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A distinct matrix representation of the planar kinematic chains and isomorphism recognition

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ABSTRACT

Structural synthesis of kinematic chains has been an indispensable area of the mechanism-design problem. The duplication may occur while developing kinematic chains. Therefore, an isomorphic test is required to eliminate duplication. For this purpose, the numbers of methods are proposed during recent years. However, most of the methods are complex and difficult to understand, and fulfil the only primary condition, but not the secondary conditions for isomorphism detection. In the present work, a new method is introduced to detect isomorphism in planar kinematic chains (KCs) fulfilling both primary and secondary conditions. First, KC's are topologically transformed into skeleton diagrams, and then skeleton matrices [S] and identification strings [IS] are formulated consequently. In order to detect isomorphism, the IS is considered as an invariant string of a KC which in turn, enables the detection of isomorphism between the KCs. The proposed method accurately recognizes isomorphism up to 12 links KCs with no counter examples found in the literature. Three examples with one degree of freedom having 10 links 12 joints, 10 links 13 joints and 12 links three degree of freedom systems are introduced to reveal the reliability and strength of the proposed method.

Keywords: Skeleton matrix; kinematic chains; mechanism; isomorphism identification string; degree of freedom.

INTRODUCTION

Recognition of kinematic chains is the part of structural synthesis, the chance of duplication increases as the no of links is increased. Isomorphic tests have been proposed to avoid duplication by many researchers. Unidentified isomorphism leads to duplicate kinematic chains, but wrongly detected isomorphism curtails the number of feasible solutions for distinct kinematic chains and mechanisms. Many new methods for detection of isomorphism have been reported so far, but this procedure demands more computational timing because of the comparison of the unique code of each kinematic chain [1] if the unique codes of each kinematic chain are same they term as the isomorphic chains otherwise non-isomorphic. Thus, recognition of isomorphism is a critical and time-consuming issue for the design of kinematic chains. In the area of structural synthesis, the most time-consuming the process is to detect isomorphism because of the adaptation of the inefficient method for enumeration of all possible kinematic chains for a given number of links and mobility. For the development of the 230 number of kinematic chains having 10-number of links, one degree of freedom accounts more than 90% computational time for isomorphism detection [2] In mechanical design, an indispensable step in the systematic synthesis of kinematic chains is isomorphism identification [3,4] and necessitates computer-aided tools for computation. In the field of computer-aided design (CAD) the detection of structural isomorphism between the various kinds of kinematic chains belonging to the same family is one of the most prominent difficulties faced while developing kinematic chains with a specified number of links and degree of freedom number synthesis of kinematic chains require characterization and development of all possible kinematic chains containing a given number of links and mobility [5]. This makes it easier for the elimination of isomorphic chains between the groups of non-identified kinematic chains. The analysis is based on the functional requirements in the structural analysis, which include the determination of the kind of mobility along the kinematic chain. Simultaneously, the structurally unique mechanisms can be obtained from the kinematic chain by fixing link one by one. There is a high possibility of duplicate kinematic chain generation, which requires testing of isomorphism [6]. The most inefficient technique in structural synthesis is the test of isomorphism. Isomorphism testing might have reached more significant levels, but the procedure still lacks efficiency as it compares every code of each new kinematic chain with the ones already existing [1]. Thus, isomorphism detection is an essential process in the field of kinematic chain design. Several approaches have been developed to overcome this issue over the last few decades. T.S. Mruthyunjaya [7] developed the code based characteristic polynomial and distance-based approaches. The code-based approach constitutes the code which is a decodable function of repeated expression of the adjacency matrices of kinematic chains. The isomorphism is detected through the comparison of that function. The polynomial approach enabled Uicker and Raicutodevelop an index of isomorphism established as the method of the link-link adjacency matrix [8]. This technique only fulfilled necessary condition, but not sufficient condition for detection of isomorphism between two topological graphs of the chain. Several variations of code based approaches, were proposed including max/min code by Ambedkar and Agrawal [9, 10], degree code by C. Tang and T.Liu [11] and standard code by J.K. Shin & S. Krishnamurthy [12,13].The primary advantages of the code-based methods included excitability and uniqueness along with the possibility of the classification of the kinematic chains. The Hamming distances-based approach was proposed by Rao [14-16]. Linkage path codes of kinematic chains were developed by Yan and Hwang [17]. Triple sequential step to test for isomorphism based on link-link distance matrix was also proposed by Yadav [18, 19]. However, the distance-based approach was considered inappropriate for computerized structural synthesis proposed by Mruthyunjaya [7]. A method based on eigenvectors and values of the adjacency matrices was proposed by Z. Chang in 2002 [20]. This method was made error-free by proposing the primary and secondary conditions of eigenvectors and eigen-value of matrices based on adjacency for isomorphic kinematic chains by J.P. Cubillo and J.Wan [21]. This method was further proved by P.S. Rajesh and C.S. Linda [22] in being successful for the identification of all non-isomorphic chains having 1,2 and three mobility, with up to 14 links having an exponential time complexity.

The canonical perimeter topological graph and characteristic adjacency matrix approach for testing isomorphism for both simple and multiple joint chains were proposed by H.Ding and Z.Huang [23, 24-26].The main disadvantage with this method was its time complexity, n(2L) is the time complexity of the proposed algorithm, where L represents the number of basic loops of the graph. Where n is the number of vertices, finding basic loops of a graph to cost time with the complexities of O(n) or O(n2) or any higher exponential order. It is possible that there are no secondary number of primary loops in a graph that represents a kinematic chain allowing this technique to function swiftly in the synthesis of the kinematic chains automatically [27, 28].

Approximate solutions proposed by the evolutionary methods [29-33] and artificial neural networks [34, 35] Isomorphism identification of kinematic chains with 50 links using novel evolutionary approaches proposed by R. Xiao et al. [29] takes little more than unity seconds.The Neural network approach [35] Improved by M. Zhang et al. But time spent in the detection of isomorphism is more than 2 s for the links 20. The main disadvantage of the approximate solution is unpredictable and unstable computation time.

A new method based on Incident matrices to identify the isomorphism of topological graphs" proposed by Y. Fei, et al. [36]. The methodology of obtaining the primary condition for detection the isomorphism is founded by the incidence matrices' calculation. Moreover, it can be executed by programs automatically. By utilizing the results obtained from the primary condition, the approach to search the correspondence of different vertices is presented, and the secondary criteria for the isomorphism identification are achieved. An algorithm for detection of isomorphism between the kinematic chain based on dividing and matching vertices proposed by K Zeng et al [37]. The time complexity of DMA is O(Mn2). Comparison between DMA and other three well-known algorithms on isomorphic and non-isomorphic kinematic chains based on runtime, DMA grabs the flag for the better performance. The experimental results show the high efficiency of DMA. Some theorems and corollaries are produced in this research paper for showing the reliability and credibility of the DMA. However, according to research most of these methods, which are complicated in methodology and hard to understand, some of them fulfil primary, but not secondary criteria in the isomorphism identification betwixt the kinematic chains.

In the proposed research, a new methodology is opted to identify the isomorphism between the kinematic chains based on the identification string (IS) which also contains the primary and secondary condition. In primary condition, the basic structural information of the KCs is preserved &secondary condition is derived from the determinant of [S] matrix established on identification string IS and skeleton matrices of specified chains are recommended the first time. By comparison, one part of an identification string IS of the chains, the primary condition is obtained. Furthermore, the last string of the Identification String of the chains has been presented as the secondary condition for the isomorphism identification among the kinematic chains. Finally, the complete methodology for detection of isomorphic chains is adequately demonstrated by few Examples.

THEORY AND BASIC CONCEPT

Degree of the rigid link D(li)

Rigid Links means no deformation in the link during operation of KCs. Rigid links are classified according to the presence of the number of joints made with each other links. And may be specified as unary, Binary, ternary --- n-nary link having 1, 2, 3 --- m provisions of joints with other links respectively. Therefore, the degree of the rigid links represents the type of link binary; ternary, etc. D(li) will be equal to 1 for unary, 2 for binary, 3 for ternary, and --- m for the n-nary link.

Type of connections (Binary Strings)

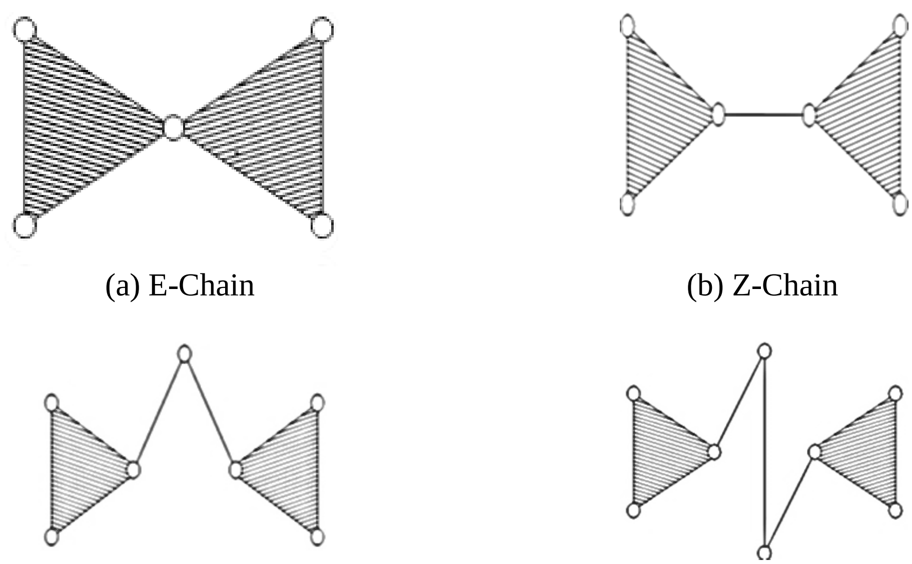
A kinematic chain is the secondary product of polygonal links (having the degree of a link more than 2) and binary links. The polygonal links are joined to each other with the help of binary links, which are called connections and designated as follows:

E – Chain: When two polygonal links are directly connected with one joint (Figure 1(a))

Z – Chain: When two polygonal links are connected with the help of one intermediate binary link (Figure 1(b))

D – Chain: When two polygonal links are connected with the help of two intermediate binary link (Figure 1(c))

V – Chain: When two polygonal links are connected with the help of three intermediate binary link (Figure 1(d))



(a) E-Chain (b) Z-Chain

(c) D-Chain (d) V-Chain

Figure 1. Kinematic chain connections.

Designation of links

Link of the kinematic chains is designated as follows:

n_2 = Number of binary links present in a kinematic chain

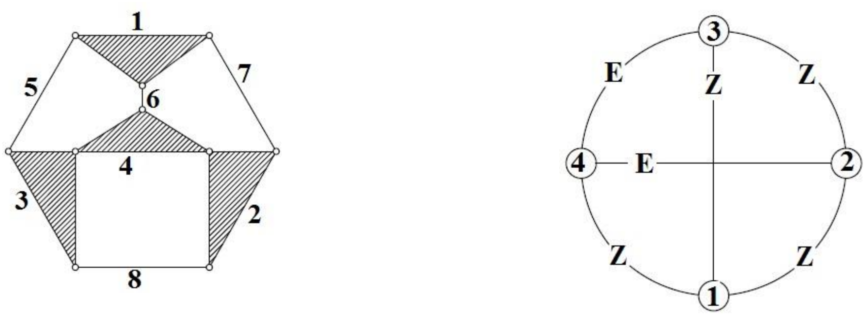
n_3 = Number of ternary links present in a kinematic chain

n_4 = Number of quaternary links present in a kinematic chain

i_{max} = maximum degree of links presents in a kinematic chain.

Skeleton Representation of the kinematic chain

Skeleton diagram is the abstract topological representation ofa given kinematic chain and was proposed by Frank [38]. A kinematic chain is the assemblage of polygonal links (links having a degree more than 2) and binary strings. The polygonal links are connected to each other by numerous combinations of E, Z, D, or V chains. In the skeleton diagram, all polygonal links are represented by the circles, and these circles are connected to each other with the help of straight/curved lines showing the types of connections (E, Z, D, or V chains). The number of lines radiated from a circle represents the degree of that polygonal link. For example, Figure 2(a) represents the 1-dof 8 link, 10 jointed kinematic chains and Figure 2(b) represents its skeleton. It contains 4 ternaries (polygonal) links connected to each other with the help of 6 –connections, comprising 2 E–chains (in between polygonal link 3-4 & 2-4) and 4 Z –chains (in between polygonal links 1-3, 1-2, 1-4, & 2-3).



(a) (b)

Figure 2. 8 link, 10 joint & 1-DOF Kinematic chain (a) and its Skeleton(b)

Matrix Representation of Skeleton [S]

The researcher has been represented the kinematic chain by (0, 1) adjacency matrix, shortest path distance matrix, joint-joint matrix and, etc. The size of the matrix is nxn where the n = number of links existing in a kinematic chain. If the number of links is high, the size of the matrix is also very large & determination of some structural invariant is the problematic and time consuming issue. In the proposed work, a new skeleton matrix is developed, the size of which is reduced to the number of polygonal links in a kinematic chain. For example; in 8-link kinematic chains, if there are four (4) ternary link and 4 binary links, then only a 4*4 size of the matrix isadequate to represent a kinematic chain instead of the 8*8 size of the matrix. Therefore, a skeleton matrix of size (n'xn') is defined as.

$$[S] = \{s_{ij}\}_{n'xn'} \text{ (n' = number of polygonal links)} \tag{1}$$

$$s_{ij} = \begin{cases} \{C_{ij}\} \\ D(l_i)^2 \\ 0 \end{cases} \tag{2}$$

Where;

s_{ij} = C_{ij};the sum of the squared values of the type of connections between the i^{th} and j^{th} polygonal links directly connected

s_{ij} = 0; if i^{th} and j^{th} polygonal links are not directly connected

s_{ij} = $d(l_i)^2$; the squared value of the degree of i^{th} polygonal link

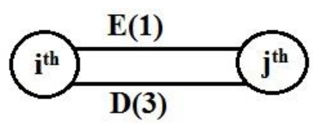
Skeleton matrices [S] shows, the polygonal link relationship among each other in a skeleton diagram and the shape of [S] matrix as given below.

$$S = \begin{bmatrix} d(l_1)^2 & \cdots & s_{1n'} \\ \vdots & \ddots & \vdots \\ s_{n'1} & \cdots & d(l_i)^2 \end{bmatrix}_{n' \times n'} \tag{3}$$

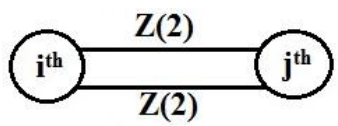
The values of the different type of connections are:
E –chain = 1; as there is one joint between two polygonal links
Z –chain = 2; as there are two joints between two polygonal links
D –chain = 3; as there are three joints between two polygonal links
V –chain = 4; as there are four joints between two polygonal links

Justification of squared values in {sij}
Let us consider the two different arrangements of connection between two polygonal links as shown in Figure 3(a) and 3(b).

The sum of the value of the connection between i^{th} & j^{th} links; it will be same, i.e. 4 in both the arrangement shown in Figure 3(a) and (b) resulting in the value of $S_{ij} = 4$ while it should be different as both arrangements are different. If we take the sum of the square value of the connections; it is 10 ($1^2 + 3^2$) for the arrangement shown in Figure 3(a) and is 8 (2^2+2^2) for the arrangement shown in Figure 3(b). Therefore, taking the square value in the element of [S] matrix makes it as a unique representation for the kinematic chains.



(a)



(b)

Figure 3. Two different arrangements of connection ith and jth links; (a) Two polygonal links connected with E & D –chains and (b) Two polygonal links connected with Z –chains.

Identification String (ISi)
The Identification String may be defined as the unique invariance string in which all the information is furnished in a single string of a given kinematic chain; it is the combination of the number of degree of freedom, link nomenclature, connection nomenclature and determinant of Skeleton Matrix [S]. It is represented as follows.

$$(\mathrm{IS_i}) = \mathrm{F, n, j/i_{max}\text{---}n_4\ n_3\ n_2/E\ Z\ D\ V/|S|} \tag{4}$$

Where F, n, j,i_{max}, n_4, n_3, n_2, E, Z, D, V, and |S| are defined as follows

Degree of freedom (F)

The degree of freedom is the number of independent variables needed to accurately position all links of the mechanism with respect to the fixed link. It is also known as mobility and generally referred to as symbol F. The first string of the Identification String is the number of degree of freedom (F) of the given kinematic chain.

Link Nomenclature String

The Link nomenclature string has the information about the total number of links as well as the number of types of links existing in the kinematic chains. It is the second string of the classification string. It is represented as follows.

$$\text{n, j/}i_{\max}\text{---}n_4 n_3 n_2\text{/} \quad (5)$$

where,
n = total number of links
j = Number of simple joints,
$i_{\max} = n/2$ max degree of link
n_4, n_3,$n_2$are the number of quaternary, ternary and binary links respectively

Connection Nomenclature String

It is the representation of the number of types of connections present in the given kinematic chain . It is the third string of classification string. It is represented as follows:

$$/E\ Z\ D\ V/ \quad (6)$$

Where, E, Z, D and V are the numbers of E-, Z-, D- and V-chains existing in the given kinematic chain

Determinant of Skeleton Matrix [S]

Last and most important position of Identification String is left for the determinant of Skeleton Matrix [S] string. It possesses secondary condition for detection of isomorphism. If the Identification String mapped for the respective chain except for the last string, it probably is an isomorphic chain as it fulfils a primary condition. It is represented as |S|

If two kinematic chains are isomorphic, their skeleton matrices may become identical by applying interchange of rows and columns at the same time [36] and matrices are equivalent. Therefore, the determinant of [S] matrix can be utilized as an invariant. If the determinants of two [S] matrices are same, then it accomplishes the secondary condition of isomorphism.

$$\text{Det}\ (S_1\ ) = \det(S_2) \text{ or } |S_1| = |S_2| \quad (7)$$

Structural Isomorphism between the kinematic chains

Two kinematic chains are said to be structurally isomorphic only if their links and the adjacent relationship between the links are one to one correspondence. From the definition of graph theory, there must exist a one-to-one correspondence between their vertices and edges that preserve incidence.

The existence of Isomorphism between the kinematic chains leads to duplicate chains, while wrongly identified isomorphism curtailed the number of feasible solutions for unique designs. If two kinematic chains are isomorphic, their skeleton matrices may become identical by applying interchange of rows and columns at the same time [36]. It can be defined by the following equation [20, 21]:

$$T_m \cdot S_1 \cdot T_m^{-1} = S_2 \tag{8}$$

Where T_m is the row transforming matrix, T_m^{-1} is the inverse matrix of T_m , S_1 and S_2 are the skeleton matrices of the two skeleton diagram respectively. Therefore, last string which is det(S) should also be same for isomorphic chains

Structural Invariant of a kinematic chain

The proposed Identification String contains all information about the kinematic chains like, number of degree of freedom, links, joints, type of connection and Determinant of Skeleton Matrix [S] of Skeleton diagram, which is the topological representation of the kinematic chains. Therefore, the Identification String is considered as an invariant string of a kinematic chain which may be used to detect isomorphism. The single string has plenty of information to fulfil the primary and secondary condition for detection of isomorphism between the kinematic chains.

Primary and Secondary Conditions

Identification string contains primary and secondary conditions as shown in Figure 4 and discuss below for detection of isomorphism.

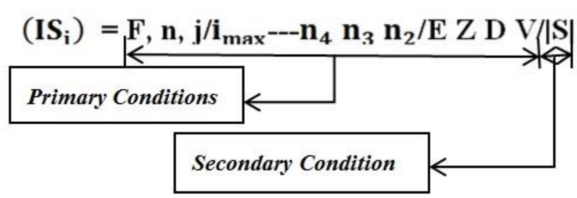


Figure 4. Primary and secondary conditions for Identification strings.

Isomorphism identification is accomplished in two parts of Testing. Initially check the primary conditions of the kinematic chains by comparing first part of identification string for respective chains if precisely mapped, then it is qualified for the next and second part of the testing of isomorphism that is a secondary condition for chains said to be isomorphism and vice versa.

Recognition of isomorphism

Case 1: Primary condition One-to-one and onto (bijection)

In the field of mathematics, a bijection, bijective function or one-to-one correspondence is a function between the elements of two sets, where each element of one set is paired with exactly one element of the other set, and each element of the other set is paired with exactly one element of the first set. There are no unpaired elements. In mathematical terms, a bijective function f: $A_1 \rightarrow A_2$is a one-to-one (injective) and onto (surjective) mapping of a set A_1 to a set A_2 [43].
Elements of two kinematic chains are represented by Identification String **F, n, j/$\mathbf{i_{max}}$---**

n_4 n_3 n_2/E Z D V/ like number of degree of freedom, number of links, number of joints, number of senary links, quinary links, Quaternary links, ternary links, binary links, unary links, and the number of E, Z, D, V chains. These values must be equal and unique in the both kinematic chains (a one-to-one injective)and onto (surjective mapping of a set).

Case 2: Secondary condition

Preservation of adjacency of vertices for the graph is to be isomorphic their adjacency of vertices must be the same. The adjacency matrix represents whether pairs of vertices are adjacent or not in the respective graph which to is compared. The adjacency matrix is a (0,1matrix) with zeros on its diagonal. Similarly, the [S] Skeleton matrix is kind of the adjacency matrix. [S] Skeleton matrix indicates whether pairs of a polygonal link (vertices) are adjacent or not in the skeleton diagram (graph). In the abstract skeleton diagram, the skeleton matrix is a $(0,S_{ij})$-matrix with d(li) on its diagonal where Sij is the summation of the squared value of the type of chain between $i_{th}$and j_{th} polygonal links those are connected, i.e. E = 1, Z = 2, D = 3 and V = 4 and d(li) is the squared value of degree of polygonal links

If the kinematic chains are isomorphic, their determinants of the skeleton matrices are independent of procedure of applying interchanges of rows and columns, the value as a determinant remains unaltered.

$$\det(S_1) = \det(S_2) \tag{9}$$

Where S_1 and S_2 are the skeleton matrices of the kinematic chains.

RESULT: ALGORITHM OF IDENTIFICATION OF ISOMORPHISM

Step 1: At first, Testing for the Primary condition, after the formation of one part of the classification string,
F/ N/j/i_{max}---n_4 n_3 n_2/E Z D V
If one major part of the Identification String is e mapped for each chain, subsequently the chains qualify for the next round of test of isomorphism. However, if the primary condition fails, then to stop the test immediately as chains are not belonging to the same family, and chains are said to be non-isomorphic.

Step 2: Convert the kinematic chains into Skeletons for respective chains.

Step 3: Develop the Skeleton matrices [S].

Step 4: Evaluate the Determinant of Skeleton [S] matrices of each kinematic chain.

Step 5: Prepare the Identification String of respective kinematic chains

$$(CS_i) = F, n, j/i_{max}\text{---}n_4\ n_3\ n_2/E\ Z\ D\ V/|S| \tag{10}$$

Step-6: At last, checking for the secondary condition. Compare the last string of the classification sting as it passes the secondary condition that is the determinant of Skeleton Matrices [S] of respective chains after the checking of Step-1 for the primary condition to be fulfilled. By comparing the IS of each kinematic chain starting from Dof and proceeds further if all the strings of classification are precisely matched with respective chain, they recognized as isomorphic chains otherwise non-isomorphic. Few Examples are provided to explain the method properly with more in depth of concepts.

Example 1

Figure 5(a) and (b) shows two co spectral graph based on the adjacency matrix, kinematic chains with three degrees of freedom, 10 links and 12 joints. In this example, the complete method will be demonstrated in identifying isomorphism.

Testing for the primary condition

Step 1: Formation of one part of the Identification String (IS) by visualizing of both kinematic chains:

$$(IS_i) = F,\ n,\ j/i_{max}\text{---}n_4\ n_3\ n_2/E\ Z\ D\ V/|S| \quad (11)$$

Part of IS for both the chains are:

$$03\ 10\ 12/05\ 00\ 00\ 04\ 06/02\ 03\ 00\ 01/ \quad (12)$$
$$03\ 10\ 12/05\ 00\ 00\ 04\ 06/02\ 02\ 00\ 02/ \quad (13)$$

Now both the chains are not eligible for the next round of testing of isomorphism because the primary condition is not fulfilled. Compare the identification string for both the chains. By comparing *(IS$_1$) ≠ ((IS$_2$)*, therefore chains are non-isomorphic, but we are not terminating test now, because we need to demonstrate our method in detail.

Step 2: Conversion of kinematic chain into Skeleton

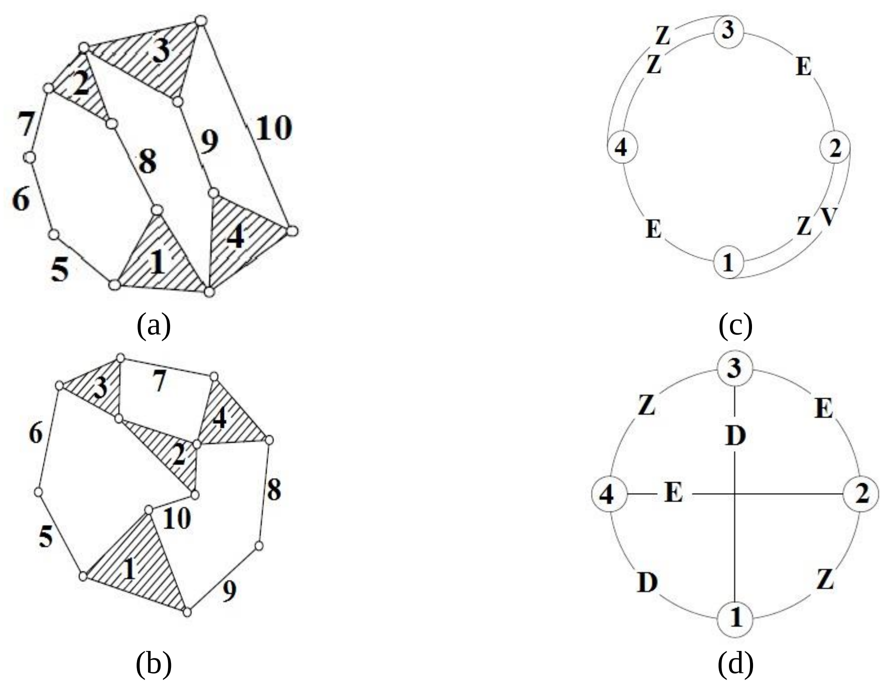


Figure 5: (a) and (b) 10-bar 3-DOF kinematic chains, (c) and (d) their skeleton diagram, respectively.

Step 3: Development of Skeleton matrices
Skeleton matrices of the kinematic chains [S] Figure 5(c) and (d)

$$S_{2a'} = \begin{bmatrix} 9 & 20 & 0 & 1 \\ 20 & 9 & 1 & 0 \\ 0 & 1 & 9 & 8 \\ 1 & 0 & 8 & 9 \end{bmatrix} \tag{14}$$

$$S_{2b'} = \begin{bmatrix} 9 & 4 & 9 & 9 \\ 4 & 9 & 1 & 1 \\ 9 & 1 & 9 & 4 \\ 9 & 1 & 4 & 9 \end{bmatrix} \tag{15}$$

Step 4: Determinants of Skeleton Matrices [S] of respective chains
$|\mathbf{S_{2a'}}| = -5.904 \text{x} 10^3$
$|\mathbf{S_{2b'}}| = -2.435 \text{x} 10^3$
Step 5: Formation of complete identification string (Cs)
Classification strings of both Kinematic chains of Figure 5(a) and (b)

$$(\mathrm{IS_i}) \quad = \mathrm{F, n, j/i_{max}\text{---}n_4\ n_3\ n_2/E\ Z\ D\ V/|S|} \tag{16}$$
$$(\mathrm{IS_a}) = 03\ 10\ 12/05\ 00\ 00\ 04\ 06/02\ 03\ 00\ 01/\text{-}5.904\times\omega \tag{17}$$
$$(\mathrm{IS_b}) = 03\ 10\ 12/05\ 00\ 00\ 04\ 06/02\ 03\ 00\ 01/\text{-}2.435\times\omega \tag{18}$$

where $\omega = 10^3$
Step 6: Checking for secondary condition.
Compare the Identification String for both the chains. By comparing $(IS_a) \neq (IS_b)$, thus the kinematic chain 5(a) is non-isomorphic with kinematic chain 5(b) as Step 1 for primary condition was not fulfilled.

Our method reports that both the KC shown in Figure 5(a) and (b) are non-isomorphic because $(\mathrm{IS_1}) \neq (\mathrm{IS_2})$they are different for both kinematic chains. Note that by using another method of the adjusted adjacency matrix of the graph approach [39] and summation polynomials [40], the same conclusion is obtained.

Example 2
The second example concerns another two KC with 10 bars, 13 joints, single degree of freedom as shown in Figure 6 (a) and (b). The task is to examine whether these two chains are isomorphic. We know they are isomorphic by the method of the artificial neural network [41]

Testing for the Primary condition
Step 1: Formation of one part of identification string (IS) by visualizing of both kinematic chains

$$\mathrm{F, n, j/i_{max}\text{---}n_4\ n_3\ n_2/E\ Z\ D\ V/} \tag{19}$$

Part of IS for both the chains are
01 10 13/00/00/01/04/05/00/03/05/00/00
01 10 13/00/00/01/04/05/00/03/05/00/00

Now both the chains are eligible for next round of testing of isomorphism.

Step 2: Conversion of kinematic chain into Skeleton

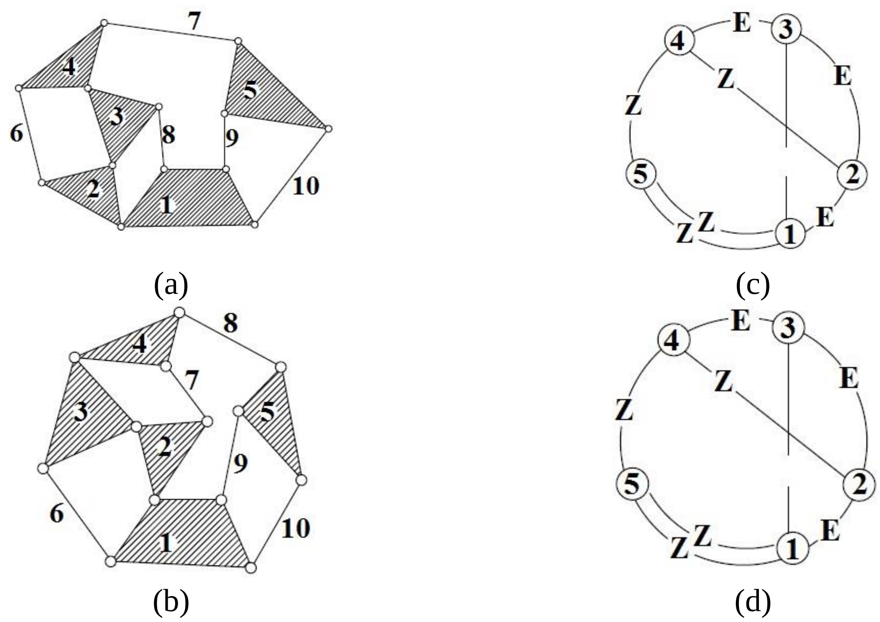


Figure 6: (a) and (b) 10-bar 1-dof kinematic chains, (c) and (d) their skeleton diagrams, respectively.

Step 3: Development of Skeleton matrices Skeleton matrices of the kinematic chains [S] Figure 6 (a) and (b)

$$S_{3a'} = \begin{bmatrix} 16 & 1 & 4 & 0 & 8 \\ 1 & 9 & 1 & 4 & 0 \\ 4 & 1 & 9 & 1 & 0 \\ 0 & 4 & 1 & 9 & 4 \\ 8 & 0 & 0 & 4 & 9 \end{bmatrix} \tag{20}$$

$$S_{3b'} = \begin{bmatrix} 16 & 1 & 4 & 0 & 8 \\ 1 & 9 & 1 & 4 & 0 \\ 4 & 1 & 9 & 1 & 0 \\ 0 & 4 & 1 & 9 & 4 \\ 8 & 0 & 0 & 4 & 9 \end{bmatrix} \tag{21}$$

Step 4: Determinants of Skeleton Matrices [S] for both chains:

$$|S3a'| = 1.46 \times 10^4 \tag{22}$$
$$|S3b'| = 1.46 \times 10^4 \tag{23}$$

Step 5: Formation of complete Identification String (IS)
Identification strings of both Kinematic chains of Figure 6(a) and (b)

Where

$$|S| = |S3a'|/|S3b'| \quad (24)$$

$$(IS_i) = F, n, j/i_{max}\text{---}n_4\ n_3\ n_2/E\ Z\ D\ V/|S| \quad (25)$$

$$(IS_a)=01\ 10\ 13/00/00/01/04/05/00/03/05/00/00/1.46\times\xi \quad (26)$$

$$(IS_b)=01\ 10\ 13/00/00/01/04/05/00/03/05/00/00/1.46\times\xi \quad (27)$$

Where $\xi = 10^4$

Step 6: Checking for the secondary condition.

Compare the Identification String for both the chains. It is found that $(IS_a) = (IS_b)$, thus the kinematic chain 6 (a) is isomorphic with 6 (b)

Example 3

The last example is shown in Figure 7. It consists of three degrees of freedom 12 bar kinematic chains (a), (b) & (c). They have the same Eigen values. The characteristic polynomial and adjacency matrix approach fails to detect isomorphism for these chains [7]. These kinematic chains are tested for isomorphism by the proposed method.

Testing for the Primary condition

Step 1: Formation of one part of the identification string (IS) by visualizing of kinematic chains

$$(IS_i) = F/\ N/j/i_{max}\text{---}n_4\ n_3\ n_2/E\ Z\ D\ V/|S| \quad (28)$$

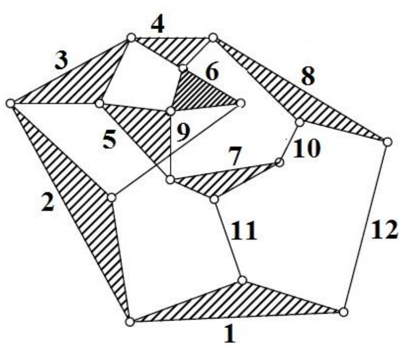
Part of IS for the respective chains is as follows.

03/12/15/06 00 00 08 04/00 08 04 00 00/ (29)

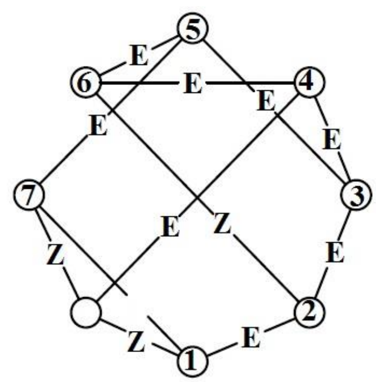
03/12/15/06 00 00 08 04/00 09 03 00 00/ (30)

03/12/15/06 00 00 08 04/00 09 03 00 00/ (31)

Step 2: Conversion of kinematic chain into skeleton



(a)



(c)

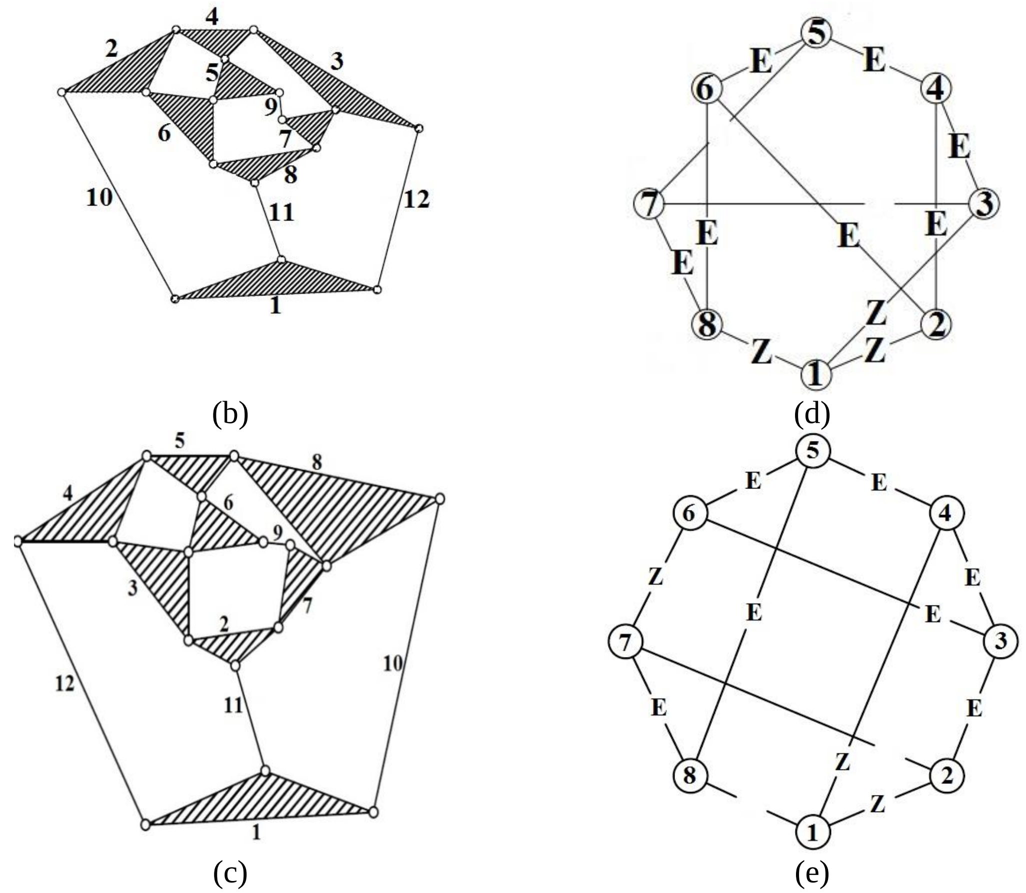


Figure 7: (a), (b) and (c) 12-bar, 3-F kinematic chains, (c), (d) and (e) their skeleton diagram, respectively.

Step 3: Development of Skeleton matrix

$$S4a' = \begin{bmatrix} 9 & 4 & 4 & 0 & 0 & 0 & 0 & 4 \\ 4 & 9 & 0 & 1 & 0 & 1 & 0 & 0 \\ 4 & 0 & 9 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 9 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 9 & 1 & 4 & 0 \\ 0 & 1 & 0 & 0 & 1 & 9 & 0 & 1 \\ 0 & 0 & 1 & 0 & 4 & 0 & 9 & 1 \\ 4 & 0 & 0 & 0 & 0 & 1 & 1 & 9 \end{bmatrix} \quad (32)$$

$$S4b' = \begin{bmatrix} 9 & 1 & 0 & 0 & 0 & 0 & 4 & 4 \\ 1 & 9 & 1 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 9 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 9 & 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 & 9 & 1 & 1 & 0 \\ 0 & 4 & 0 & 1 & 1 & 9 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 & 9 & 4 \\ 4 & 0 & 0 & 4 & 0 & 0 & 4 & 9 \end{bmatrix} \quad (33)$$

$$S4c' = \begin{bmatrix} 9 & 4 & 4 & 0 & 0 & 0 & 0 & 4 \\ 4 & 9 & 0 & 1 & 0 & 1 & 0 & 0 \\ 4 & 0 & 9 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 9 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 9 & 1 & 4 & 0 \\ 0 & 1 & 0 & 0 & 1 & 9 & 0 & 1 \\ 0 & 0 & 1 & 0 & 4 & 0 & 9 & 1 \\ 4 & 0 & 0 & 0 & 0 & 1 & 1 & 9 \end{bmatrix} \quad (34)$$

Skeleton matrices of the kinematic chains [S] Figure 7(c), (d) and (e) respectively

Step-4: Determinants of [S] matrix of Figure 7(a), (b) and (c)

$$|S4a'| = 1298*\Phi \quad (35)$$
$$|S4b'| = 1137*\Phi \quad (36)$$
$$|S4c'| = 1137*\Phi \quad (37)$$

Above are the determinants of all three kinematic chains respectively where the value of Φ is 10^4 in this case.

Step 5: Identification strings of the Kinematic chains of Figure 7(a), (b) and (c)

$$(IS_i) = F/\ N/j/i_{max}\text{---}n_4\ n_3\ n_2/E\ Z\ D\ V/|S| \quad (38)$$
$$\text{Where } |S| = |S4a'/4b'/4c'| \quad (39)$$
$$(IS_a) = 03/12/15/06\ 00\ 00\ 08\ 04/00\ 08\ 04\ 00\ 00/1298 \quad (40)$$
$$(IS_b) = 03/12/15/06\ 00\ 00\ 08\ 04/00\ 09\ 03\ 00\ 00/1137 \quad (41)$$
$$(IS_c) = 03/12/15/06\ 00\ 00\ 08\ 04/00\ 09\ 03\ 00\ 00/1137 \quad (42)$$

Step 6: Checking for the secondary condition.
Above are the identification strings of three kinematic chains respectively.

$$(IS_a) \neq (IS_b) = (IS_c) \quad (43)$$

It is concluded that kinematic chains of Figure 7(b) and (c) are isomorphic but (a) & (b) are non-isomorphic and (a) & (c) are also non-isomorphic. These results are in accordance with the result of Chang et al. [20].

CONCLUSIONS

The referred identification string is the unique invariants of the kinematic chain that are reliable torecognizing isomorphism not only among the kinematic chains and even trustworthy for detection isomorphism among co-spectral graph efficiently. The methodology is applied to 6 links, 8 links single degree of freedom and 10 links and 12 links multi-degree of freedom simple jointed kinematic. No counterexample has been found for the detection of isomorphism in 6- link, 8- link, one dof and 12 links multi degree of freedom the examples cited shows the results in accordance with [39-41] and [20]. The kinematic structural synthesis is the systematic development of kinematic chains and mechanisms derived from the kinematic chains. The kinematic chains can be represented by their skeleton, which is an abstract topological graph of kinematic chains. During enumeration of all possibly kinematic chains, duplication may occur. To eliminate this duplication, an isomorphic test is required. For this purpose, numbers of methods are proposed during recent years. However, most of the methods are complex and difficult to understand, are an only necessary condition, but not sufficient for the isomorphism detection. The new approach addresses the critical issue of isomorphism using identification string.

Highlights of the proposed methods are as follows:

1. The Skeleton diagram and matrix of the kinematic chain are the unique representation and enable isomorphism identification easy
2. The size of the skeleton matrix[S] is equal to the number of polygonal links instead of the total number of links existing in the kinematic chains as generally proposed by other researchers [3,4], resulting in reducing the computational time to evaluate the invariant of the kinematic chain.
3. Single classification string for the kinematic chain has plenty of information to fulfill both primary and secondary conditions to identify isomorphism.
4. The approach is reliable and efficient for both simple and complex kinematic chains.

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