JOURNAL OF MECHANICAL ENGINEERING AND SCIENCES
ISSN: 2289-4659 e-ISSN: 2231-8380
VOLUME 19, ISSUE 2, 2025, 10620 - 10630
DOI: https://doi.org/10.15282/jmes.19.2.2025.4.0832

RESEARCH ARTICLE

Enhanced performance of step slider bearings with couple stress and magnetic fluid lubrication incorporating sinusoidal magnitude: A theoretical investigation

Dipali K. Chauhan[1,2], Jimit R. Patel[1]*, G. M. Deheri[3]

[1] Department of Mathematical Sciences, P. D. Patel Institute of Applied Sciences, Charotar University of Science and Technology (CHARUSAT), Changa - 388 421, Anand, Gujarat, India

Phone: +91 2697265190, Fax.: +91 2697265192

[2] Department of Mathematics,Shri Alpesh N. Patel Postgraduate Institute of Science & Research, Sardar Patel University, Anand - 388 001, Gujarat, India

[3] Department of Mathematics, Sardar Patel University, Vallabh Vidyanagar - 388 120, Anand, Gujarat, India

ABSTRACT – Step slider bearings play a critical role in tribological applications, particularly within high-precision mechanical systems, where their operational efficiency is significantly influenced by the nature of the lubricant and its rheological response to external field variations. This theoretical investigation examines the behavior of a step slider bearing, taking into account the effects of couple stress and magnetic fluid as a lubricant. A sinusoidal model is employed to modulate the magnetic field's intensity, aiming to optimize bearing performance. The Reynolds equation governing the step slider bearing is adapted to incorporate the Neuringer and Rosensweig theory for magnetic fluid flow and the Stokes micro-continuum principle for couple stress effects. Through the solution of the modified Reynolds equation under appropriate boundary conditions, parameters such as pressure, load carrying capacity (LCC), center of pressure, and frictional force are quantified. Visual displays are used to present the study's results. The outcomes indicate a significant increase in load compared to systems without magnetic fluid. In addition, the increase in this couple stress and the parameters of magnetization will increase the load capacity and friction force. On the other hand, a decrease in the coefficient of friction is observed. The effect of magnetic fluid lubrication is found to increase the LCC by at least 28.48% with the inclusion of couple stress. In precision bearing systems used in aerospace, robotics, and micro-mechanical devices, enhanced load-carrying capacity and friction regulation can be achieved by utilizing ferrofluids subjected to well-tuned magnetic fields.

ARTICLE HISTORY
Received : 01st Sept. 2024
Revised : 26th Feb. 2025
Accepted : 02nd May 2025
Published : 30th June 2025

KEYWORDS
Step slider bearing
Load carrying capacity
Magnetic fluid
Couple stress
Sinusoidal magnitude of magnetic field

1. INTRODUCTION

The difficulties related to slider bearings have been widely studied recently, as they are well-suited to the mathematical analysis of a simple problem. The slider bearing is a common type of bearing used in various machines, including engines, compressors, turbines, electric motors, and generators. These bearings are structural components used to provide transverse loads within an engineering arrangement. Ramanaiah and Sarkar [1] investigated the effects of couple stress and found that the coefficient of friction was lower than that of Newtonian fluids, the load-carrying capacity showed an increase, and the frictional force also increased. Magnetic fluid lubrication in porous step bearings was investigated by Shah and Patel [2], including the influence of random porosity and the strength of a variable magnetic field. Rao and Agrawal [3] explored the effects of surface roughness, demonstrating that LCC increases with morphological parameters in porous structures. Naduvinamani et al. [4] analyzed Rayleigh step slider bearings with Rabinowitch fluid, finding that dilatant fluids enhance the LCC, while pseudoplastic fluids exhibit the opposite trend. Similarly, Naduvinamani and Siddangouda [5] reported that couple-stress lubricants improve LCC and reduce friction compared to Newtonian cases. Andharia and Pandya [6] investigated the impacts of surface roughness, showing that LCC increases with standard deviation but decreases with aspect ratio and variance. Maiti [7] examined micropolar fluid lubrication, concluding that these fluids enhance load capacity, particularly in composite slider bearings. Deheri et al. [8] analyzed the performance of characteristics of a Shlimois model-based ferrofluid (FF) lubrication of a rough porous convex pad slider bearing, finding that the LCC decreased due to porosity. Hanumagowda et al. [9] discussed the effect of couple stress and magneto-hydrodynamics (MHD) on a secant slider bearing. It was concluded that the bearing characteristics, both steady and dynamic, would show an increasing trend in their values when the magnetic field was applied to the bearing structure and when fluids with couple stress were used as lubricants. Nada [10] studied the performance of a finite hydrodynamic slider bearing lubricated by ferrofluid, finding that the magnetic effect of the ferro-lubricant has a significant improving impact on the overall static characteristics of the slider bearing. Anthony and Elamparithi [11] examined the performance of the rough porous Rayleigh step slider bearing lubricated with couple stress fluids with the effect of magneto-hydrodynamics. It was observed that the workload and frictional force increase when compared with the plate without roughness, and decrease for the coefficient of friction.

*CORRESPONDING AUTHOR | Jimit R. Patel | ✉ patel.jimitphdmarch2013@gmail.com

With advancements in bearing technology, the incorporation of magnetic fluids, also known as ferrofluids, has opened new avenues for sealing, lubrication, cooling, and vibration reduction in mechanical systFerrofluidsluids, consisting of stable colloidal suspensions with dispersed ferromagnetic particles in a liquid base, exhibit distinctive responses to magnetic fields, enabling their application across various domains such as biomedical engineering and industrial operations. Recently, numerous theoretical investigations [12-15] have been conducted on the ferrofluid lubricated bearing design system due to its various advantages, including long life, silent operation, and reduced wear. Deheri and Patel [16] investigated magnetic fluid-based squeeze films in porous parallel plate slider bearings, showing that negatively skewed roughness, particularly with negative variance, enhances performance when optimizing magnetization and aspect ratio. Patel and Deheri [17] further examined transversely rough porous bearings with slip velocity, finding that magnetic fluid lubrication significantly improves performance, with negatively skewed roughness boosting LCC. Shukla and Deheri [18] analyzed porous rough step bearings with slip velocity, concluding that while transverse roughness negatively affects performance, magnetization can partially offset these effects, especially when minimizing slip in cases of negatively skewed roughness. Shimpi and Deheri [19] analyzed the performance of a magnetic fluid-based squeeze film between infinitely long, porous, and rough rectangular plates. It was revealed that the adverse effects of standard deviation and porosity can be partially compensated for by the impactful effects of magnetization parameters in the case of negatively skewed roughness, through adjusting the aspect ratio. Shimpi and Deheri [20] investigated the magnetic fluid-based squeeze film behavior between a curved, rough, porous circular plate and a flat, rough, porous circular plate. It was observed that magnetic fluid lubrication enhances the performance of bearings with an optimum curvature and negative variance, providing intensity in the LCC as well. The combined effect of longitudinal surface roughness and deformation has been presented by Patel et al. [21] in the behavioral view of a ferrofluid-based squeeze film in conical plates, revealing that the use of magnetization can enhance performance. Still, the depth of deformation of the bearing should be carefully considered, as the strength of the magnetic field can provide one more control over the design.

The notion of couple stress plays a central role in studying mechanical behavior, particularly in structures with complex geometries or with small features. Couple stress refers to the tensions within a substance due to microscopic instants or pairs, which influence its mechanical properties and habit of deformation. The framework offers an in-depth understanding of material responses, particularly in the context of analyzing microstructure, biological materials, and small-scale mechanical systems. Couple stress fluids are of paramount importance in improving the performance of slider bearings, especially in magnetic and slip conditions [22-26]. Lin et al. [27] studied the effect of non-Newtonian couple stresses on Rayleigh step slider bearings, stating that the impact on LCC, dynamic stiffness, and damping coefficients was improved. Naduvinamani and Angadi [28] have discovered that the LCC and dynamic properties can be enhanced by using surface roughness in couple-stress-lubricated Rayleigh step bearings. A double-layered porous Rayleigh step slider bearing, presented by Naduvinamani and Ganachari [29], demonstrated better efficiency in lubrication due to an increase in LCC and a minimization of friction. Naduvinamani et al. [30] and Naduvinamani and Siddangouda [31] analyzed the effect of surface unevenness and found that negatively skewed roughness had a positive impact, resulting in increased load and reduced friction. Due to the importance of Magnetic fields in engineering, Barik et al. [32] have shown that a sinusoidal magnetic field augments load capacity by 38.3 percent. According to Patel and Deheri [33], magnetization enhances thin-film lubrication in slider bearings with rough and porous surfaces. Patel et al. [34] have described how the optimization of the step size enhances the load-bearing capability in magnetic fluid-lubricated porous step bearings.

Slider bearings play a significant role in reducing friction and loading activities in machinery; hence, optimizing the performance of the bearings is crucial in various types of mechanical and industrial applications. Conventional lubrication systems often encounter challenges in achieving high load capacity and low friction. Recent improvements have made it possible to incorporate ferrofluids and consider the couple stress effect to counteract these shortcomings. Ferrofluids, with their peculiar properties —i.e., responsiveness to magnetic fields —have been observed to enhance the performance of bearings. Additionally, the concept of couple stress is believed to aid in mitigating internal stresses within the material, particularly in materials with microstructures. Earlier studies have demonstrated that the application of magnetic fluids may improve bearing performance. Nonetheless, a thorough examination is required to combine the magnetic fluid lubrication effects with couple stress. Through this integration, it may be possible to bring considerable changes to LCC and reduce frictional forces, consequently increasing efficacies and extending the life of the machinery systems.

The present work was a necessary study because very few publications exist in the literature on magnetic fluid lubrication in the presence of couple stress. Therefore, considering this background, the current work examines the movement of a step slider bearing taking into account the magnitude of ferrofluid and couple stress lubrication. The sinusoidal model of the magnetic field and the adoption of the Reynolds equation by the theory of Neuringer-Rosensweig (NR) to describe magnetic fluid flow, along with the Stokes micro-continuum model that describes couple stress, can be used to model the effect of these factors on bearing performance in great detail, as current research suggests. The study will focus on the most crucial performance parameters, including pressure distribution, load-carrying capacity, center of pressure, and frictional force. The results should provide useful information and design rules to develop and optimize high-performance bearing systems, and will ultimately lead to the realization of more efficient mechanical components.

2. MATERIALS AND METHODS

2.1 Mathematical Formulation

Various theories have been developed to elucidate the anomalous behaviour of fluids with substructure, such as polymeric fluids [35, 36]. The micro-continuum theory extends classical fluid dynamics to include polar phenomena, accounting for couple stress, body couples, and asymmetric stress tensors as proposed by Stokes [37-39]. In this study, we employ an analytical method to solve the mathematical equations governing pressure, load-carrying capacity, center of pressure, and frictional force, thereby observing the performance of step slider bearings. The fundamental equations governing the motion of fluids with couple stress, expressed in Cartesian tensor notation, are as follows:

$$\dot{\rho} + \rho v_{k,k} = 0 \tag{1}$$

$$\rho \dot{v_i} = \rho b_i + T_{ji,j} \tag{2}$$

$$\rho g_i + e_{ijk} T_{jk} + M_{ji,j} = 0 \tag{3}$$

The superscript dot indicates a material time derivative, and a subscript followed by a comma indicates partial differentiation.

The constitutive equations governing the stress tensor and the couple stress tensor are provided as follows:

$$T_{ij} = \left(-p + \lambda v_{k,k}\right)\delta_{ij} + \mu\left(v_{i,j} + v_{j,i}\right) - \frac{\rho}{2} e_{ijk} g_k + \eta \nabla^2 (v_{i,j} - v_{j,i}) \tag{4}$$

$$M_{ij} = 2\eta e_{j\alpha\beta} v_{\beta,\alpha i} + 2\eta' e_{i\alpha\beta} v_{\beta,\alpha j} \tag{5}$$

Substituting T_{ij} and M_{ij} from Eqs. (4) and (5) into Eq. (3) is true for all values of the variable involved. Eq. (2) yields the field equation for velocity:

$$\rho \dot{v_i} = -p_{,i} + (\lambda + \mu + \eta \nabla^2) v_{k,ki} + (\mu - \eta \nabla^2) v_{i,jj} + \rho b_i + \frac{1}{2} e_{ijk} (\rho g_k)_{,j} \tag{6}$$

Building on the foundational work by Ramanaiah and Sarkar [1], this study focuses on evaluating the performance of slider bearings lubricated by magnetic fluids exhibiting couple stress properties. The geometry of the two-dimensional slider bearings, depicted in Figure 1, comprises two closely spaced rigid surfaces undergoing relative motion. The lubricant filling the gap is modelled as an incompressible fluid with couple stress, assuming no body forces and body couples. Figure 2 shows the configuration of the step slider bearing.

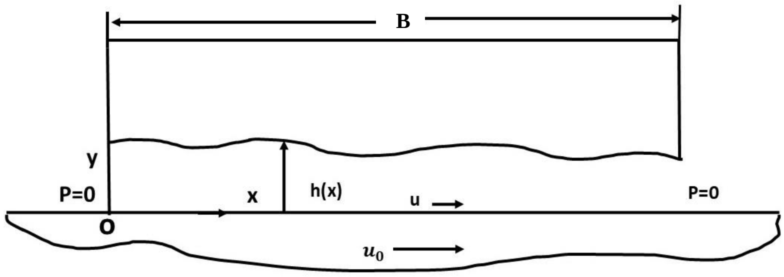

Figure 1. The geometry of the slider bearing

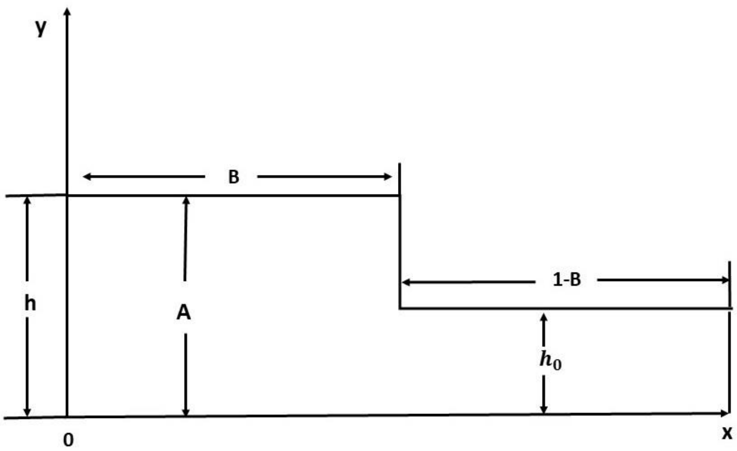

Figure 2. Configuration of the Step slider bearing

Under these conditions, and with the velocity vector $(u, v, 0)$, the fundamental Eqs. (1) and (6) simplify to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$\mu\frac{\partial^2 u}{\partial y^2}-\eta\frac{\partial^4 u}{\partial y^4}=\frac{dp}{dx} \tag{8}$$

The relevant components of the stress tensor and the couple stress tensor are derived as:

$$T_{21}=\mu\frac{\partial u}{\partial y}-\eta\frac{\partial^3 u}{\partial y^3} \tag{9}$$

$$M_{23}=-2\eta\frac{\partial^2 u}{\partial y^2} \tag{10}$$

2.2 Boundary Conditions

Boundary conditions for Eqs. (7) and (8) include standard no-slip conditions and no-couple stress conditions, respectively:

$$u(y=0)=u_0; u(y=h)=0 \tag{11}$$

$$\frac{\partial^2 u}{\partial y^2}(y=0)=0;\ \frac{\partial^2 u}{\partial y^2}(y=h)=0 \tag{12}$$

Solving Eq. (8) with these boundary conditions, the velocity profile, u is determined as (shown in the Appendix):

$$u=\frac{1}{2\mu}\frac{dp}{dx}\left(y^2-hy+2l^2\left[1-\frac{\cosh\{(2y-h)/2l\}}{\cosh(h/2l)}\right]\right)+u_0(1-\frac{y}{h}) \tag{13}$$

where, $l=(\eta/\mu)^{1/2}$.

The volume flow rate q is determined as :

$$q=\int_0^h u\,dy=\frac{u_0 h}{2}-\frac{1}{12\mu}\frac{dp}{dx}\frac{1}{s(l,h)} \tag{14}$$

where $s(l,h)=\left\{h^3-12l^2h+24l^3 tanh\left(\frac{h}{2l}\right)\right\}^{-1}$

Integrating Eq. (7) across the fluid film with Eq. (14) and boundary conditions:

$$v(y=0)=0;\ v(y=h)=0 \tag{15}$$

One can obtain the Reynolds equation for the bearings as:

$$\frac{d}{dx}\left\{\frac{1}{s(l,h)}\frac{dp}{dx}\right\}=6\mu u_0\frac{dh}{dx} \tag{16}$$

Combining Eqs. (9) and (13) provide,

$$T_{21}=\left(y-\frac{h}{2}\right)\frac{dp}{dx}-\frac{\mu u_0}{h} \tag{17}$$

The friction force on the bearing surface $(y=0)$ is expressed as:

$$f=-\int_0^b T_{21}dx=\int_0^b\left(\frac{h}{2}\frac{dp}{dx}+\frac{\mu u_0}{h}\right)dx \tag{18}$$

Step bearings have garnered significant attention in tribology due to their superior LCC compared to other bearing geometries. This study extends the theoretical framework proposed by Ramanaiah and Sarkar [1] and Cameron [40] to investigate step bearings, modeling the non-dimensional film thickness, $H(X)$, as defined by:

$$H\left(\frac{x}{B}\right)=H=\frac{h}{h_0}=\begin{cases}A>1;\ \text{when } 0<\frac{x}{B}<1\\ 1;\quad \text{when } 1<\frac{x}{B}<\frac{1}{B}\end{cases} \tag{19}$$

A simplified steady flow model for ferrofluid subjected to sinusoidally varying external magnetic fields, as proposed by Neuringer and Rosensweig [41], is employed. The model's equations contain:

$$\rho(\bar{q}\nabla)\bar{q}=-\nabla p+\mu\nabla^2\bar{q}+\mu_0(\bar{M}\nabla)\bar{H} \tag{20}$$

$$\nabla\bar{q} = 0 \tag{21}$$

$$\nabla \times \bar{H} = 0 \tag{22}$$

$$\bar{M} = \bar{\mu}\bar{H} \tag{23}$$

$$\nabla(\bar{H} + \bar{M}) = 0 \tag{24}$$

Utilizing Eqs. (21) - (24), Eq. (20) transforms to:

$$\rho(\bar{q}\nabla)\bar{q} = -\nabla\left(p - \frac{\mu_0\bar{\mu}}{2}M^2\right) + \mu\nabla^2\bar{q}. \tag{25}$$

The lubricant within the step slider bearing is treated as an isoviscous and incompressible fluid, exhibiting laminar flow characteristics. To model the film pressure in such a system, we use a modified Reynolds-type equation, which accounts for the effects of a magnetic field discussed in Prajapati [42] and Bhat [43]:

$$\frac{d}{dx}\left\{\frac{1}{s(l,h)}\frac{d}{dx}\left(\mathrm{p} - \frac{\mu_0\bar{\mu}}{2}\mathrm{M}^2\right)\right\} = 6\mu u_0\frac{dh}{dx} \tag{26}$$

where $M^2 = ksin\left(\left(\frac{x}{B}\right) - \left(\frac{x}{B}\right)^2\right)$, where, k is a constant adjusted for dimensional consistency.

2.3 Non-Dimensionlization

Dimensionless quantities are introduced:

$$X = \frac{x}{B};\ L = \frac{2l}{h_0};\ Q = \frac{2q}{u_0h_0};\ P = \frac{ph_0^2}{6\mu u_0B};\ F = \frac{fh_0}{6\mu u_0B};\ \mu^* = \frac{\mu_0\bar{\mu}kBh_0^2}{6\mu u_0} \tag{27}$$

$$S(L,H) = \left[H^3 - 3L^2H + 3L^3tanh\left(\frac{H}{L}\right)\right]^{-1} \tag{28}$$

where, h_0 is the minimum film thickness.

The boundary conditions for the dimensionless pressure distribution within the lubrication film are:

$$\mathrm{P}(X = 0) = 0; \mathrm{P}(X = 1) = 0 \tag{29}$$

with the help of Eqs. (27) - (28) and the boundary conditions Eq. (29), the dimensionless pressure, P, load capacity, W, the center of pressure, $\bar{X}$ and dimensionless frictional force, F are found as:

$$P = \frac{\mu^*sin(X - X^2)}{2} + \int_0^X[(H - Q)S(L,H)]dX \tag{30}$$

where,

$$Q = \frac{\int_0^1 HS(L,H)dX}{\int_0^1 S(L,H)dX} \tag{31}$$

$$W = \int_0^1 PdX = \frac{\mu^*}{2} \times 0.165479 + \int_0^1[X(Q - H)S(L,H)]dX \tag{32}$$

$$\bar{X} = \frac{1}{W}\int_0^1 XPdX = \frac{\mu^*}{2W} \times 0.0827396 + \frac{1}{2W}\int_0^1[X^2(Q - H)S(L,H)]dX \tag{33}$$

and,

$$F = \frac{1}{6}\int_0^1\left[3H(H - Q)S(L,H) + \frac{1}{H}\right]dX \tag{34}$$

The basic characteristics of the slider bearing, $W, \bar{X}, F$ and C are obtained by evaluating the integrals Eqs. (32)-(34) provided the function, $H(X)$ is given by

$$\begin{aligned} W &= \frac{\mu^*}{2} \times 0.165479 + \frac{1}{2}B(1 - B)(A - 1) \\ &\times \left[B\left\{1 - 3L^3\left(\frac{1}{L} - \tanh\frac{1}{L}\right)\right\} + (1 - B)\left\{A^3 - 3L^3\left(\frac{A}{L} - \tanh\frac{A}{L}\right)\right\}\right]^{-1} \end{aligned} \tag{35}$$

$$\bar{X} = \frac{1 + B}{3} + \frac{\mu^*}{2W}(0.0827396 - \frac{0.165479}{3}(1 + B)) \tag{36}$$

$$F = (A-1)\left(W - \frac{\mu^*}{2} \times 0.165479\right) + \frac{1}{6}\left(\frac{B}{A} + 1 - B\right) \tag{37}$$

$$C = \frac{F}{W} \tag{38}$$

where, pressure, P denotes the hydrodynamic force generated within the lubricant film, affecting the bearing's capacity to support loads. The LCC (W) represents the total force the bearing can endure due to the pressure distribution in the lubricant. The frictional coefficient, C indicates the resistance to motion between bearing surfaces, with a lower value signifying improved lubrication and efficiency.

3. RESULTS AND DISCUSSION

Step slider bearings, utilized in precision machinery, enhance load capacity, reduce friction, and improve stability in high-performance applications.The combined impact of couple stress and ferrofluid as lubricant on the step slider bearing's performance is analysed in this comparative study. The length of the coupling stress molecules is a characteristic of the additives in non-Newtonian lubricants. By setting $l \to 0$ with $\mu^* \to 0$, the current study transitions to the dimensionless form of the Reynolds equation for a conventional fluid-based step bearing system. When varying l with $\mu^* = 0$, this study aligns with the findings of Ramanaiah and Sarkar [1], discussing a step bearing system without magnetization effects. The current work expands that discussion to remarkable effect in the bearing performance by taking into account change of both l and μ^*. Specifically, the load-carrying capacity increases by:

$$\frac{0.165479\mu^*}{2} \tag{39}$$

while the frictional force increases by:

$$\frac{0.165479(A-1)\mu^*}{2} \tag{40}$$

due to the presence of couple stress and ferrofluid. These effects are not observed in the studies by Ramanaiah and Sarkar [1] or in conventional fluid-based bearing systems. The theoretical message from Eq. (35) is that W increases with increasing μ^* because the expression involved in Eq. (35) is mathematically linear with respect to μ^*.

The step height ratio, A, defines the proportion of the step height to the overall bearing film thickness, which impacts pressure distribution and load-bearing efficiency. The magnetic fluid parameter, μ^* measures the effect of an external magnetic field on ferrofluid lubricants, enhancing LCC and minimizing wear. The couple stress parameter, L accounts for microstructural effects in lubricants, contributing to increased load support, lower friction, and improved lubrication film stability. The graphical analysis of parameters, such as magnetic fluid, couple stress aspect ratio effect on LCC, friction coefficient, and center of pressure, is presented. Figures 3 to 6 illustrate the LCC and frictional coefficient concerning various values of couple stress and magnetic fluid parameters.

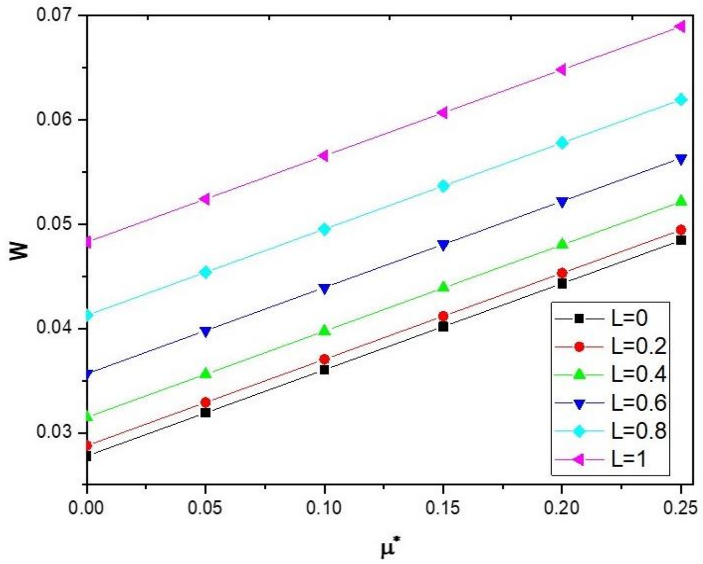

Figure 3. W versus μ^* and L for the step height ratio $(A = 2)$

Figures 3 and 4 depict the variation of LCC (W), as a function of ferrofluid parameter, μ^* for different values of the couple stress parameter, L. The results indicate that W follows distinct trends in bearings lubricated with Newtonian fluids and those incorporating increased W due to couple stress and ferrofluid lubrication. Notably, for aspect step height ratios $A = 2$ and $A = 3$, the presence of couple stress and magnetic fluid parameters enhances LCC. Additionally, the load-carrying capacity is higher for $A = 2$ compared to $A = 3$. It is observed that the LCC decreases as the step height ratio increases. The effect of couple stress is to say the least, nominal for lower values of $L(0 < L < 2)$ which becomes more manifest when the step height ratio decreses. The effect of μ^* On LCC, the effect is more pronounced as the step height ratio decreases.

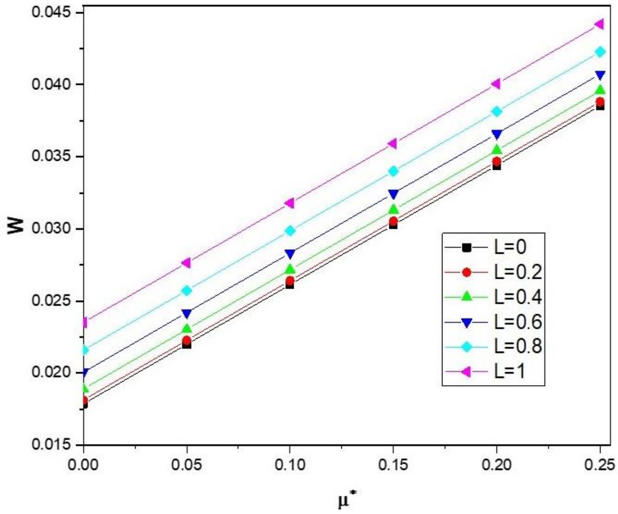

Figure 4. W versus μ^* and L for the step height ratio ($A = 3$)

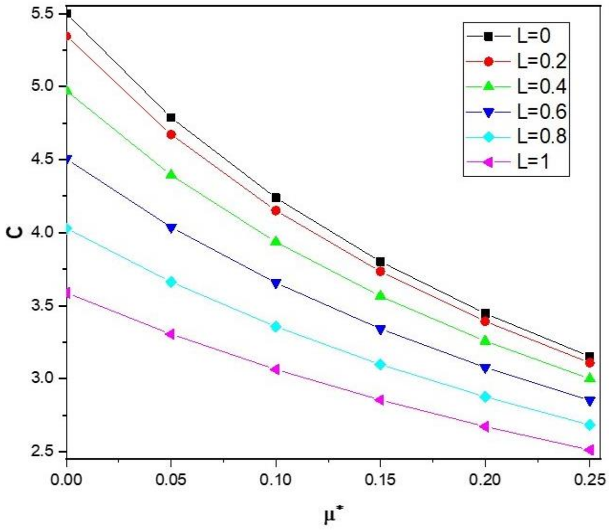

Figure 5. C versus μ^* and L for the step height ratio ($A = 2$)

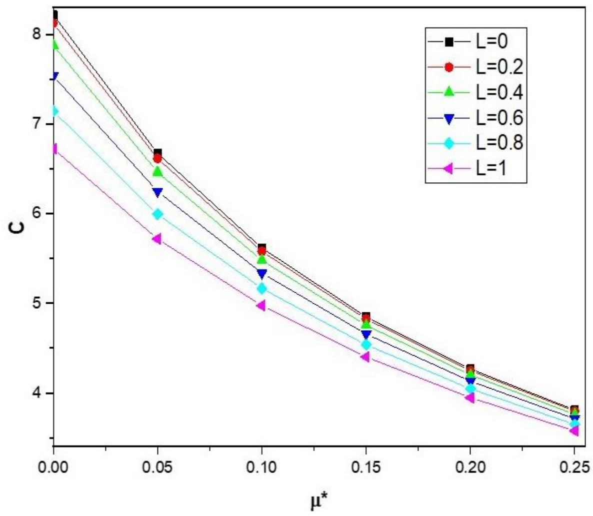

Figure 6. C versus μ^* and L for the step height ratio ($A = 3$)

Figures 5 and 6 present the variation of the frictional coefficient, C, with respect to μ^* for the influence of L and follows distinct trends in Newtonian and ferrofluid-lubricated bearings. The results indicate that for $A = 2$ and $A = 3$, the presence of couple stress and magnetic fluid parameters leads to a reduction in C. Furthermore, the frictional coefficient is lower when compared to $A = 2$ compared to $A = 3$. One can very well see that C increses with increasing step height ratio. The couple stress effect is not significant for lower values, but it becomes more visible as the step height ratio decreases. The impact of μ^* On C, it gets increased with an increasing step height ratio.

Figure 7 illustrates the LCC concerning the step height ratio A under different conditions: (i) $L = 0, \mu^* = 0$ representing the conventional fluid-based step bearing; (ii) $L = 0.1, \mu^* = 0$ indicating the presence of only a couple stress; (iii) $L = 0$, $\mu^* = 0.1$ representing solely magnetic fluid; and (iv) $L = 0.1, \mu^* = 0.1$ denoting the combined effect of couple stress and magnetic fluid. Upon meticulous examination of Figure 7, it becomes apparent that W exhibits similar trends across all cases. However, cases (iii) and (iv) notably augment the load-carrying capacity of the step-bearing system. Moreover, minimal deviation in W is observed for higher values of A across all scenarios, although Case (iv) demonstrates a higher load due to the concurrent presence of couple stress and ferrofluid. Accordingly, the LCC experiences an increase with the augmentation of couple stress and magnetic fluid parameters compared to scenarios without magnetic fluid and couple stress. Considerable load reduction is caused when A moves up. However, the situation remains better due to the combined effect of μ^* and L.

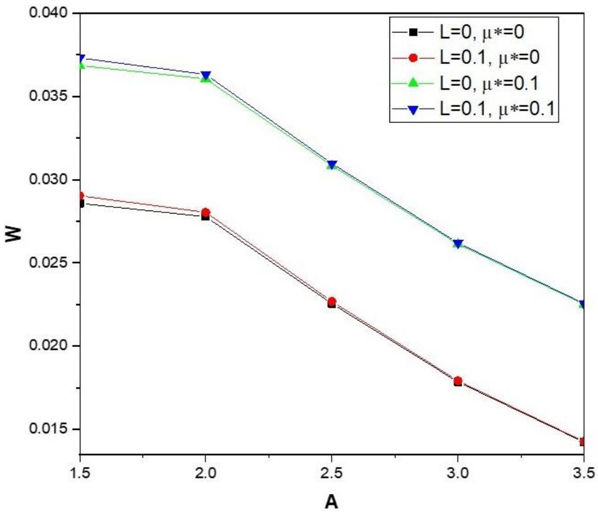

Figure 7. W and step height ratio for different L and μ^*

4. CONCLUSIONS

Based on the investigation into the impact of a sinusoidal magnitude of magnetic field on a step slider bearing with couple stress and ferrofluid lubrication, the following scientific conclusions are drawn. A sinusoidal magnitude of magnetic field induces an increase in the LCC by a factor $(1 - sin\ (1))\frac{\mu^*}{2}$, while for a magnetic field whose magnitude is determined by a linear expression, it is $\frac{\mu^*}{2}$. Here, the LCC is reduced but remains closer to the linear case. The load-carrying capacity demonstrates an increase corresponding to the augmentation of the couple stress parameters across various magnetic field parameters. The positive effect of μ^* is relatively more as compared to the impact of L. An increase in the step height ratio contributes to the performance of the bearing system. Throughout the study, the friction force maintains consistency with the rising couple stress parameter. The friction coefficient exhibits a decline with the augmentation of a couple stress parameters. In comparison to systems devoid of magnetic fluid, our findings indicate an enhancement in load-carrying capacity. Additionally, the load capacity increases, and the frictional coefficient decreases as the couple's stress and magnetization parameters increase. A future direction of research could be analyzing the impact of couple stress and magnetic fluid on the performance of a step-porous, rough slider bearing. Even if the flow is absent, a significant amount of load is supported by the bearing system which is unlikely in the case of traditional lubrication. If properly designed, this piece of work could be employable in the industry. These conclusions contribute to a deeper understanding of the intricate interplay between magnetic fields, couple stress, and lubrication in slider-bearing systems, offering insights into optimizing their performance in various engineering applications.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the constructive suggestions, critical remarks, and in-depth comments of the reviewers and editors, which resulted in an improvement in the paper's presentation. This study was not supported by any grants from funding bodies in the public, private, or not-for-profit sectors.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

AUTHORS CONTRIBUTION

Dipali K. Chauhan (Methodology; Data curation; Writing - original draft; Formal analysis)

Jimit R. Patel (Methodology; Data curation; Conceptualization; Formal analysis; Visualisation; Supervision)

G. M. Deheri (Supervision)

AVAILABILITY OF DATA AND MATERIALS

The data supporting this study's findings are available on request from the corresponding author.

ETHICS STATEMENT

Not applicable.

REFERENCES

[1] G. Ramanaiah, P. Sarkar, "Slider bearings lubricated by fluids with couple stress," *Wear*, vol. 52, no. 1, pp. 27-36, 1979.

[2] R. C. Shah, N. I. Patel, "Impact of various and arbitrary porous structure in the study of squeeze step bearing lubricated with magnetic fluid considering the variable magnetic field," *Journal of Engineering Tribology*, vol. 229, no. 5, pp. 646-659, 2015.

[3] P. S. Rao, S. Agarwal, "Theoretical study of couple stress fluid film in rough step slider bearing with assorted," *Journal of Nanofluids*, vol. 7, no. 1, pp. 92-99, 2018.

[4] N. B. Naduvinamani, S. Patil, S. S. Siddapur, "On the study of Rayleigh step slider bearings lubricated with non-Newtonian Rabinowitsch fluid," *Industrial Lubrication and Tribology*, vol. 69, no. 5, pp. 666-672, 2017.

[5] N. B. Naduvinamani, A. Siddangouda, "A note on porous Rayleigh step bearings lubricated with couple stress fluids," *Proceedings of the Institution of Mechanical Enginers, Part J: Journal of Engineering Tribology*, vol. 221, no. 5, pp. 615-621, 2007.

[6] P. I. Andharia, H. M. Pandya, "Effect of longitudinal surface roughness on the performance of Rayleigh step bearing," *International Journal of Applied Engineering Research*, vol. 13, no. 21, pp. 14935-14941, 2018.

[7] G. Maiti, "Composite and step slider bearings in micropolar fluid," *Japanese Journal of Applied Physics*, vol. 12, no. 7, pp. 1058-1064, 1973.

[8] G. M. Deheri, J. R. Patel, N. D. Patel, "Shliomis model-based ferrofluid lubrication of a rough porous convex pad slider bearing," *Tribology in Industry*, vol. 38, no. 1, pp. 57-65, 2016.

[9] B. N. Hanumagowda, T. Cyriac, H. S. Doreswamy, A. Salma, "Analysis of static and dynamic characteristics of Secant slider bearing with MHD and couple stress fluid," *Malaya Journal of Matematik*, vol. 8, no. 2, pp. 581-587, 2020.

[10] G. S. Nada, "A study of finite hydrodynamics slider bearings lubricated with ferrofluids," *Engineering Research Journal- Faculty of Engineering*, vol. 52, no. 2, pp. 16-25, 2023.

[11] J. Anthony, S. Elamparithi, "Effect of MHD and surface roughness on porous step slider bearing lubricated with couple stress fluid," *International Journal of Heat and Technology*, vol. 41, no. 1, pp. 135-142, 2023.

[12] M. M. Munshi, A. R. Patel, G. M. Deheri, "Analysis of rough porous inclided silder bearing lubricated with a ferrofluid considering sip velocity," *International Journal of Research in Advent Technology*, vol. 7, no. 1, pp. 387-396, 2019.

[13] R. C. Shah, "Ferrofluid lubrication of porous- rough circular squeeze film bearings," *The European Physical Journal Plus*, vol. 137, no. 2, pp. 1-16, 2022.

[14] R. C. Shah, R. B. Shah, "Static and dynamic performance of ferrofluid lubricated long journal bearing," *Zeitschrift für Naturforschung A*, vol. 76, no. 6, pp. 493-506, 2021.

[15] R. C. Shah, D. B. Patel, "On the ferrofluid lubricated exponential squeeze film bearings," *Zeitschrift für Naturforschung A*, vol. 76, no. 3, pp. 209-215, 2021.

[16] G. M. Deheri, J. R. Patel, "Magnetic fluid based squeeze film in a rough porous parallel plate slider bearing," *International Journal of Engineering*, vol. 9, no.3, pp. 443-448, 2011.

[17] N. D. Patel, G. M. Deheri, "Based parallel plate porous slider bearing with slip velocity," *Journal of the Serbian Society for Computational Mechanics*, vol.5, no. 1, pp. 104-118, 2011.

[18] S. Shukla, G. M. Deheri, "Effect of slip velocity on magnetic fluid lubrication of rough porous Rayleigh step bearing," *Journal of Mechanical Engineering and Sciences*, vol. 4, no.1, pp. 532-547, 2013.

[19] M. E. Shimpi, G. Deheri, "Analysis of squeeze film performance in porous rough rectangular plates under the presence of a magnetic fluid lubricant," *Acta technica corviniensis- Bulletin of Engineering*, vol. 4, no. 1, pp. 49-54, 2011.

[20] M. E. Shimpi, G. M. Deheri, "Surface roughness effect on a magnetic fluid-based squeeze film between a curved porous circular plate and a flat circular plate," *Journal of the Brazilian Society Mechanical Sciences and Engineering*, vol. 36, no. 2, pp. 233-243, 2014.

[21] J. R. Patel, M. E. Shimpi, G. M. Deheri, "Ferrofluid-based squeeze film for a rough conical bearing with deformation effect," *International Conference on Research Invoations in Science, Engineering & Technol - Kalpa Publications in Computing*, vol. 2, no. 6, pp. 119-129, 2017.

[22] V. Badescu, "Two classes of sub-optimal shapes for one-dimensional slider bearings with couples stress lubricants," *Applied Mathematical Modelling*, vol. 81, pp. 887-909, 2020.

[23] T. Cyraic, B.N. Hanumagowda, M. Umeshaiah, V. Kumar, J. Chohan, R. Kumar, et al., "Performance of rough secant slider bearing lubricated with couple stress fluid in the presence of magnetic field," *Modern Physics Letters B*, vol. 38, no. 19, pp. 2450140-2450143, 2023.

[24] V. Jyothi, B. N. Hanumagowda, A. Verma, "Analysis of pressure load-carrying capacity and squeezing time on magnetohydrodynamic couple stress fluid flow between sphere and flat plate with slip velocity," *International journal of Modern physics B*, vol. 38, no. 26, pp. 2450349-2450350, 2023.

[25] R. Vinutha, B. N. Hanumagowda, K. R. Vasanth, "Effect of magnetic field and slip velocity on the performance of bearings lubricated by couple stress fluid on long cylinder and infinte plate- A theoretical analysis," *Modern Physics Letters B*, vol. 38, no. 30, pp. 2450176-2450259, 2024.

[26] U. Devani, J. Patil, S. Bilal, B. N. Hanumagowda, V. B. Trimbak, J. Tawade, et al., "Study of MHD on porous flat and curved circular plate lubricated with couple stress fluid- a slip velocity model," *Results in Engineering*, vol. 24, pp. 102914-1-12, 2024.

[27] J. R. Lin, L. M. Chu, W. L. Liaw, "Effects of non-Newtonian couple stresses on the dynamic characteristics of wide Rayleigh step slider bearings," *Journal of Marine Science and Technology*, vol. 20, no. 5, pp. 547-553, 2012.

[28] N. B. Naduvinamani, A. Angadi, "Surface roughness influence on the dynamic performance of Rayleigh step bearing lubricated with couplestress fluid," in *Proceeding of the Natinoal Academy of Sciences, India Section. A - Physical Sciences*, vol. 94, pp. 235-247, 2024.

[29] N. B. Naduvinamani, R. Ganachari, "Double-layered porous Rayleigh step slider bearings lubricated with couplestress fluids," *Indian Journal of Science and Technology*, vol. 15, no. 28, pp. 1389-1398, 2022.

[30] N. B. Naduvinamani, S. T. Fathima, P. S. Hiremath, "Hydrodynamic lubrication of rough slider bearings with couple stress fluids," *Tribology International*, vol. 36, no. 12, pp. 949-959, 2003.

[31] N. B. Naduvinamani, A. Siddangouda, "Effect of surface roughness on the hydrodynamic lubrication of porous step-slider bearings with couple stress fluids," *Tribology International*, vol. 40, no. 5, pp. 780-793, 2007.

[32] M. Barik, S. R. Mishra, G. C. Dash, "Effect of sinusoidal magnetic field on a rough porous hyperbolic slider bearing with ferrofluid lubrication and slip velocity," *Tribology – Materials, Surfaces and Interfaces*, vol. 10, no. 3, pp. 131-137, 2016.

[33] J. R. Patel, G. M. Deheri, "A study of thin film lubrication at nanoscale for a ferrofluid based infinitely long rough porous slider bearing," *Facta Universitatis Series: Mechanical Engineering*, vol. 14, no. 1, pp. 89-99, 2016.

[34] D. A. Patel, M. Joshi, D. B. Patel, "Design of porous step bearing by considering different ferro fluid lubrication flow models," *Journal of Manufacturing Engineering*, vol. 16, no. 4, pp. 108-114, 2021.

[35] T. Ariman, M. A. Turk, N. D. Sylvester, "Microcontinuum fluid mechanics-A review," *International Journal of Engineering Science*, vol. 11, no. 8, pp. 905-930, 1973.

[36] T. Ariman, M. A. Turk, N. D. Sylvester, "Applications of microcontinuum fluid mechanics," *International Journal of Engineering Science*, vol. 12, no. 6, pp. 273-293, 1974.

[37] V. K. Stokes, "Couple stresses in fluids," *Physics of Fluids*, vol. 9, pp. 1709-1715, 1966.

[38] V. K. Stokes, "Effects of couple stresses in fluids on hydromagnetic channel flows," *Physics of Fluids*, vol. 11, pp. 1131-1133, 1968.

[39] V. K. Stokes, "Effects of couple stresses in fluids on the creeping flow past a sphere," *Physics of Fluids*, vol. 14, pp. 1580-1582, 1971.

[40] A. Cameron. Basic Lubrication Theory. 1st Ed. London: Longman, 1971.

[41] J. L. Neuringer, R. E. Rosensweig, "Ferrohydrodynamics," *Physics of Fluids*, vol. 7, no.12, pp. 1927-1937, 1964.

[42] B. L. Prajapati, "On certain theoretical studies in hydrodynamic and electro-magneto hydrodynamic lubrication," *Ph.D. Thesis*, Sardar Patel University, India, 1995.

[43] M. V. Bhat. Lubrication with Magnetic fluid. 1st Ed. India: Team Spirit (India) Pvt. Ltd., 2003.

NOMENCLATURE

v_i	Velocity vector	$\bar{\mu}$	Magnetic susceptibility of the ferrofluid
b_i	Body force per unit mass	μ_0	Free Space Permeability
b	Width of the bearing	h	Film Thickness
g_i	Body couple per unit mass	h_0	Dimensionless Pressure
T_{ij}	Stress Tensor	W	Dimensionless load-carrying capacity
M_{ij}	Couple stress Tensor	$\bar{X}$	Dimensionless centre of pressure
p	Pressure	F	Dimensionless frictional force
λ, μ	Classical viscosity coefficient	A	Step height ratio
η, η'	New material constants specific to the fluid	B	Step riser location
q	Volume flow rate	C	Frictional coefficient
l	Couple stress parameter	ρ	Density
f	Friction force	$\delta_{i,j}$	Kronekar Delta
$\bar{q}$	Fluid velocity	e_{ijk}	Permutation tensor
$\bar{H}$	External magnetic field		

APPENDICES

Eq. (8) can be solved using:

$$\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} = \frac{dp}{dx} \tag{A.1}$$

where,

$$u = A_0 + A_1 y + \frac{1}{2\mu}\frac{dp}{dx} y^2 + B_1 \cos h\left(\frac{y}{l}\right) + B_2 \sin h\left(\frac{y}{l}\right) \tag{A.2}$$

Using boundary conditions,

$$u(y=0) = u_0; u(y=h) = 0 \tag{A.3}$$

$$\frac{\partial^2 u}{\partial y^2}(y=0) = 0; \frac{\partial^2 u}{\partial y^2}(y=h) = 0 \tag{A.4}$$

From Eq. (A.2), one can find A_0, A_1, B_1 and B_2 as follows:

$$\begin{aligned} A_0 &= u_0 + \frac{l^2}{\mu}\frac{dp}{dx} \\ A_1 &= \frac{1}{h}\left[-u_0 - \frac{h^2}{2\mu}\frac{dp}{dx}\right] \\ B_1 &= -\frac{l^2}{\mu}\frac{dp}{dx} \\ B_2 &= \frac{l^2}{\mu}\frac{dp}{dx}\left[\frac{-1 + \cos h\left(\frac{h}{l}\right)}{\sin h\left(\frac{h}{l}\right)}\right] \end{aligned} \tag{A.5}$$

Substituting all the above values in Eq. (A.2), Eq. (A.6) is obtained:

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left(y^2 - hy + 2l^2\left[1 - \frac{\cosh\left\{\frac{2y-h}{2l}\right\}}{\cosh\left(\frac{h}{2l}\right)}\right]\right) + u_0\left(1 - \frac{y}{h}\right) \tag{A.6}$$