ORIGINAL ARTICLE

Numerical investigation of heat transfer characteristics for blood/water-based hybrid nanofluids in free convection about a circular cylinder

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ABSTRACT – This paper investigates hybrid nanofluids flowing around a circular cylinder of free convection under the constant surface heat flux. Nanoparticles of copper oxides, Gold, and Aluminum (CuO, Au, Al) are considered to support the heat transfer performance of blood/water-based hybrid nanofluids. The governing model for hybrid nanofluids which is in form of non-linear partial differential equations (PDEs) are first transformed to a more convenient form by similarity transformation approach then approximated numerically by the Keller box method. Several comparatives are performed in this work resulting in the superiority of the hybrid-nanofluid over regular nanofluid in terms of heat transfer rate, velocity, and local skin friction coefficient. Findings confirmed that the surface temperature and temperature field are augmented, with increasing volume fraction for nanoparticles. Also, Gold nanoparticles give a higher result for all examined physical properties than Aluminum and copper oxides nanoparticles.

INTRODUCTION

Thermal properties of heat transfer fluids have recently gained more attention because of its numerous applications in physics, engineering, in addition to thier importance in the industry. Numerous literatures have shown that majority of the convectional fluids used for heat transfer including; oil, ethylene glycol, water, and many more, are known to limits the enhancement of heat transfer due to their low thermal conductivity. In recent years, several researchers across the globe have increased their interest in nanofluids to improve heat transfer performance, fluid flow characteristics, the thermal conductivity of the fluid, and many more. Nanofluid was first introduced by Choi and Eastman [1], where the author explained it as being a mixture of the base fluid such as glycol, engine oil, water, and dispersed nanometer-sized particles. The dispersed nanometer-sized particles with an approximate diameter less than 100 nm are from a type of materials made from metals (Cu, Ag), oxides (Al2O3, CuO), carbides (SiC), or carbon nanotubes (CNTs, MWCNTs, diamond) [2].

The concept of nanofluids by Choi and Eastman [1] and a study on the pattern of nanofluids flow for heat conduction by Kumar et al. [3] has led to numerous studies most of which expand the concept in different directions [4, 5]. A recent study by Buongiorno [6] on heat transfer enhancement of convective transport in nanofluid has garnered considerable attention. The author explored the flow in a porous medium filled with nanofluid. Nield and Kuznetsov [7] extended the Buongiorno model to a porous medium problem by Cheng and Minkowycz [8] and present the impact of nanoparticles on natural convection on the vertical plate of a porous medium. Nield and Kuznetsov [9, 10] further extended their results from [7] to free convection of a porous medium immersed past a vertical plate by a nanofluid. Besides, Tiwari and Das [11] investigation that provided a mathematical model highlighting the effect of nanoparticles volume fraction has received widespread employment lately. Sheremet et al. [12] studied the flow characteristics of nanofluids generated by free convection in a square cavity utilizing Tiwari-Das's model. Waini et al. [13] Modeled the heat transport through nanofluid under the influences of Dufour and Soret on a moving thin needle via Tiwari-Das's model. See also these comprehensive references where the governing models have been constructed with the aid of Tiwari-Das's model [14-17].

Several works of the literature suggest that surface charge or surfactant technology are some of the techniques used to suspend nanoparticles in the nanofluid [18]. Diverting from the vertical plate approach, recent studies considered the convection boundary layer flow over a horizontal circular cylinder, solid sphere, etc. For example, some researchers [19-22] studied the convection boundary layer flow in nanofluid past a solid sphere and horizontal circular cylinder. More studies on the boundary layer flow in a nanofluid over cylinder are given in the following references [23-26].

Lately, the area of nanofluids has been expanding into more extensive, rigorous, and exciting disciplines, with numerous applications in industries such as microelectronics, air-conditioning, refrigeration, mobile computers processors, coolant in machining, and many more. The development of numerical simulation methods was a response to

the continuous demand of numerical computation. These methods usually base on constructive existence proofs. Based on the current trend of research in this area, several authors are now investigating the possibilities of considering fluids containing more than one type of nanoparticle. This process is called hybrid nanofluids. These types of nanofluids are employed in several disciplines for enhancing heat transfer. Suresh et al. [27] are among the first researchers to study the hybrid nanofluid that possesses improved heat transfer advantages with rheological conduct in addition to enhanced thermo-physical features. Other studies on the hybrid nanofluids are presented by Devi and Devi [28] on the flow of hydro-magnetic hybrid Cu-Al2O3/water nanofluid. Hayat et al. [29] analyzed the rotating hybrid flow of Ag- CuO/H2O nanofluid under radiation and partial slip boundary. Khashi'ie et al. [30] present a study on the Riga plate of cylinder surfaces. Waini et al. [31] studied the application of Al2O3-Cu/water in the boundary layer flow for thin needle and sensor surfaces. More studies on hybrid nanofluids can be referred to [32-36].

Human blood is classified as a non-Newtonian fluid that carries physical properties. This fluid is studied as a nanofluid and gives physic behaviors compares with other fluids when using it as base fluid. In addition, there are many studies that consider the human blood as base fluid in convection boundary layer flow. Dash et al. [37] studied the model of flow properties in a Casson fluid in a tube filled with the presence of a homogeneous porous medium using human blood as a base fluid. The problem of magneto-hydrodynamics (MHD) free convection in Casson nanofluid utilizing Carbon Nanotubes (CNTs) suspended in human blood was investigated by Alkasasbeh et al. [38]. The theoretical study for the physical features of a two-dimensional incompressible in the existence of human blood as a hybrid was considered by Sadaf and Abdelsalam [39]. The model of human blood flow has been improved for carbon nanotubes with the effect of thermal radiation and chemical reaction by Kalita et al [40]. Besides that, a lot of researchers examined the convection boundary layer flow using human blood base fluid, such as [41-43].

Motivated by the contributions of researchers as Swalmeh [44] and Alwawi [45], as well as, depending on Suresh et al. [27], and the list of literature discussed above, this article concern on filling the research gap by studying the problem of steady laminar free convection boundary layer flow in presence of an incompressible hybrid nanofluid, on a horizontal circular cylinder. The governing nonlinear partial differential equations were solved using Keller-box numerical method but under some boundary conditions. Keller-box method [46] is a well-known solver that results in the numerical solution of convection boundary layer flow problems. Preliminary results of the involved parameter have been plotted and discussed. Based on the literature considered in this study, the present problem is yet to be explored by researchers, and thus, our results are new.

PROBLEM FORMULATION

As illustrated in Figure 1, a free convection flow of water /human blood containing hybrid nanoparticles about a circular cylinder was assumed. The symbol x refers to the axis measured on the circumference of the cylinder starting from the point of stagnation and, y refers to the axis that measures the perpendicular distance to its surface. q_w indicates constant heat flux, T_∞ is the surrounded temperature of the liquid which remains fixed, and g stands for gravity vector.
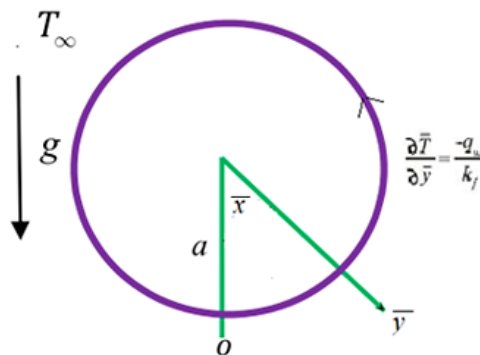

Figure 1. Schematic physical model

Based on the above consideration, the mathematical model governing flow for our problem is (see Tiwari and Das [11]):

$$\frac{\partial(\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{v})}{\partial\bar{y}} = 0, \tag{1}$$

$$\rho_{hnf}\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = \left(\mu_{hnf} + \kappa\right)\frac{\partial^2\bar{u}}{\partial\bar{y}^2} + \beta_{hnf}\,g\left(T - T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right), \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial\bar{x}} + \bar{v}\frac{\partial T}{\partial\bar{y}} = \alpha_{hnf}\frac{\partial^2 T}{\partial\bar{y}^2}, \tag{3}$$

alongside the following conditions:

$$\overline{u} = \overline{v} = 0, \frac{\partial T}{\partial \overline{y}} = \frac{-q_w}{k_f} \text{ at } \overline{y} = 0,$$

$$\overline{u} \to 0, T \to T_\infty, \text{ as } \overline{y} \to \infty,$$

(4)

The symbols used in this study follow from the notations section. Table. 1, present the properties of hybrid nanofluid/nanofluid.

Table 1. Properties of hybrid nanofluid / nanofluid (addressed by Manjunatha et al. [47])

Properties of Nanofluid	Properties of Hybrid Nanofluid
$\rho_{nf} = (1-\gamma_2)\rho_f + \gamma_2\rho_f,$	$\rho_{hnf} = (1-\gamma_2)[(1-\gamma_1)\rho_f + \gamma_1\rho_{s1}] + \gamma_2\rho_{s2},$
$(\rho c_p)_{nf} = (1-\gamma_2)(\rho c_p)_f + \gamma_2(\rho c_p)_{s1},$	$(\rho c_p)_{hnf} = (1-\gamma_2)[(1-\gamma_1)(\rho Cp)_f + \gamma_1(\rho Cp)_{s1}] + \gamma_2(\rho Cp)_{s2},$
$\beta_{nf} = (1-\gamma_2)\beta_f + \gamma_2\beta_s$	$\beta_{hnf} = (1-\gamma_2)[(1-\gamma_1)\beta_f + \gamma_1\beta_{s1}] + \gamma_2\beta_{s2}.$
$\mu_{nf} = \dfrac{\mu_f}{(1-\gamma_2)^{2.5}},$	$\mu_{hnf} = \dfrac{\mu_f}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}},$
$\dfrac{k_{nf}}{k_f} = \dfrac{(k_s+2k_f)-2\gamma_2(k_f-k_s)}{(k_s+2k_f)+\gamma_2(k_f-k_s)},$	$\dfrac{k_{nf}}{k_{bf}} = \dfrac{k_{s2}+2k_{bf}-2\gamma_2(k_{bf}-k_{s2})}{k_{s2}+2k_{bf}+\gamma_2(k_{bf}-k_{s2})},$
$\alpha_{nf} = \dfrac{k_{nf}}{(\rho c_p)_{nf}},$	$\dfrac{k_{bf}}{k_f} = \dfrac{k_{s1}+2k_f-2\gamma_1(k_f-k_{s1})}{k_{s1}+2k_f+\gamma_1(k_f-k_2)},$
	$\alpha_{hnf} = \dfrac{k_{hnf}}{(\rho cp)_{hnf}},$

where γ_2 is the volume fraction for the CuO, γ_1 is the volume fraction for nanoparticles (Au-Al). $\gamma_1 = \gamma_2 = 0$ represents a regular Newtonian fluid.

To non-dimensionlization process, we'll present the following variables:

$$x = \frac{\overline{x}}{a}, y = Gr^{1/5}\left(\frac{\overline{y}}{a}\right), u = \left(\frac{a}{v_f}\right)Gr^{-2/5}\overline{u}, v = \left(\frac{a}{v_f}\right)Gr^{-1/5}\overline{v}, \theta = Gr^{1/5}\left(\frac{T-T_\infty}{aq_w/k_f}\right),$$

(5)

where $Gr = g\beta_f(aq_w/k)(a^3/v_f^2)$ is the Grashof number.

By utilizing Eq. (5), the governing model become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

(6)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\rho_f}{\rho_{hnf}}\left(\frac{1}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}}\right)\frac{\partial^2 u}{\partial y^2} + \frac{1}{\rho_{hnf}}\left((1-\gamma_2)[(1-\gamma_1)\rho_f + \gamma_1\frac{\rho_{s1}\beta_{s1}}{\beta_f}] + \gamma_2\frac{\rho_{s2}\beta_{s2}}{\beta_f}\right)\theta\sin x,$$

(7)

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\left[\frac{k_{hnf}/k_f}{(1-\gamma_2)[(1-\gamma_1)+\gamma_1(\rho Cp)_1/(\rho Cp)_f] + \gamma_2(\rho Cp)_{s2}/(\rho Cp)_f}\right]\frac{\partial^2 \theta}{\partial y^2},$$

(8)

where $Pr = v_f/\alpha_f$ is the Prandtl number.

Subject to:

$$u = v = 0, \theta' = -1, \text{ at } y = 0,$$

$$u \to 0, \theta \to 0, \text{ as } y \to \infty.$$

(9)

Next, Eq. (6)-(8) are reduced depending on the following transformation:

$$\psi = xf(x.y), \quad \theta = \theta(x, y), \tag{10}$$

and ψ refers to the stream function defined as:

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}, \tag{11}$$

consequently, the simplified Eq. (6)-(8) are:

$$\frac{\rho_f}{\rho_{hnf}}\left(\frac{1}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}}\right)\frac{\partial^3 f}{\partial y^3} + \frac{1}{\rho_{hnf}}\left((1-\gamma_2)[(1-\gamma_1)\rho_f + \gamma_1\frac{\rho_{s1}\beta_{s1}}{\beta_f}] + \gamma_2\frac{\rho_{s1}\beta_{s2}}{\beta_f}\right)\frac{\sin x}{x}\theta$$
$$+ f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right), \tag{12}$$

$$\frac{1}{\Pr}\left[\frac{k_{hnf}/k_f}{(1-\gamma_2)[(1-\gamma_1)+\gamma_1(\rho Cp)_1/(\rho Cp)_f]+\gamma_2(\rho Cp)_{s2}/(\rho Cp)_f}\right]\frac{\partial^2 \theta}{\partial y^2} + f\frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right) \tag{13}$$

with boundary conditions:

$$f = \frac{\partial f}{\partial y} = 0, \theta' = -1 \text{ at } y = 0,$$
$$\frac{\partial f}{\partial y} \to 0, \ \theta \to 0, \text{ as } y \to \infty. \tag{14}$$

When x approaches zero $(x \approx 0)$, i.e., at the lower stagnation point, the previous model turns into:

$$\frac{\rho_f}{\rho_{hnf}}\left(\frac{1}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}}\right)\frac{\partial^3 f}{\partial y^3} + f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{1}{\rho_{hnf}}\left((1-\gamma_2)[(1-\gamma_1)\rho_f + \gamma_1\frac{\rho_{s1}\beta_{s1}}{\beta_f}] + \gamma_2\frac{\rho_{s1}\beta_{s2}}{\beta_f}.\right)\theta = 0, \tag{15}$$

$$\frac{1}{\Pr}\left[\frac{k_{hnf}/k_f}{(1-\gamma_2)[(1-\gamma_1)+\gamma_1(\rho Cp)_1/(\rho Cp)_f]+\gamma_2(\rho Cp)_{s2}/(\rho Cp)_f}\right]\frac{\partial^2 \theta}{\partial y^2} + f\frac{\partial \theta}{\partial y} = 0 \tag{16}$$

and boundary conditions are:

$$f(0) = f'(0) = 0, \theta'(0) = -1 \text{ as } y = 0,$$
$$f' \to 0, \ \theta \to 0 \text{ as } y \to \infty, \tag{17}$$

where the prime symbol indicates differentiation with respect to y

The local skin friction coefficient C_f, and local wall temperature θ_w can be written as

$$C_f = \frac{Gr^{-1/5}a^2}{\mu_f \nu_f}\tau_w, \qquad \theta_w = \theta(x,0), \tag{18}$$

where $\tau_w = \mu_{hnf}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}$, is the wall shear stress.

Applied non-dimensional variables (5) and boundary conditions (9), C_f becomes:

$$C_f = Gr^{-1/5}\left(\frac{1}{(1-\gamma_1)^{2.5}(1-\gamma_2)^{2.5}}\right)x\frac{\partial^2 f}{\partial y^2}(x,0), \tag{19}$$

NUMERICAL SOLUTION AND VALIDATION OF RESULTS

There are several reliable numerical methods for solving systems of equations for issues related to heat transfer in the boundary strata region, including the Keller Box method, which was introduced by Keller [46] in 1970. This method became very popular when Cebeci and Bradshaw [48] gave a detailed explanation of it. Later, many researchers relied on it in their studies, such as Nazar et al. [49, 50], Tham et al. [51, 52], Alkasasbeh et al. [53], Alwawi et al. [21, 45] and others. This method begins by transforming the systems of equations to the first degree and then finding the difference equations through the central differences method. Then, the system is linearized using Newton's method and finally written in the form of a matrix-vector and solved using the tri-diagonal elimination method. Also, calculations are also performed via the Matlab program. Numerical and graphical results are obtained for the effect of the parameter related to the flow characteristics. To emphasize the accuracy of our findings, they were compared with previously published outcomes dealt with by Merkin and Pop [54], and Nazar et al. [55] as it can be seen an excellent match of our results with those results in Tables 2 and 3.

Table 2. Comparison of local skin friction coefficient θ_w with Newtonian fluid ($\gamma_1 = \gamma_2 = 0$) at Pr = 0.7

x^o	Merkin and Pop [54]	Nazar et al. [55]	Present
0.0	1.996	1.996	1.996
0.2	1.999	1.999	1.999
0.4	2.005	2.004	2.004
0.6	2.014	2.013	2.012
0.8	2.026	2.026	2.025
1.0	2.043	2.044	2.044
1.2	2.064	2.065	2.066
1.4	2.089	2.091	2.090
1.6	2.120	2.123	2.123
1.8	2.158	2.161	2.158
2.0	2.202	2.207	2.206
2.2	2.256	2.262	2.260
2.4	2.322	2.329	2.327
2.6	2.403	2.413	2.411
2.8	2.510	2.523	2.519
3.0	2.660	2.681	2.677
π	2.824	2.828	2.825

Table 3. Comparison of local skin friction coefficient C_f with Newtonian fluid ($\gamma_1 = \gamma_2 = 0$) at Pr = 0.7

x^o	Merkin and Pop [54]	Nazar et al. [55]	Present
0.0	0.000	0.000	0.000
0.2	0.274	0.273	0.274
0.4	0.541	0.540	0.541
0.6	0.793	0.795	0.794
0.8	1.031	1.027	1.029
1.0	1.241	1.235	1.237
1.2	1.422	1.413	1.417
1.4	1.567	1.555	1.559
1.6	1.671	1.657	1.668
1.8	1.732	1.714	1.722
2.0	1.744	1.723	1.735
2.2	1.704	1.680	1.692
2.4	1.608	1.580	1.601
2.6	1.451	1.418	1.443
2.8	1.225	1.188	1.149
3.0	0.913	0.868	0.881
π	0.613	0.574	0.597

RESULTS AND DISCUSSION

The effect of the volume fraction parameters (γ_1, γ_2) on different physical quantities, such as local wall temperature, and local skin friction, as well as temperature and velocity profiles, have been schemed through Figures 2-9, where the values of volume fraction parameters (γ_1, γ_2) used ranged between 0.005 and 0.1. Table 4 illustrates the thermo-physical properties of ultrafine particles and base fluids.

Table 4. Different values of thermo-physical properties of nanoparticles and base fluids, [22, 56, 57]

Physical properties	Based fluid		Nanoparticles		
	HB	W	Au	Al	CuO
k (W/mK)	0.492	0.613	314.4	237	18
ρ (kg/m³)	1053	997.1	19320	2701	6510
ρc_p (J/kgK)	3594	4179	128.8	902	540
$\beta \times 10^{-5}(K^{-1})$	0.18	21	1.416	2.31	0.85
Pr	24	6.2	…	…	---

Figure 2 generally shows that the increment of γ_2 on local skin friction C_f in presence of Au-CuO, Al-CuO suspended in human blood hybrid nanofluid, also CuO suspended in human blood regular nanofluid, augmentation in γ_2 leads to a decrease in its curves values, which mean γ_2 and C_f have an inverse correlation. We could notice that the hybrid nanofluids have a clear curves elevation in their local skin fraction than the regular nanofluid if we take them in comparison. This implies that there is a noticeable difference between their properties and the role of adding other nanoparticles to the base fluid in getting effective results. Adding ultrafine particles of gold (Au) element shows a higher response in properties difference than adding ultrafine particles of Aluminium (Al) element. But their behaviour in response to the varying of x and γ_2 in this Figure could be considered close. Figure 3 confirms the inverse relation between γ_2 and local skin friction but for different fluid bases human blood and water. In both hybrid nanofluids with Au-CuO the increment of volume fraction γ_2 decreases C_f, but using the water as based fluid reflects on rising the curves higher than human blood with respect to *x*.

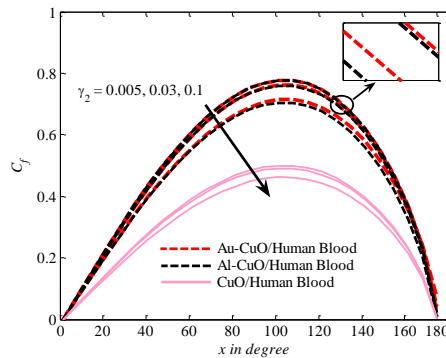**Figure 2.** Variation of local skin friction of Au-CuO, Al-CuO/human blood hybrid nanofluids and CuO- human Blood nanofluid for various values of *x* and γ_2
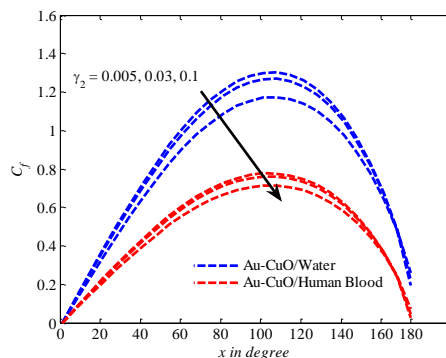

Figure 3. Variation of local skin friction of Au-CuO /water and Au-CuO / human blood hybrid nanofluids for various values of *x* and γ_2

Enhancement of the volume fraction $\gamma 2$ with the surface temperature θ_w is presented in Figure 4. It is found that the local wall temperature generally increased by the increase of the volume fraction γ_2 for Au-CuO, Al-CuO /human blood hybrid nanofluid, and CuO/ human blood nanofluid. The hybrid nanofluid supports the local wall temperature's values and makes this relationship stronger, and we notice adding Al or Au as a second nanoparticle to the nanofluid does not exhibit much contrast in their results, so they are close (or near) in their effect. The effect (action) of changing the base fluid to the relation of γ_2 versus θ_w is illustrated in Figure 5, which indicates that the water base fluid has the greater values of local wall temperature than the Human blood base fluid as x and γ_2 is getting increased. This may be as a result of the difference between the thermal conductivities values between the water and human blood (see table 4). In fact, the increase in nanoparticle volume fraction led to an increase in thermal conductivity and, as a result, an increase in surface temperature has occurred.
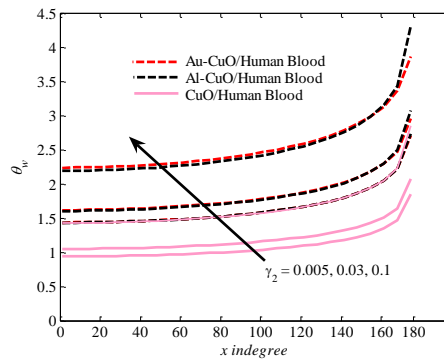

Figure 4. Variation of local wall temperature of Au-CuO,Al-CuO /human blood hybrid nanofluid and CuO/ human blood nanofluid for various values of x and γ_2
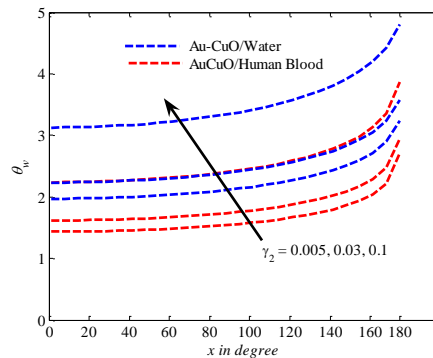

Figure 5. Variation of local wall temperature of Au-CuO/water and Au-CuO/human blood hybrid nanofluids for various values of x and γ_2

The temperature Field $\theta(0, y)$ is the third physical quantity taken into account as the volume fraction γ_2 varies. From Figure 6, we conclude that γ_2 and $\theta(0, y)$ have a direct relationship with all the nanofluids. CuO, Al-CuO /human blood hybrid nanofluids have a stronger correlation to the temperature field increment than CuO/ Human Blood nanofluid when the volume fraction and the x are increasing. Using water as a base fluid gives higher temperature curves than the curves of human blood as a base fluid which is seen obviously in Figure 7 for the same hybrid nanoparticles Au-CuO. At this point, we mention that the heat capacity of the water is greater than the human blood according to Table 4 as a possible main reason for the interpretation of the figure. Increased heat transfer from the cylinder's surface to the liquid is caused by an increase in the volume fraction of nanoparticles, which assists in increasing the thickness of the thermal layer, hence raising the temperature of the fluid.
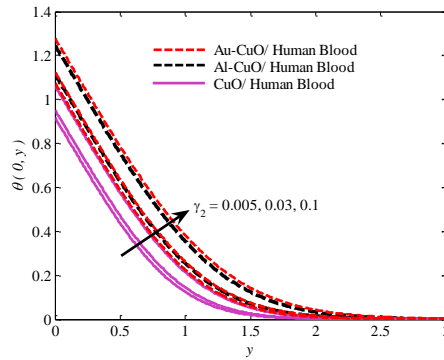
Figure 6. Variation of temperature for human blood hybrid nanofluid with Au-CuO, Al-CuO and human blood nanofluid with CuO for various values of x and γ_2
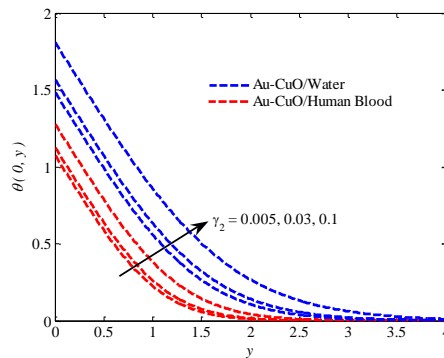

Figure 7. Variation of temperature for water and human blood hybrid nanofluids with Au-CuO for various values of x and γ_2

Another inverse correlation investigated between γ_2 and other physical quantities in this study is the velocity field $(df/dy)(0, y)$. Figures 8 and 9 clearly show this relation. In Figure 8, the curves values of $(df/dy)(0, y)$ get lower and lower along with the increase of γ_2 for human blood hybrid nanofluid with Au-CuO, Al-CuO, and Human blood nanofluid with CuO. This is maybe attributed to the role which γ_2 plays in increasing both the density and the viscosity of the hybrid nanofluid so that the velocity field is decreased. Figure 9 shows the different results obtained for the hybrid nanofluid when using different fluid base water and human blood with Au-CuO nanoparticles. The water base hybrid nanofluid keeps getting the higher values of velocity field during the decrement of γ_2 more than the curves of human blood base hybrid fluid, which seems sensible returning to the lower values of dynamic viscosity of water compared with human blood.
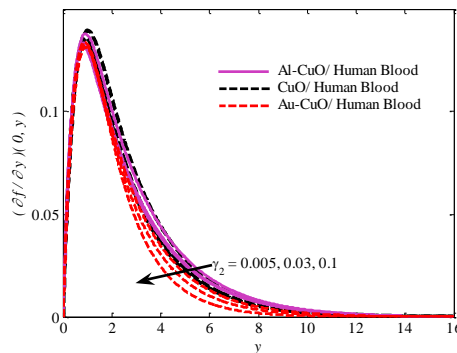

Figure 8. Variation of velocity for human blood hybrid nanofluid for Au-CuO, Al-CuO and Human blood nanofluid with CuO for various values of x and γ_2
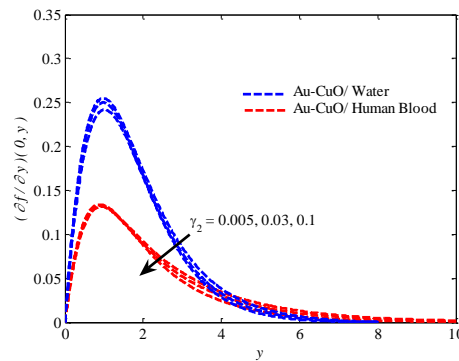
Figure 9. Variation of velocity for water and human blood hybrid nanofluids with Au-CuO for various values of x and γ_2

CONCLUSION

This paper investigated the problem of natural convection heat transfer in hybrid nanofluid over a horizontal circular cylinder. Besides, we also considered the constant heat flux boundary condition. From this consideration, the variation of volume fraction of (γ_1, γ_2) has been checked for its impact on the following physical quantities, local skin friction Cf , local wall temperature θ_w , temperature $\theta(0, y)$ and velocity profiles $(df / dy)(0, y)$ for CuO/human blood nanofluid, Au-CuO, Al-CuO suspended in Human Blood /water hybrid nanofluid. The following points conclude the results:

1. The hybrid nanofluid that contains two nanoparticles (CuO and either Au orAl) get clear different properties compared with nanofluid with one component (CuO only) of nanoparticles; Au-CuO, Al-CuO/human blood hybrid nanofluids demonstrate higher values of the physical properties $C_f, \theta_w, \theta(0, y)$ and velocity profiles $(df / dy)(0, y)$ than CuO/ human blood nanofluid.

2. The water base fluid with hybrid nanoparticle Au-CuO manifested an increase of the four physical quantities, namely $C_f, \theta_w, \theta(0, y)$, and velocity profiles $(df / dy)(0, y)$ compared with the human blood base fluid with hybrid nanoparticle Au-CuO.

3. The increase of volume fraction for the γ_2 leads to an increment in the local wall temperature θw and temperature field $\theta(0, y)$ in all cases while decreasing the local skin friction C_f and velocity field $(df / dy)(0, y)$.

4. *Au* nanoparticle gives higher results with $C_f, \theta_w, \theta(0, y)$ and velocity field $(df / dy)(0, y)$ than *Al* nanoparticle as a hybrid nanoparticle due to the difference of thermo-physical properties between these elements.

NOTATIONS

a	Radius of cylinder	θ	Temperature of nanofluid
C_f	Local skin friction coefficient	μ_f	Dynamic viscosity
Gr	Grashof number	ρ	Density
g	Gravity vector	(ρc_p)	Heat capacity
k	Thermal conductivity	τ_w	Wall shear stress
M	Magnetic parameter	γ_1	Volume fraction of *Au / Al*
Pr	Prandtl number	γ_2	Volume fraction of *CuO*
q_w	surface heat flux	ψ	Streem function
T	Temperature of the fluid	θ_w	Local wall temperature
T_∞	surrounded temperature		
u	x-component of velocity		Subscript
v	y-component of velocity	hnf	Hybrid – nanofluid
v_f	Kinematic viscosity of base fluid	s	Nanoparticles
α	Thermal diffusivity	nf	Nanofluid
β_f	Thermal expansion	f	host fluid

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