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ORIGINAL ARTICLE

Representation of bolted joints in a structure using finite element modelling and model updating

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ABSTRACT – Efficient and accurate finite element (FE) modelling of bolted joints is essential for increasing confidence in the investigation of structural vibrations. However, modelling of bolted joints for the investigation is often found to be very challenging. This paper proposes an appropriate FE representation of bolted joints for the prediction of the dynamic behaviour of a bolted joint structure. Two different FE models of the bolted joint structure with two different FE element connectors, which are CBEAM and CBUSH, representing the bolted joints are developed. Modal updating is used to correlate the two FE models with the experimental model. The dynamic behaviour of the two FE models is compared with experimental modal analysis to evaluate and determine the most appropriate FE model of the bolted joint structure. The comparison reveals that the CBUSH element connectors based FE model has a greater capability in representing the bolted joints with 86 percent accuracy and greater efficiency in updating the model parameters. The proposed modelling technique will be useful in the modelling of a complex structure with a large number of bolted joints.

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Finite element modelling; experimental modal analysis; dynamic behaviour; model updating; bolted joints

INTRODUCTION

Bolted joints are the most common joining method widely used in numerous industrial sectors, such as the automotive and construction industries. The key advantages of using bolted joints are due to its low cost, fast assembling work and easy to disassemble. Bolted joints are observed to be the essential elements in increasing structural damping and minimising the magnitude of the resonance of bolted structures [1–3].

The predictions of the dynamic behaviour of bolted structures can likely be performed efficiently using appropriate finite element (FE) modelling. However, the modelling generally has limited factors in accurately predicting the dynamic behaviour of the bolted structure due to especially the difficulty of creating reliable FE models of the bolts itself [4, 5]. Therefore, an appropriate and reliable modelling scheme of the bolted structure is significantly required and crucial in ensuring the accuracy of the predicted results of the dynamic behaviour of the bolted joint structure.

One of the main criteria for the development of a reliable FE model of the bolted structure is about accurate modelling of the bolts. A few FE based approaches, for instance, using FE element connectors such as CBAR [6, 7], CELAS [6, 7], CBEAM [8–10] and CBUSH [11–13] have been extensively utilised to represent the bolts. However, many of the reported studies do not reach a conclusion on the most appropriate elements that are applicable to represent the bolts in the bolted joint structure. Therefore, this research will investigate the most appropriate representation of the bolts in the bolted joint structure via the FE CBEAM and CBUSH element connectors.

This study put forwards a modelling scheme for developing an efficient FE model for a bolted joint structure, concentrating particularly on the bolts modelling. A normal modes analysis is performed, and the predicted natural frequencies and mode shapes are compared with the experimental modal analysis (EMA) counterparts. The most appropriate representation of bolts is selected based on the total error recorded from the comparison. FE model updating is used to update the initial FE model of the bolted joint structure in the light of experimental data and to improve the accuracy and reliability of the FE model. The updated FE model of the bolted joint structure could be used confidently for subsequent analyses.

FE MODELLING AND ANALYSIS

The FE modelling and analysis of the bolted structure was performed using the NX11 Simcenter 3D design software. In this work, the bolted joint structure consisted of two plates with equal dimensions, namely Plate A and Plate B, which are made of steel. The plates were joined together by using stainless-steel bolts and nuts of size M10. Each plate has a length of 380 mm, a width of 45 mm and a thickness of 6 mm. Figure 1 shows the 3D CAD model of the bolted structure.

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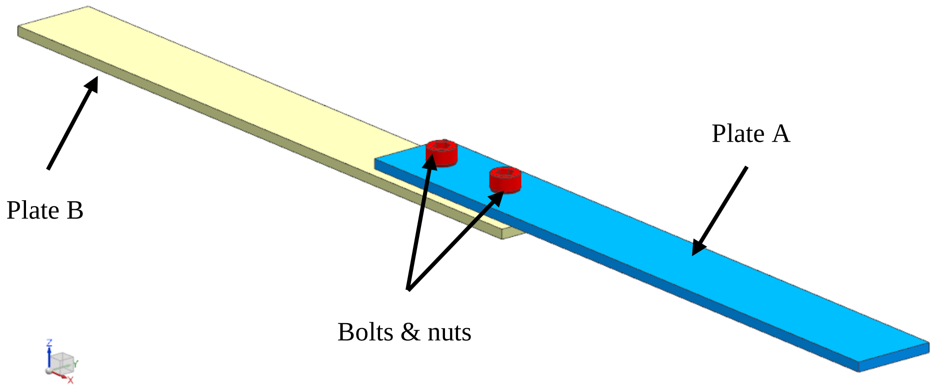

Figure 1. 3D CAD model of the bolted structure.

Table 1 shows the material properties of Plate A, Plate B, and bolt and nut. The tabulated assigned values of Plate A and Plate B are slightly varied as compared with that of the literature [14]. The material properties of Plate A and Plate B are systematically adjusted using the FE model updating method adopted by previous researchers [15–18]. The use of the FE model updating technique is mainly to improve the accuracy of FE models of Plate A and Plate B before they are assembled to form a bolted structure.

Table 1. Material properties for Plate A, Plate B and, bolts and nuts

Property	Plate A	Plate B	Bolt and nut
Young's modulus (N/mm^2)	206 400	209 400	193 000
Shear modulus (N/mm^2)	75 000	75 000	75 000
Poisson's ratio	0.302	0.316	0.27
Mass density (kg/mm^3)	7.458×10^{-6}	7.45×10^{-6}	7.86×10^{-6}

The mid-surfaces of the 3D CAD models of Plate A and Plate B were extracted. The mid-surfaces, which are in the form of 2D thin shell elements, were discretised into finite elements. Each meshed plate was created by using 992 CQUAD8 elements and 6 CTRI6 elements. CQUAD8 and CTRI6 elements were chosen for the meshing due to their capabilities of producing more accurate results than the other element types [19]. The element size adopted in this study was 5 mm. The size was selected after evaluating several convergent tests performed on the FE model with different mesh sizes [20]. Figure 2 shows the FE model of the bolted structure.

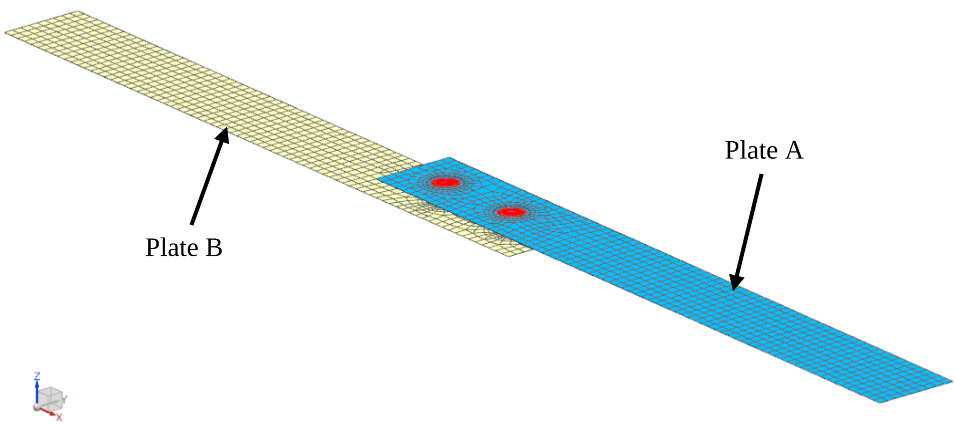

Figure 2. FE model of the bolted structure

In the modelling of the representation of the physical stainless-steel bolts and nuts, two types of elements namely the CBEAM and CBUSH elements used in this study, are compared to find the most appropriate representation of the bolt's shank [21, 22]. CBEAM is a beam element connector that supports the extension, torsion, bending in two perpendicular planes, and the associated shear. On the other hand, CBUSH is a structural scalar element connecting two non-coincident grid points, or two coincident grid points, or one grid point. Meanwhile, the bolt's head and nut are presented by RBE3. Figure 3 shows the cross-section of the bolted joint, which illustrates the representation of the bolt's shank, bolt's head and nut.

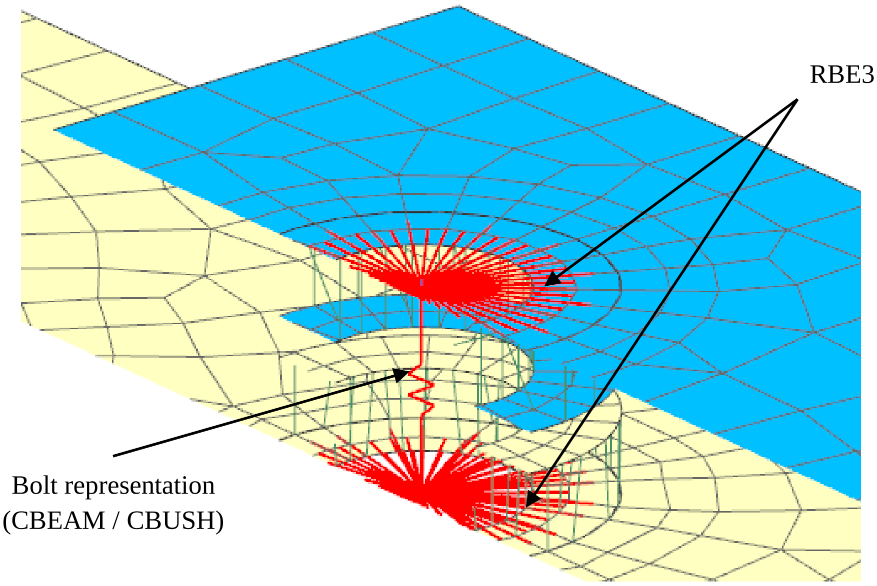

Figure 3. Cross-section of the bolted joints

A rod-shaped fore section with a radius of 5 mm was used in modelling the bolt's shank using CBEAM element. Table 2 shows the physical properties of the CBEAM element. The CBEAM element was assigned with stainless-steel properties as tabulated in Table 1.

Table 2. Physical properties entry for CBEAM

Parameter	Value
Radius	5 mm
Area (A)	78.54 mm^2
Moment of Inertia (I_z, I_y)	490.87 mm^4
Torsional Constant (K)	981.76 mm^4

Modelling of the bolt's shank using CBUSH requires a spring-type element with six-degree of freedom components. Figure 4 shows the assignment of stiffness components' numbering consist of axial stiffness K_1, shear stiffness K_2 and K_3, rotational stiffness K_4, K_5 and K_6. Swift's flexibility formula was used to calculate the shear stiffness K_2 and K_3 [11, 23–27]. Table 3 shows the calculated value of the stiffness components of CBUSH element used in this work [28].

Table 3. Value of stiffness components of CBUSH element

Stiffness components	Value
Axial stiffness, K_1	9.474×10^5 N/mm
Shear Stiffness, $K_{2,3}$	5.403×10^5 N/mm
Rotational Stiffness, K_4	1.000×10^2 N.mm
Rotational Stiffness, $K_{5,6}$	3.458×10^9 N.mm

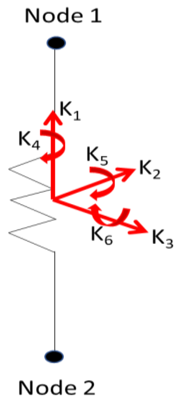

Figure 4. Stiffness components of CBUSH

Normal modes analysis of the bolted structure was performed by using the solution type of SOL 103 Real Eigenvalues of NX11 Simcenter to characterise the first ten natural frequencies and mode shapes. The predicted results were used as the initial FE results in this study. The initial FE results were compared and correlated with the results of EMA.

EXPERIMENTAL MODAL ANALYSIS (EMA)

EMA is used to measure the characteristics of structural dynamics, which are the natural frequencies and mode shapes, of the physically assembled bolted structure. In this work, the bolted joint structure was set-up for the experiment, as shown in Figure 5. The EMA procedure and setup were adopted based on the previous similar studies done by other researchers [29–31]. The bolted joint structure was assembled by joining two equally dimensional steel plates named as Plate A and Plate B with stainless steel bolts and nuts. The bolted joint structure was suspended from a test rig by using rubber bands to simulate free-free boundary conditions.

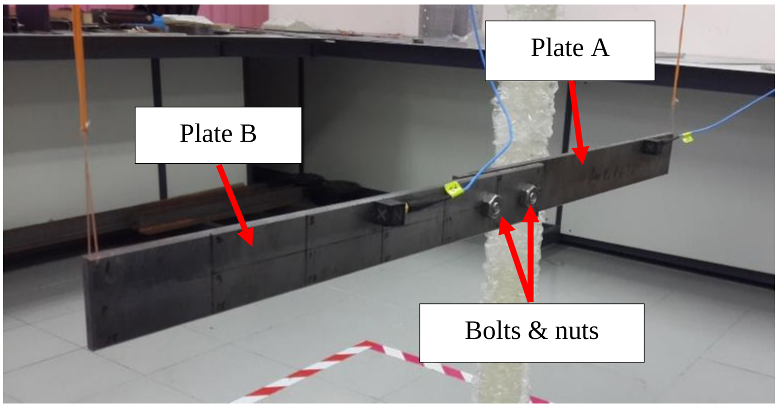

Figure 5. Set-up of the bolted joint structure for EMA

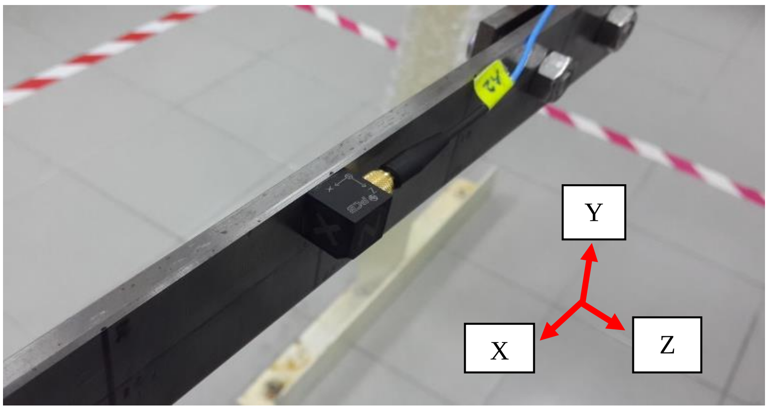

Figure 6. Triaxial accelerometer and directions

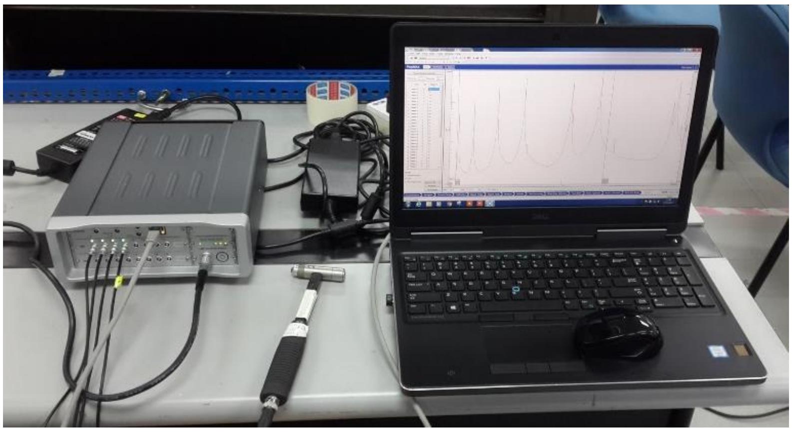

Figure 7. Test equipment – LMS Test.Lab and impact hammer

Triaxial accelerometers, each with the sensitivity of 100 mV/g, were used to measure the responses in the x, y and z directions, as shown in Figure 6, to extract complete responses of the bolted structure. The other equipment used in EMA is the 16-channels LMS SCADAS Mobile system with the LMS Test Lab 16A software packages, as well as an impact hammer with a sensitivity of 0.23 mV/N (Figure 7). The impact hammer was used to excite the bolted structure at a fixed reference point.

FE MODEL UPDATING

FE model updating is an analytical method by which the accuracy and reliability of an FE model can be improved by systematically adjusting the parameters of the FE model within an identified range [32, 33]. The method was employed in this work to reduce the uncertainties introduced in the initial FE models as a result of invalid assumptions about model properties. The selection of the parameters used for the FE model updating procedure is based on the results of the sensitivity analysis [34].

In this work, the SOL 200 Model Update of NX 11 Simcenter 3D software was utilised in the effort to update the initial FE models of the bolted structure. The first ten natural frequencies were incorporated in the objective function to improve the correlation between the predicted and measured natural frequencies. The objective function used in this work is as in Eq. (1) [16, 35]:

$$min \sum_{i=1}^{m} W_i \left(\frac{\omega_i^n}{\omega_i^e} - 1 \right)^2 \tag{1}$$

where ω_i^n is the i-th predicted frequency, ω_i^e is the i-th measured frequency, the weightage, W_i is set to unity and i = 1, 2,…, 10.

RESULTS AND DISCUSSION

In this study, the first step is to select the most appropriate FE model for the representation of a bolted joint structure. Two different FE models were developed and analysed for the bolted joint structure. On the first FE model, the bolts were modelled using CBEAM element connectors and the other one, CBUSH element connectors were used for the bolts. The selection of the most appropriate FE model was made by evaluating the discrepancies, in terms of total errors, recorded from the comparison of the natural frequencies and mode shapes obtained from EMA and the FE models [36, 37]. The discrepancies recorded from the comparison were reduced using FE model updating by systematically adjusting the pre-selected updating parameters to match the initial FE model to the test model [15, 16].

The comparison of the natural frequencies between EMA and the initial FE model of the bolted joint structure-based CBEAM and CBUSH element connectors is shown in Table 4 and Table 5 respectively. The total error recorded from the FE model with CBEAM element connectors is 20.93%, whereas the total error of the FE model with CBUSH element is 14.56%. The Modal Assurance Criterion (MAC) values calculated are in the well acceptable range which is of 0.91 to 0.99 for both FE models, indicating EMA and the initial FE models are in close agreement with each other (Figure 8 and Figure 9).

The evaluation of the total error shows that the most appropriate FE model to represent the bolted structure is the FE model with CBUSH element connectors. This finding is in line with the previous research conducted by [28]. Meanwhile, the comparison of the total error between the FE model with CBEAM element connectors and the one with CBUSH element is about 6.3%. The result of the comparison reveals that the FE model with CBUSH element connectors is the most capable of being used to predict the dynamic behaviour of a bolt joint structure accurately.

Table 4. Comparison of EMA and initial FE of the bolted joint structure using CBEAM element connectors

Mode	I. EMA (Hz)	II. Initial FE with CBEAM (Hz)	III. Error between I & II (%)	IV. MAC
1	75.1	74.3	1.07	0.98
2	200.8	202.0	0.60	0.94
3	404.9	396.7	2.03	0.99
4	469.3	447.0	4.75	0.95
5	630.8	626.8	0.63	0.97
6	674.4	648.7	3.81	0.96
7	1034.6	1006.6	2.71	0.94
8	1053.2	1060.4	0.68	0.91
9	1277.0	1250.6	2.07	0.95
10	1435.1	1472.2	2.59	0.94
Total error			20.93	

Table 5. Comparison of EMA and initial FE of the bolted joint structure using CBUSH element connectors

Mode	I. EMA (Hz)	II. Initial FE with CBUSH (Hz)	III. Error between I & II (%)	IV. MAC
1	75.1	75.3	0.27	0.98
2	200.8	202.4	0.80	0.94
3	404.9	401.6	0.82	0.99
4	469.3	446.9	4.77	0.95
5	630.8	632.6	0.29	0.97
6	674.4	661.3	1.94	0.96
7	1034.6	1018.9	1.52	0.94
8	1053.2	1067.5	1.36	0.91
9	1277.0	1278.4	0.11	0.96
10	1435.1	1473.8	2.70	0.94
Total error			14.56	

Mode 1 EMA

Mode 1 FE

Mode 1 MAC 0.98

Mode 2 EMA Mode 2 FE

Mode 2 MAC 0.94

Mode 3 EMA Mode 3 FE

Mode 3 MAC 0.99

Mode 4 EMA Mode 4 FE

Mode 4 MAC 0.95

Mode 5 EMA Mode 5 FE

Mode 5 MAC 0.97

Figure 8. Comparison of mode shapes between EMA and FE method with CBUSH element connectors for Mode 1 until Mode 5

Mode 6 EMA Mode 6 FE

Mode 6 MAC 0.96

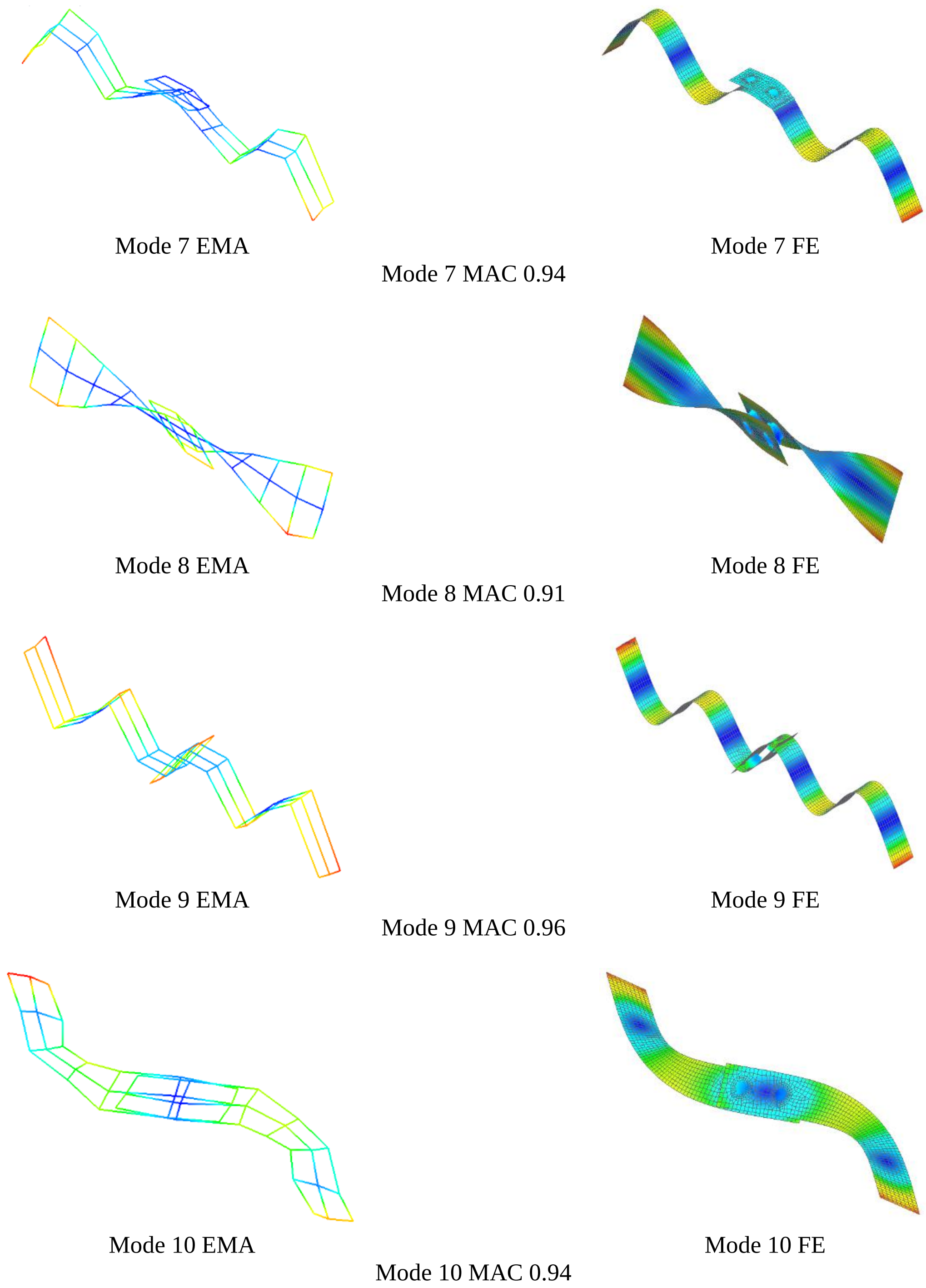

Figure 9. Comparison of mode shapes between EMA and FE method with CBUSH element connectors for Mode 6 until Mode 10

The total error calculated from the comparison of EMA and FE model with CBUSH element connectors of the bolted joint structure was further reduced using FE model updating [32, 38]. As explained in the previous section, the sensitivity analysis was carried out to select a few parameters of the FE model to be used to improve the natural frequencies of the bolted joint structure systematically.

The results of the sensitivity analysis are summarised in Figure 10. The components $K_{2\ CBUSH}$ and $K_{3\ CBUSH}$ of the CBUSH element connectors were selected as the updating parameters based on the value of sensitivity coefficient [35]. The FE model updating method of the bolted structure uses the Genetic Algorithms as the optimizer. The comparison of results between EMA and the updated FE model is tabulated in Table 6. The total error recorded has reduced to 14.37% with the parameters $K_{2\ CBUSH}$ and $K_{3\ CBUSH}$ being used as the updating parameters.

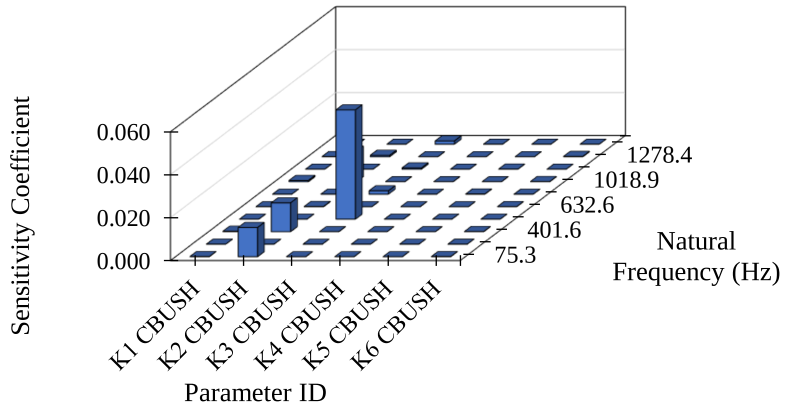

Figure 10. Sensitivity matrix of potential updating parameters

Table 6. Comparison of EMA and updated FE of the bolted joint structure using CBUSH element connectors

Mode	I. EMA (Hz)	II. Updated FE (Hz)	III. Error between I & II (%)
1	75.1	75.3	0.27
2	200.8	202.4	0.80
3	404.9	401.6	0.82
4	469.3	447.8	4.58
5	630.8	632.6	0.29
6	674.4	661.3	1.94
7	1034.6	1019	1.51
8	1053.2	1067.5	1.36
9	1277.0	1278.4	0.11
10	1435.1	1473.9	2.70
	Total Error		14.37

The values of the updated parameters of $K_{2\ CBUSH}$ and $K_{3\ CBUSH}$ are shown in Table 7. It was observed that the values of the shear stiffnesses $K_{2\ CBUSH}$ and $K_{3\ CBUSH}$ of the bolt's CBUSH element connectors have increased from their initial values. A slight increment of 1.1% has been recorded in $K_{2\ CBUSH}$, as well as an increment of 8.2% in $K_{3\ CBUSH}$. These results show that the shear stiffnesses of the bolted joints in the bolted structure have a significant influence over the dynamic behaviour of the bolted joint structure. In other words, successfully identifying these two parameters using the sensitivity analysis and systematically adjusting them in the light of experimental results leads to a significant improvement in the accuracy of the updated FE model of the bolted joint structure, with 86 percent accuracy.

Table 7. Updated parameters for the FE model of the bolted joint structure

Parameter ID	Initial Value	Updated Value	Unit
$K_{2\ CBUSH}$	540 314	546 257	N/mm
$K_{3\ CBUSH}$	540 314	584 394	N/mm

CONCLUSIONS

Representation of a bolted joint structure using the finite element method is presented. The dynamic behaviour of the bolted joint structure in terms of natural frequencies and mode shapes are determined and validated using the finite element method and experimental modal analysis. The sensitivity analysis has been successfully used to determine the most influential updating parameters, which are the stiffnesses of the bolts. Results show that the CBUSH element connectors based FE model has a greater capability over CBEAM element connectors based FE model in representing the bolted joints with 86 percent accuracy and greater efficiency in updating the model parameters. The developed representation of the bolted joints can be used for modelling other complex bolted joint structures.

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REFERENCES

[1] R. A. Ibrahim and C. L. Pettit, "Uncertainties and dynamic problems of bolted joints and other fasteners," *Journal of Sound and Vibration*, vol. 279, no. 3–5, pp. 857–936, 2005.

[2] H. Ahmadian and H. Jalali, "Identification of bolted lap joints parameters in assembled structures," *Mechanical Systems and Signal Processing*, vol. 21, no. 2, pp. 1041–1050, 2007.

[3] I. Zaman, A. Khalid, B. Manshoor, S. Araby, and M. I. Ghazali, "The effects of bolted joints on dynamic response of structures," in *IOP Conf. Series: Materials Science and Engineering*, 2013, vol. 50, p. 012018.

[4] S. Daouk, F. F. Louf, C. Cluzel, O. Dorival, L. Champaney, and S. Audebert, "Study of the dynamic behavior of a bolted joint under heavy loadings," *Journal of Sound and Vibration*, vol. 392, pp. 307–324, 2017.

[5] S. Shokrollahi and F. Adel, "Finite element model updating of bolted lap joints implementing identification of joint affected region parameters," *Journal of Theoretical and Applied Vibration and Acoustics*, vol. 2, no. 1, pp. 65–78, 2016.

[6] M. H. N. Izham and M. S. M. Sani, "Model updating of frame structure with bolted joints," *Journal of Physics: Conference Series*, vol. 1262, p. 012023, 2019.

[7] A. Rutman, C. Boshers, L. Pearce, and J. Parady, "Fastener modeling for joining parts modeled by shell and solid elements," in *Americas Virtual Product Development Conference*, 2007, pp. 1–26.

[8] R. Grzejda, "Modelling bolted joints using a simplified bolt model," *Journal of Mechanical and Transport Engineering*, vol. 69, no. 1, pp. 29–37, 2017.

[9] N. Tanlak, F. O. Sonmez, and E. Talay, "Detailed and simplified models of bolted joints under impact loading," *Journal of Strain Analysis for Engineering Design*, vol. 46, no. 3, pp. 213–225, 2011.

[10] R. Omar, M. N. A. Rani, M. A. Yunus, A. A. M. Isa, and Z. A. Rahman, "Updating of the dynamic behaviour of an assembled structure with bolted joints in light of EMA results," *Journal of Physics: Conference Series*, vol. 1262, no. 1, p. 012016, 2019.

[11] I. Uysal, M. Suer, and I. Ozkol, "Finite Element Modelling and Analysis of Simple Lap Joints," in *Proceedings of 5th International Conference on Recent Advances in Space Technologies - RAST2011*, 2011, pp. 68–72.

[12] R. Omar, M. N. Abdul Rani, M. A. Yunus, W. I. I. Wan Iskandar Mirza, and M. S. Mohd Zin, "Investigation into joint stiffness for finite element modelling of the dynamic behaviour of a structure with bolted joints," in *IOP Conference Series: Materials Science and Engineering*, 2019, vol. 506, p. 012009.

[13] R. Omar, M. N. Abdul Rani, and M. A. Yunus, "Finite element modelling and updating of a bolted structure using elements representing the stiffness members, bolts and affected areas of the joints," *Journal of Physics: Conference Series*, vol. 1262, no. 1, p. 012005, 2019.

[14] R. C. Hibbeler, *Mechanics of Materials*, 8th ed. Prentice Hall, 2011.

[15] M. N. Abdul Rani, H. Ouyang, M. A. Yunus, and B. A. Aminudin, "Model updating for a thin steel sheet welded structure," in *20th International Congress on Sound and Vibration 2013*, 2013, pp. 1–8.

[16] M. N. Abdul Rani, S. Kasolang, M. H. Othman, M. A. Yunus, W. I. I. Wan Iskandar Mirza, and H. Ouyang, "Finite element modelling and modal based updating for the dynamic behaviour of a laser spot welded structure," in *23rd International Congress on Sound and Vibration*, 2016, pp. 315–322.

[17] R. Omar, M. N. Abdul Rani, W. I. I. Wan Iskandar Mirza, M. A. Yunus, and M. H. Othman, "Finite element modelling and updating for bolted lap joints," *Journal of Mechanical Engineering*, vol. SI 4, no. 3, pp. 202–222, 2017.

[18] R. Omar, M. N. Abdul Rani, M. A. Yunus, W. I. I. Wan Iskandar Mirza, and M. S. Mohd Zin, "Finite element modelling and updating of a non-identical built-up structure with bolted joints," in *25th Internatinal Congress on Sound and Vibration (ICSV 25)*, 2018, pp. 4624–4631.

[19] R. Omar, M. N. Abdul Rani, M. A. Yunus, W. I. I. Wan Iskandar Mirza, and M. S. Mohd Zin, "Efficient finite element modelling for the investigation of the dynamic behaviour of a structure with bolted joints," *AIP Conference Proceedings*, vol. 1952, no. 1, p. 020082, Apr. 2018.

[20] R. Omar *et al.*, "Investigation of mesh size effect on dynamic behaviour of an assembled structure with bolted joints using finite element method," *International Journal of Automotive and Mechanical Engineering*, vol. 15, no. 3, pp. 5695–5708, 2018.

[21] Siemens, *Siemens Element Library Reference*. 2014.

[22] SIEMENS, "Introduction to NX FE model updating," 2011.

[23] H. Huth, "Influence of Fastener Flexibility on the Prediction of Load Transfer and Fatigue Life for Multiple-Row Joints," in *Fatigue in Mechanically Fastened Composite and Metallic Joints, ASTM STP 927*, J. M. Potter, Ed. Philadelphia: American Society for Testing and Materials, 1986, pp. 221–250.

[24] L. Jarfall, "Shear loaded fastener installations," *International Journal of Vehicle Design*, vol. 7, pp. 337–380, 1986.

[25] Z. L. Whitman, "The effect of fatigue cracks on fastener flexibility load distribution and fatigue crack growth," University of Utah, 2012.

[26] F. Gunbring, "Prediction and Modelling of Fastener Flexibility Using FE," Linkoping University Institute of Technology, 2008.

[27] S. Chandregowda and C. R. Ganga Reddy, "Evaluation of fastener stiffness modelling methods for aircraft structural joints," *AIP Conference Proceedings*, vol. 1943, p. 020001, 2018.

[28] R. Omar, M. N. Abdul Rani, M. A. Yunus, D. P. Starbuck, W. I. I. Wan Iskandar Mirza, and M. S. Mohd Zin, "Different flexibility formulae for finite element modelling and analysis of the dynamic behaviour of a structure with bolted joints," *AIP Conference Proceedings*, vol. 2059, p. 020004, 2019.

[29] F. Adel, S. Shokrollahi, M. Jamal-Omidi, and H. Ahmadian, "A model updating method for hybrid composite/aluminum bolted joints using modal test data," *Journal of Sound and Vibration*, vol. 396, pp. 172–185, 2017.

[30] S. Saeed, N. Ahmad, and A. Farhad, "Finite element model updating of interfaces in single-lap bolted joints using modal test data," in *21st International Congress on Sound and Vibration*, 2014.

[31] W. I. I. Wan Iskandar Mirza, M. N. Abdul Rani, M. A. Yunus, R. Omar, and M. S. Mohd Zin, "Alternative scheme for frequency response function measurement of experimental- analytical dynamic substructuring," *Journal of Mechanical Engineering and Sciences*, vol. 13, no. 2, pp. 4946–4957, 2019.

[32] J. E. Mottershead and M. I. Friswell, "Model updating in structural dynamics: A survey," *Journal of Sound and Vibration*, vol. 167, no. 2, pp. 347–375, 1993.

[33] S. Sehgal and H. Kumar, "Structural Dynamic Model Updating Techniques: A State of the Art Review," *Archives of Computational Methods in Engineering*, vol. 23, no. 3, pp. 515–533, 2016.

[34] J. E. Mottershead, M. Link, and M. I. Friswell, "The sensitivity method in finite element model updating: A tutorial," *Mechanical Systems and Signal Processing*, vol. 25, no. 7, pp. 2275–2296, 2011.

[35] H. Ahmadian, J. E. Mottershead, S. James, M. I. Friswell, and C. A. Reece, "Modelling and updating of large surface-to-surface joints in the AWE-MACE structure," *Mechanical Systems and Signal Processing*, vol. 20, no. 4, pp. 868–880, 2006.

[36] M. S. Mohd Zin, M. N. Abdul Rani, M. A. Yunus, M. A. Ayub, M. S. M. Sani, and M. A. S. Aziz Shah, "Modal based updating for the dynamic behaviour of a car trunk lid," *AIP Conference Proceedings*, vol. 2059, no. January, p. 020001, 2019.

[37] M. S. Mohd Zin, M. N. Abdul Rani, M. A. Yunus, W. I. I. Wan Iskandar Mirza, M. A. Ayub, and R. Omar, "Frequency Response Function Based Updating of a Laser Spot Welded Structure using Synthesised Frequency Response Function," *IOP Conference Series: Materials Science and Engineering*, vol. 506, no. 1, p. 012012, 2019.

[38] M. I. Friswell and J. E. Mottershead, *Finite element model updating in structural dynamics*. London: Kluwer Academic Publishers, 1995.