \in R_n \end{gathered} \tag{7}$$

Lyapunov Theory for Controller Design

The Lyapunov second method, that is now known as the Lyapunov stability criterion, which apply of a Lyapunov function $\boldsymbol{V(x)}$. It is analog to the potential function in the classical dynamics. The equation for a system is represented in Eq. (8).

$$\dot{x} = f(x) \tag{8}$$

where $\dot{x}$ is dynamic variable quantity, $f(x)$ is dynamic function of x, having a point of equilibrium at x = 0. It is a function where:

$V(x): R_n \to R$
$V(x) = 0$ if and only if $x = 0$
$V(x) > 0$ if and only if $x \neq 0$

such that

$$\dot{V}(x) = \frac{dV(x)}{dt} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(x) \le 0 \tag{9}$$

for all the values of $x \neq 0$;
Important remark: for asymptotic stability, $\dot{V}(t) < 0$ for $x \neq 0$ is needed (negative definite).

Optimization Methods for Obtaining the Adjustable Gain

The controller is designed by using Eq. (10).

$$u_i(t) = -\gamma \boldsymbol{F}^{\boldsymbol{T}}(x_i(t))\boldsymbol{P}(\sum_{j=1}^{q} a_{ij}(e_{ij}) + a_{i0}(e_{i0})) \tag{10}$$

where γ is gain, $\boldsymbol{F}^{\boldsymbol{T}}(x_i$(t)) is transpose of $\boldsymbol{F}(x_i)$, a_{ij} is adjacency matrix of the i^{th} agent and j^{th} agent, simulation solution matrix $\boldsymbol{P}$, e_{ij} is state error of the i^{th} agent and j^{th} agent, $e_{ij} = x_i - x_j$ and γ is the gain to be designed and $\boldsymbol{P}$ is the matrix to be computed. The γ in this design needs to be solved by an optimization method.

Bisection Algorithm Approach

The first step is to search for a lower boundary on the decision variable (any feasible value is acceptable). The second step is to search for an upper boundary on the decision variable (e.g., decrease the upper bound until a feasible problem for a fixed decision variable is infeasible). The third step is to start bisection procedure, i.e., check the value of the decision variable between the lower and the upper boundary. If it is feasible, update the lower boundary, otherwise update the upper boundary. The fourth step is to repeat the process until the bounds are sufficiently closed to the optimal value.

Authentication Method

For the research of non-linear multi-agent system consensus, MATLAB (simulation and Simulink) was used to verify the design results. First, the simulation diagram of a distributed structure is developed. The second step is to write programming code with the calculated parameters. The third step is to compile and run the program. In the process of running the program, whether the matrix $\boldsymbol{P}$ can be computed, if there is a matrix $\boldsymbol{P}$, it indicates the design has achieved initial success. The fourth step is to study the simulation result of the non-linear multi-agent system. If the five agents have converged to its steady state after a period of motion in the chart, it indicates the consensus control of nonlinear multi-agent has been successfully designed.

Control Design Derivation

For simplicity, the system has been rewritten as:

$$\dot{x}(t) = (\boldsymbol{I_q} \otimes \boldsymbol{M})x(t) + (\boldsymbol{I_q} \otimes \boldsymbol{N})\overline{f}(x(t)) + \overline{F}(x)u(t) \tag{11}$$

where

$$\boldsymbol{I_q} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}$$
$$x(t) = x_1(t) + x_2(t) + \cdots + x_q(t)$$
$$\overline{f}(x(t)) = f(x_1(t) + f(x_2(t) + \cdots + f(x_q(t))$$
$$u = u_1 + u_2 + \cdots + u_q$$
$$\overline{F}_{(x)} = F_{(x_1)} + F_{(x_2)} + F_{(x_3)} + F_{(x_4)} + \cdots + F_{(x_q)}$$
$$\sum_{i=1}^{q} \boldsymbol{M} = (\boldsymbol{I_q} \otimes \boldsymbol{M})$$
$$\sum_{i=1}^{q} \boldsymbol{N} = (\boldsymbol{I_q} \otimes \boldsymbol{N})$$

and then the control input is that

$$u(t) = -\gamma \overline{F}^{T}{}_{(x(t))}(\boldsymbol{O} \otimes \boldsymbol{P})e_{(t)} \tag{12}$$

where

$$\sum_{i=1}^{q} (a_{ij} + a_{i0}) = 0$$
$$\sum_{i=1}^{q} (e_{ij} + e_{i0}) = e_{(t)}$$

Note that the tracking error is denoted as

$$e_i(t) = x_i(t) - x_0(t) \tag{13}$$

and the following can be obtained

$$\dot{e}_i(t) = \boldsymbol{M}e_i(t) + \boldsymbol{N}(e_{fxi0}) + F(x_i(t))u_i(t) \tag{14}$$

$$e_{fxi0} = f\big(x_i(t)\big) - f\big(x_0(t)\big) \tag{15}$$

then

$$\dot{e}(t) = \dot{x}(t) - \dot{x}'(t) \tag{16}$$

According to Kronecker product algorithm

$$\sum_{i=0}^{q} \dot{x}_0(t) = (\boldsymbol{I_q} \otimes q)\dot{x}_0 = \dot{x}_0'(t) \tag{17}$$

and then

$$\dot{e}(t) = (\boldsymbol{I_q} \otimes \boldsymbol{M})e(t) + (\boldsymbol{I_q} \otimes \boldsymbol{N})F'(x(t)) + \overline{F}(x)(-\overline{F}^{T}(x(t))(O \otimes P)e(t)) \tag{18}$$

where

$$F'_{(x(t))} = \overline{f}_{(x(t))} - f_{(x_0(t))}$$
$$\overline{f}(x(t)) = f(x_1(t)) + \cdots + f(x_q(t))$$

Select the Lyapunov condition as

$$V(t) = e^{T}(t)(\boldsymbol{O} \otimes \boldsymbol{P})e(t) \tag{19}$$

The differentiation is

$$\dot{V}(t) = e^T(t)\big(\boldsymbol{O} \otimes (\boldsymbol{PM} + \boldsymbol{M^T P})\big)e(t) + 2e^T(t)(\boldsymbol{O} \otimes \boldsymbol{PN})F'(x(t)) \\ -2e^T(t)(\boldsymbol{O} \otimes \boldsymbol{P})\overline{F}(x)(\overline{F}^T(x)(\boldsymbol{O} \otimes \boldsymbol{P})e(t)) \tag{20}$$

Note that

$$2e^T(t)(\boldsymbol{O} \otimes \boldsymbol{PN})F'(x(t)) \leq 2\gamma(\boldsymbol{N})e^T(t)(\boldsymbol{O} \otimes \boldsymbol{P})e(t) \tag{21}$$

and the third item is negative definite.

$$\dot{V}(t) \leq e^T(t)(\boldsymbol{O} \otimes (\boldsymbol{PM} + \boldsymbol{M}^T\boldsymbol{P} + 2\gamma(\boldsymbol{N})\boldsymbol{P}))e(t) \\ -2e^T(t)(\boldsymbol{O} \otimes \boldsymbol{P})^T\overline{F}(x)(\overline{F}^T(x)(\boldsymbol{O} \otimes \boldsymbol{P})e(t)) \tag{22}$$

Thus, if the following condition satisfies,

$$\boldsymbol{PM} + \boldsymbol{M}^T\boldsymbol{P} + 2\gamma(\boldsymbol{N})\boldsymbol{P} < 0 \tag{23}$$

where $\gamma(N)$ is the constant matrix N's maximum singular value. Then

$$\dot{V}(t) < 0 \tag{24}$$

This means

$$\lim_{t\to\infty} e(t) = 0 \tag{25}$$

where the consensus error will converge to 0.

For the controller design, these problems need to be solved, and the adjustment of the design gain and different γ values need to be checked and verified. The main objective is to investigate the minimum energy γ, which will be defined as the best value in our approach.

Numerical Simulation

In this section, Simulink which is a MATLAB-based graphical programming environment was applied for the simulation to verify the theoretical results in the previous section.

Setup of the Non-Linear Leader-Follower Multi-Agent Systems

The dynamics of the leader is described by Eq. (5), where $x_o \in R_n$ denotes the states of the leader agent. It is required to show that the states of the leader just evolve in its own way, demonstrate that it is not affected by any followers. More importantly, the leader will provide the information of its trajectory for the other agents to follow. The dynamics of the followers are described by Eq. (4), where $x_i \in R_n$ is the state vector of agent $i, i = 1,2, \dots, q$, and $u_i \in R_m$ is the control input, $f(x_i(t)) \in R_r$ and $F(x_i) = (F_1(x_i), F_2(x_i), \dots\dots, F_m(x_i)) \in R_{n\times m}$. $M \in R_{n\times n}$ and $N \in R_{n\times r}$. For example, some non-linear function as some multi-agent functions. In the simulation, the non-linear term $f(x_i)$ in the system considered in this section is a non-linear cosine function which satisfies the Lipschitz condition that there exists a constant $\theta > 0$, described by Eq. (7), and θ is equal one. In the simulation, the network of multi-agent is consisting of a leader and its five followers. The network diagram is shown in Figure 2.

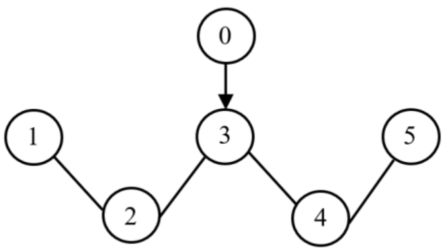

Figure 2. Non-linear multi-agent distributed working principal diagram

The controller to be designed with Eq. (10), where $e_{ij} = x_i - x_j$and γ is the gain to be designed and $\boldsymbol{P}$ is the matrix to be computed. The aim of consensus of the multi agents is to guarantee the states of q agents, which tend to be identical according to the states of the leader x_o. Specifically, for initial condition $x_i(0)$ of the q agents, $i = 0, 1, 2, \dots, q$, the system's leader following consensus is achieved if the following condition is satisfied by applying the control input described by Eq. (6).

First, the optimization problem can be solved by Eq. (23). This equation is used to obtain γ and P. For adjusting gain design, it is required to be checked with different values of γ. The main objective is to investigate the minimum energy γ, which will be defined as the best value. In the Simulink module as shown in Figure 3, Subsystem5 is the leader. Subsystem to Subsystem4 are the followers. Subsystem6 receives all information from the leader (Subsystem5) and serves the function of follow (Subsystem to Subsystem4). Scope5 receives the information from the leader (Subsystem5) and displays the leader (Subsystem5) motion status. Scope4 saves all the schematic diagrams (Scope5, Scope, Scope2). To Workspace block saves each input program. Scope and Scope2 display the final results. In the simulation diagram, Scope represents X_1 simulation diagram, Scope2 represents X_2 simulation diagram.

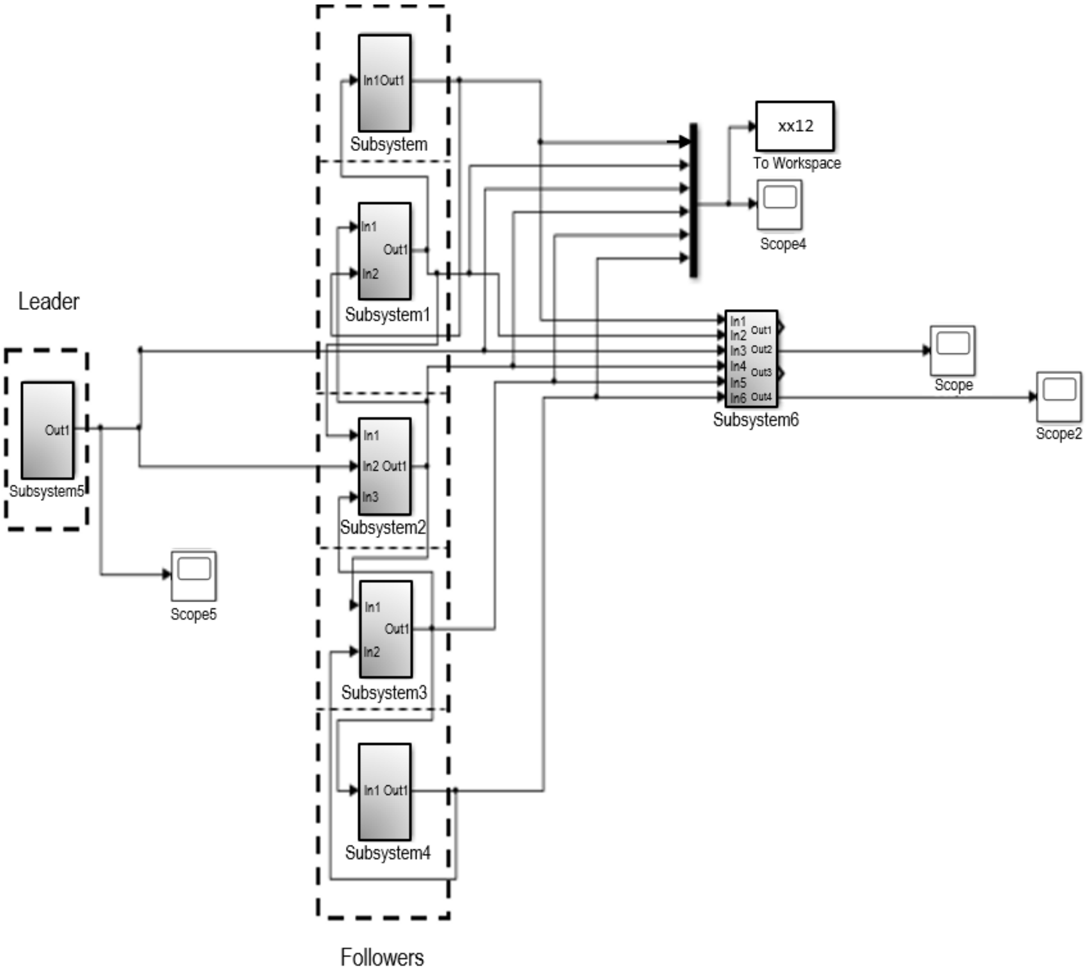

Figure 3. Non-linear multi-agent distributed Simulink module

SIMULATION RESULTS

$F(x) = \cos(x)$ was used to simulate and verify the stability of the new developed model as discussed earlier. Initially, γ was assumed as 5.0, then the simulation was repeated with γ as 1.0, 0.5 and 0.022 respectively. In the case of using γ as 5.0 for $f(x) = \cos(x)$, the system matrix was given as:

$$M = \begin{bmatrix} 2 & -5 \\ 8 & -5 \end{bmatrix} \qquad N = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

In this case, consider the non-linear function of $f(x) = \cos(x)$. "V" stands for speed. "S" stands for location. Assuming γ = 5.0 in the first simulation:

$$P = \begin{bmatrix} 0.6109 & -0.2474 \\ 0.2474 & 0.3481 \end{bmatrix}$$

Figure 4 shows the error of the state X_1 between the leader and the following agents' error, the simulation of the non-linear multi-agent system is a two-dimensional space simulation model, the abscissa represents the time of the non-linear multi-agent system, and the ordinate represents the velocity or displacement of the initial state. From Figure 4, it can be said that the designed distributed control system performs satisfactorily since the five agents have converged to its steady state after a period. Figure 5 shows the error of the state X_2 between the leader and the following agents' error, the simulation of the non-linear multi-agent system is a two-dimensional space simulation model, so the X_2 of the non-linear multi-agent system is simulated by the knowledge. The abscissa represents the time of the non-linear multi-agent system. In addition, the ordinate represents the velocity or displacement of the initial state. From Figure 5, it can be said that the designed distributed control system performs satisfactorily since the five agents have converged to its steady state after a period.

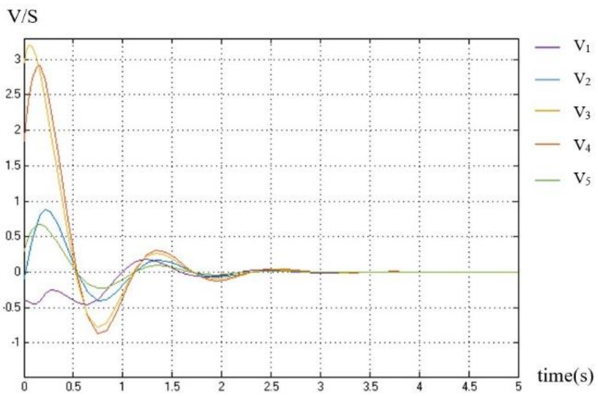

Figure 4. Simulation diagram X_1 of non-linear multi-agent (γ = 5.0)

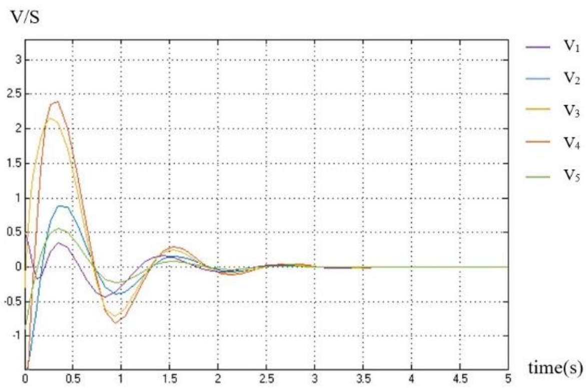

Figure 5. Simulation diagram X_2 of non-linear multi-agent (γ = 5.0)

Next, the simulation was repeated with $\gamma = 1.0$. It was found that

$$P = \begin{bmatrix} 60.2812 & -25.0021 \\ -25.0021 & -25.0021 \end{bmatrix}$$

When the simulation was repeated with $\gamma = 0.5$. It was found that

$$P = \begin{bmatrix} 1.4523 & -0.6287 \\ -0.6287 & 0.9362 \end{bmatrix}$$

Finally, the simulation was repeated with $\gamma = 0.022$. It was found that

$$P = \begin{bmatrix} 5.3846 & -2.5751 \\ -2.5751 & 3.3866 \end{bmatrix}$$

Figures 6, 8 and 10 show the errors of the state X_1 between the leader and the following agents' errors respectively to γ = 1.0, 0.5 and 0.022. Figures 7, 9 and 11 show the errors of the state X_2 between the leader and the following agents' errors respectively to γ = 1.0, 0.5 and 0.022. From Figures 6 to 11, it can be said that the designed distributed control system performs satisfactorily since the five agents have converged to its steady state after a period. Based on the simulation results, the minimum energy value found when $\gamma = 0.022$. In addition, this result is the optimal solution for our non-linear multi-agent system.

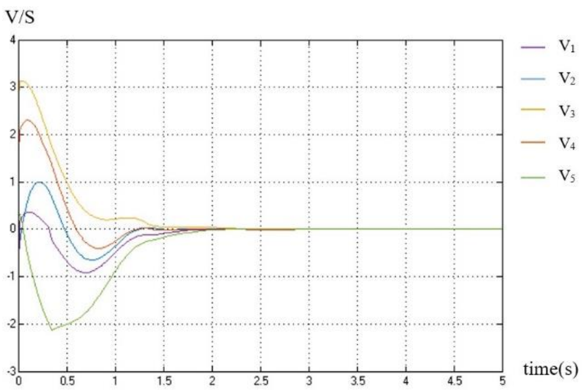

Figure 6. Simulation diagram X_1 of non-linear multi-agent (γ = 1.0)

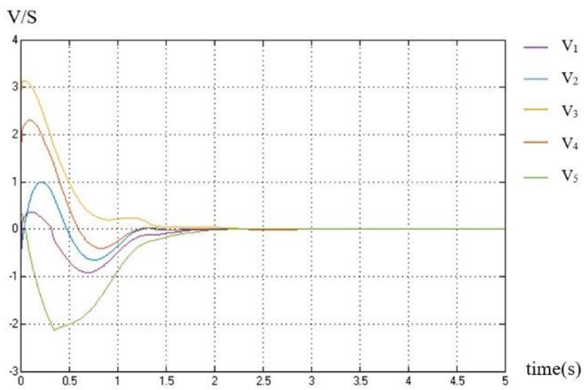

Figure 7. Simulation diagram X_2 of non-linear multi-agent (γ = 1.0)

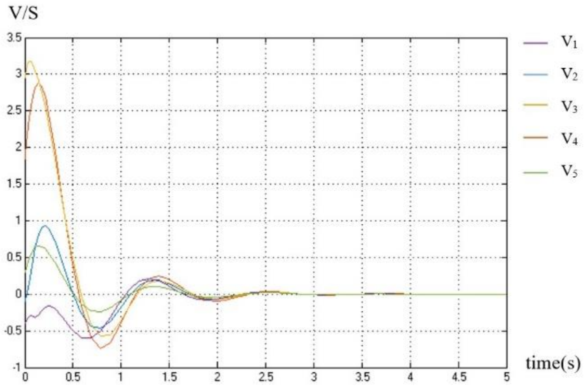

Figure 8. Simulation diagram X_1 of non-linear multi-agent ($\gamma = 0.5$)

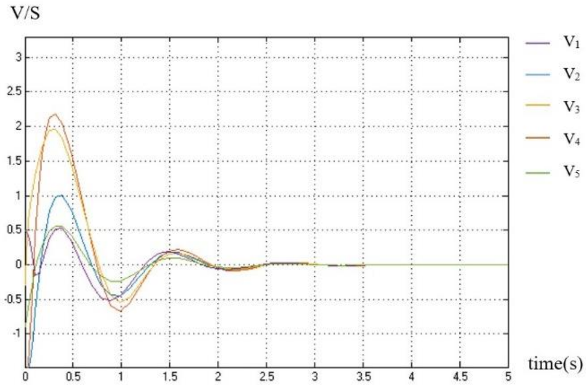

Figure 9. Simulation diagram X_2 of non-linear multi-agent ($\gamma = 0.5$)

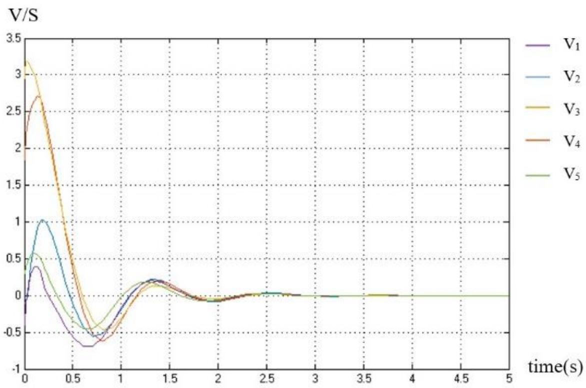

Figure 10. Simulation diagram X_1 of non-linear multi-agent ($\gamma = 0.022$)

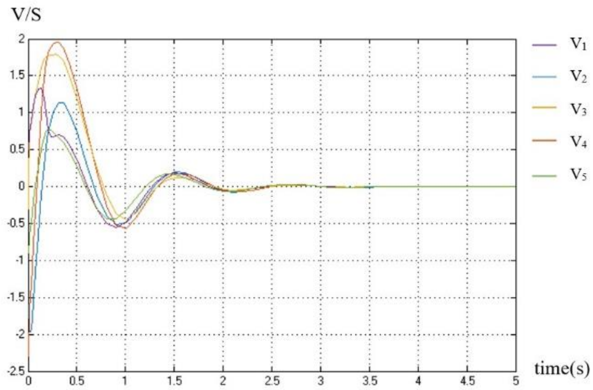

Figure 11. Simulation diagram X_2 of non-linear multi-agent ($\gamma = 0.022$)

CONCLUSION

A mathematical model for consensus control system has been developed based on Lyapunov Theory and nonlinear dynamics functional equations. This paper mainly focuses on a class of nonlinear multi-agent systems, using the Lyapunov method to develop a distributed controller with dynamic nonlinearity and leader-following communication characteristics. From the simulation results, it can be concluded that the designed distributed control system performs satisfactorily since the five agents have converged to its steady state after a period. Numerical simulations demonstrate the feasibility of the developed mathematical model for consensus control system. A new type of distributed controller for this type of nonlinear multi-agent system was developed. It has more advantages compared with the linear multi-agent system. The distributed controller designed in this paper is only for a special class of non-linear multi-agent systems. When designing a distributed controller, the modeling errors and other factors of the system model are not considered. Thus, the method is not expected to be applied in all kinds of non-linear multi-agent systems.

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