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RESEARCH ARTICLE

Structural optimization of 2-dimensional steel truss beams with different truss members using finite element analysis

N. L. Rahim[1,2]*, J. Ang[1], M. Z. A. M. Zahid[3], T. Imjai[4], S. Salehuddin[1], N. M. Ibrahim[1,2], A. R. A Karim[5], M. A. Rahim[1], L. A. Sofri[1]

[1] Faculty of Civil Engineering & Technology, Kompleks Pusat Pengajian Jejawi 3, Universiti Malaysia Perlis, 02600 Arau, Perlis, Malaysia
Phone: +049798626, Fax.: +049798636
[2] Sustainable Environment Research Group, Centre of Excellence Geopolymer and Green Technology, Universiti Malaysia Perlis, 01000 Kangar, Perlis, Malaysia
[3] School of Civil Engineering, Engineering Campus, Universiti Sains Malaysia, 14300 Nibong Tebal, Pulau Pinang, Malaysia
[4] School of Engineering and Technology, Walailak University, Nakhon Si Thammarat, 80161, Thailand
[5] Faculty of Engineering, University of Malaysia, Sarawak, 94300 Kota Samarahan, Sarawak, Malaysia

ABSTRACT - Trusses have long been integral to structural design due to their efficiency in bearing loads. However, lacking clear guidelines for optimizing truss beams often forces engineers to compromise performance and cost. This study addresses this gap by optimizing the shape and size of two-dimensional steel truss beams using Finite Element Analysis with ABAQUS software under specified conditions. The analysis considered uniform vertical loads of 200 kN, 500 kN, and 200 kN applied at strategic joints, with pinned supports as boundary conditions. Four truss configurations, V-structure, V-structure with vertical members, N-structure, and K-structure, were examined for stress, displacement, and critical buckling. Least deflection (-12.28 mm) and maximum stress (178.85 MPa) were found for the K-structure, suggesting a suitable design. Additionally, the High Edge A (HEA) 240 steel cross-section was determined to be the ideal alternative from a range perspective, yielding a construction with enhanced stability and financial aspects. This work shows how to have a much better-performing truss with the largest saving potential and the least material. The result is expected to have significant implications for the design of safer and more economical steel trusses, which are important components of many modern buildings.

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1. INTRODUCTION

Structural optimization is a key consideration in structural engineering, as it helps ensure that all buildings and infrastructure are specifically designed to be safe, efficient, and durable. Structural design looks at the shape of walls, the kind of roof, the type of floors, beams, and foundations, and material quality. It also influences the overall safety of a structure and how long it will last. In past years, steel structures have become more popular in construction because of their strength, durability, and versatility. One commonly used structure is the steel truss beam used in bridges, roofs, and many load-bearing applications. Nevertheless, there are few detailed guidelines for the design of steel trusses to ensure material efficiency, cost-effectiveness, and structural safety. These gaps illustrate the need for research to overcome these limitations, investigating truss configurations under different conditions and constraints [1, 2]. Finite Element Analysis (FEA) is a powerful tool that allows structural engineers to optimize their designs by simulating different structural behaviors under various scenarios. The current research uses FEA to examine displacement, stress, and buckling resistance of steel truss beams subjected to various diagonal member arrangements.

One of the greatest challenges in contemporary engineering is optimising structural designs for both performance and cost. This has led to inefficiencies in the design of truss beams regarding material usage and structural performance, especially since there are no detailed guidelines. Now, the latest study is on load behavior and deflection for steel beams. However, it has not sufficiently addressed the impact of varying diagonal member shapes on stress distribution and buckling resistance in truss structures. Additionally, the collapse of roof structures, particularly those utilizing steel trusses, has raised concerns about design flaws related to buckling length, stability, and load combinations [3]. Given the increasing use of steel truss beams in critical structures, optimised designs are urgently needed to ensure safety, material efficiency, and cost-effectiveness. This research aims to fill this gap by systematically analysing different truss beam configurations to determine the most effective design. This research contributes to the field of structural engineering by providing insights into the optimal design of steel truss beams, ensuring safer and more cost-effective construction practices. By identifying the most efficient truss configurations, the study promotes the sensible use of materials, reducing costs while maintaining structural integrity. Applying finite element analysis in this context offers a precise and reliable method for optimizing truss designs, ultimately aiding engineers in developing better-performing structures.

*CORRESPONDING AUTHOR | N. L. RAHIM | ✉ nurliza@unimap.edu.my

Steel structure is one of the most common construction materials widely used in structural design. Steel structures have superior durability, formability, high yield, tensile strength, and thermal conductivity [4]. As a result, steel is by far the most useful material for building structures, with strength approximately ten times that of concrete due to its high strength and uniformity [5]. Structural steel beams are available in a variety of forms, including I-beam, T-beam, L-beam (angle), Z-shape, structural channel (C-beam cross-section), High Edge A (HEA) shape, Hollow Steel Section shape, bar, rod, open web steel joist, rail profile, and plate [6]. I-beams, also called H-beams, have a cross-section with an "I" or "H" shape. As illustrated in Figure 1, the horizontal component is a "flange", while the vertical compost acts as a "web." Also, I-Beams are most frequently constructed of structural steel, although they may also be manufactured from aluminium to meet specific requirements. I-beams are the most common beam used in construction and can be utilised in beams and columns. I-Beams provide more design flexibility than other structural elements in steel structures. Consequently, they are often used as structural elements in long-span steel structures with no columns, such as aircraft hangars, arenas, and steel-concrete composite bridge girders [7].

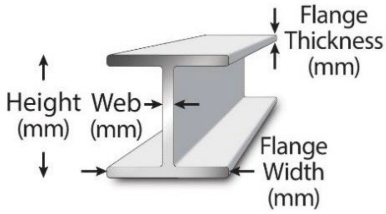


Figure 1. I-beam

Trusses are structural members used to build large-scale structural systems, such as buildings, towers, and bridges. A truss is a structure comprising three or more members that are generally recognised to be pinned or hinged at the ends of the structure. Combining one-dimensional linear elements to generate a triangular pattern results in a basic two-dimensional (2D) truss structure. The process starts with a triangular element and then expands with a pair of members to create another triangular element. The process continues until the whole structure is complete and ready to use [8]. Many diverse constructions, such as bridges, towers, cranes, poles, and stocks, use steel trusses. Unlike welded girder, less material is required when using trusses as load-bearing components. The truss is also a kind of intelligent load-bearing component because it can transmit tension and compression force simultaneously, and less material results in lower costs and lower self-weight [9]. Besides, all the truss components should be used effectively, while the loads need to be transmitted efficiently and safely. When designing a structure, structural materials should be selected based on the life cycle cost, which considers design, construction, operation, and maintenance expenses throughout its useful life [10]. The design of trusses involves two crucial criteria. First, the shape optimization identifies the most suitable geometrical layout for bars and nodes. Subsequently, in the size optimization process, the best cross-sections that can withstand external loads effectively need to be determined [11]. This research focuses on a two-dimensional truss beam structure containing four diagonal structures. The optimal truss beam structure is identified by evaluating stress and displacement among the four trusses under specific conditions. Once the best shape is determined, the study proceeds with that shape, and the different sizes of cross-section members are considered design variables. Then, the displacement and truss properties are re-evaluated in the next optimisation step. Such a process is repeated until the optimal shape for the truss and the right size of the I-beam are ascertained. This study focuses on improving steel truss beams' structural performance in roof applications. These consist of using ABAQUS to analyse the stress, displacement, and buckling behaviour of different truss beam configurations with different diagonal members, comparing the result with manual calculations, and providing recommendations on the best shape and size to create a truss beam structure.

2. MATERIALS AND METHODS

2.1 Materials Properties

S275 steel is a common type of structural steel for construction; it is balanced and compliant with the European Types (EN 1993-1-1:2005) [12]. In this study, the S275 steel grade was chosen as it possesses good mechanical properties (yield strength of 275 N/mm²), and therefore it can be used to study the structural behavior of steel truss beams. That makes it ideal for truss beams and other structural elements, especially in load-bearing applications where it balances strength and ductility well. Material coefficients for structural steels, as specified in EN 1993-1-1:2005, have been considered in this study. The coefficients used are shown in Table 1 [12].

Table 1. Material coefficients [12]

Steel Grade	Modulus of Elasticity, E	Yield strength, f_y	Shear Modulus, G	Poisson's ratio, v	Specific weight, γ
S275	210000 N/mm²	275 N/mm^2	81000 N/mm²	0.3	78.5 kN/m^3

2.2 Types of Steel Beam

Table 2 presents the proposed dimensions of the steel truss beam for this research. The flanges are designed to withstand moments for I-girders, while the diagonals, functioning as the web, are primarily designed to resist shear forces.

Consequently, the cross-sectional area of the diagonals is often smaller than that of the flanges. The HEA profile is commonly used for larger truss structures and supporting columns in buildings [13]. Therefore, the HEA profile was selected as the cross-section for the I-beam in this study. The connection between flanges and the diagonals is a critical element in the design of a truss beam. Therefore, it was assumed that the connection was welded. Figure 2 presents truss beam connections using HEA profiles that use diagonal welds to the flanges. The joints like these are designed to be quick and simple to manufacture.

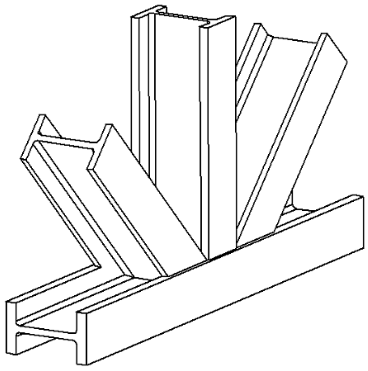


Figure 2. Connection in a truss beam constructed with HEA profiles

Table 2. Dimension of steel beam

Type of steel beam	Identification	Width (mm)	Height (mm)	Thickness (mm)		Area of cross-section (cm^2)
				Flange thickness	Web thickness	
I-Beam	HEA 220	220	210	11	7	64.34

2.3 Geometry of the Truss Structure Design

Table 3 displays the proposed geometries for the truss beam structure design. As per Davison and Owens [14], the most economical span range falls between 6 m to 12 m. Additionally, a study by Mustafa [15] focusing on optimising plane trusses across various cross-sections has suggested a truss with a length of 12 m and a height of 1.5 m. Considering the similarities between the current research and the study on structural optimization of trusses, it has been decided to adopt a total span length of 12 m with a height of 1.5 m for the proposed truss. The study focused on analysing 12-meter span truss beams using ABAQUS software. Static analysis was conducted on each model, and manual calculations were performed to validate the model's accuracy by comparing them with the static analysis results. In line with Eurocode guidelines (roof loading), the ultimate limit capacity of each truss configuration was determined and analyzed using ABAQUS. The impact of imperfections, such as stress and displacement, in the most critical compressed members was examined. The overall behavior of the truss beam was thoroughly investigated by studying and comparing the ABAQUS analysis results.

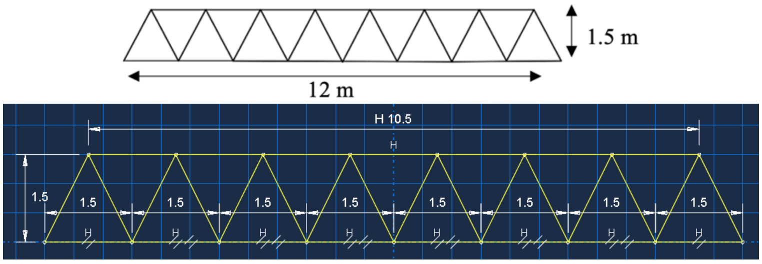
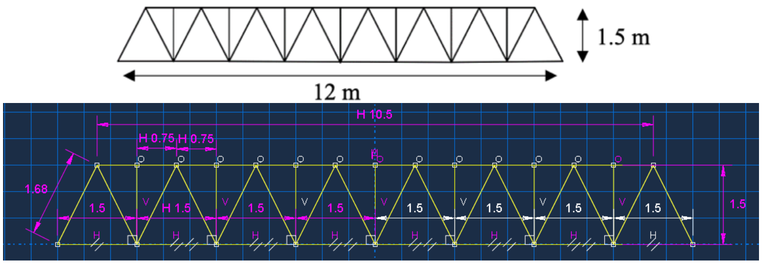
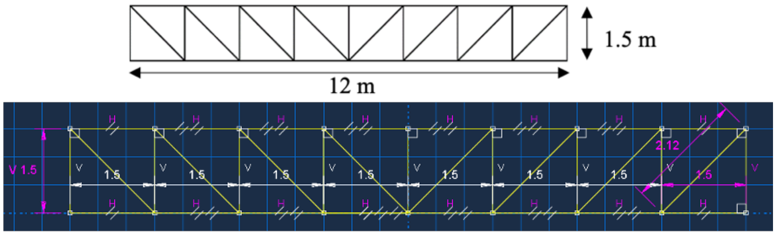
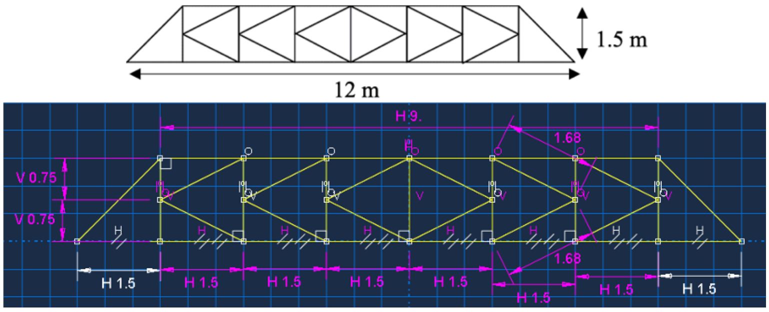
2.4 Boundary Condition

Boundary conditions are used in structural studies for those parts of the model where displacements or rotations are known. The truss in this research was pinned and supported at points A and I for the boundary conditions, as shown in Figure 3. Pinned support refers to fastening in the x and y directions, which makes the structure rigid. However, the support cannot prevent rotational movements of the member since it can only exert a force on the member operating in either direction and prevent translational movements or relative displacement of the member ends in all directions.

2.5 Loading Application

A linear isotropic truss material model was developed for the steel truss material in this study, with the elastic modulus and Poisson's ratio used to define the isotropic material properties. The loading conditions for this study were determined as vertical loads of 200 kN, 500 kN, and 200 kN acting downward at points C, E, and G, as shown in Figure 3. These loads were chosen considering the similarity between this research and the study by Reddy and Nagaraju [16], which also focused on applying vertical loads at similar joints for all trusses. In this study, the finite element technique employed is a linear static analysis using ABAQUS software, ideal for evaluating the structural performance of steel trusses under static loading conditions. The analysis assumes linear elastic behavior, where stress and strain remain proportional within the elastic limit of S275 steel, focusing on stress distribution, displacement, and buckling resistance under service loads. However, unlike Reddy and Nagaraju [16], who emphasized optimizing trusses for minimum weight under specific constraints, the present study expands the scope by incorporating additional performance metrics, including stress distribution, displacement, and buckling resistance.

Table 3. Shape of truss beam configurations

Shape	Truss Beam Configurations
V-structure	
V-structure with vertical members	
N-structure	
K-structure	

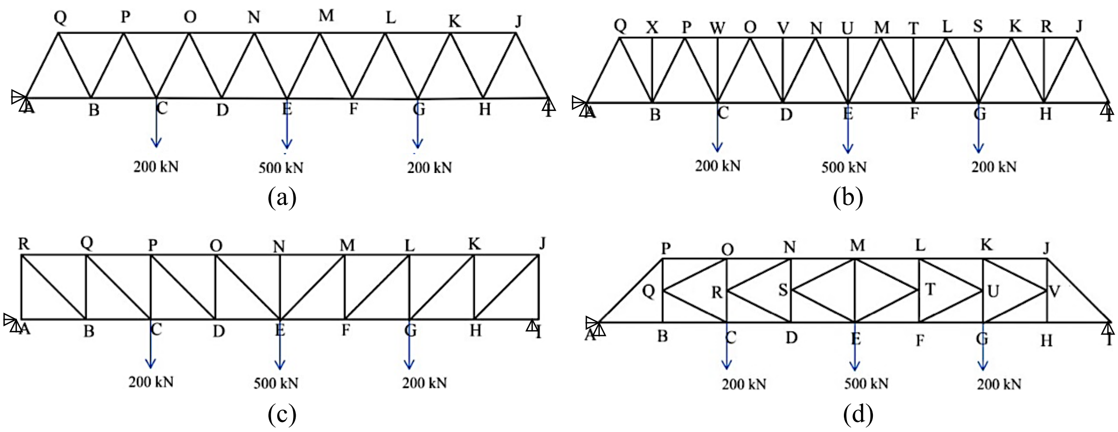


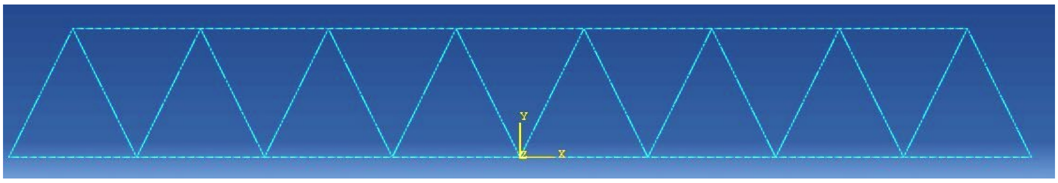
Figure 3. Geometries of the truss design and loading application for (a) V-structure, (b) V-structure with vertical bars, (c) N-structure and (d) K-structure

2.6 Sizing Optimization

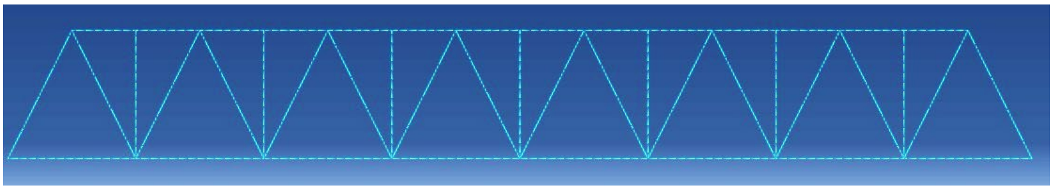
Following the determination of the best truss shape among the four trusses for this specific condition through the shape optimization process, the chosen shape is utilized in the sizing optimization process. In this process, the truss members' cross-sectional dimensions are considered the design variables to optimize structural performance while minimizing material usage. Various steel sections, specifically HEA profiles, were selected for this purpose due to their widespread application in structural engineering for load-bearing components. HEA profiles are commonly chosen for their high load-carrying capacity, stability, and versatility, which makes them suitable for truss structures. Supporting this selection, Jurčíková and Rosmanit [17] proposed using the HEA 160 profile for the bottom chord in joints modeled with RHS and HEA profiles, demonstrating its effectiveness in joint stability. Similarly, numerical studies [18, 19] on steel support structures using finite element methods suggested employing HEA 200, HEA 220, and HEA 240 profiles due to their high structural efficiency. Additionally, Vila Real et al. [20] recommended the HEA 500 section for scenarios requiring lateral-torsional buckling resistance under fire conditions, highlighting its suitability for heavy-duty applications. Based on these findings, the HEA profiles selected for this study include HEA 160, HEA 200, HEA 220, HEA 240, and HEA 500, covering a range of dimensions to address varying structural requirements (as detailed in Table 4). This selection allows the study to evaluate the performance of truss members across different sizes and loading conditions, ensuring that the chosen cross-section optimally balances performance and material efficiency. Each model underwent static analysis to evaluate the performance of the truss beam structures. Various parameters, such as stress, displacement, tensile strength, compressive strength, and compression buckling, were used to assess their behavior. Manual calculations were also performed to validate the model's accuracy, and the results were compared to those obtained from the static analysis. Furthermore, the impact of imperfections, specifically stress and displacement, in the most critical compressed members was studied.

Table 4. HEA profiles

Optimization	HEA profile	Width (mm)	Height (mm)	Thickness(mm)		Area of cross-section (cm^2)	Justification/ References
				Flange thickness	Web thickness		
1	HEA 160	152	160	9.0	6.0	38.77	Recommended for light truss elements [17]
2	HEA 200	190	200	10.0	6.5	53.83	Suitable for medium loads [18]
3	HEA 220	210	220	11.0	7.0	64.34	Frequently used for support structures [19]
4	HEA 240	230	240	12.0	7.5	76.84	Ideal for heavy-duty truss systems [19]
5	HEA 500	490	300	13.0	12.0	197.54	High resistance for critical loads [20]



(a)



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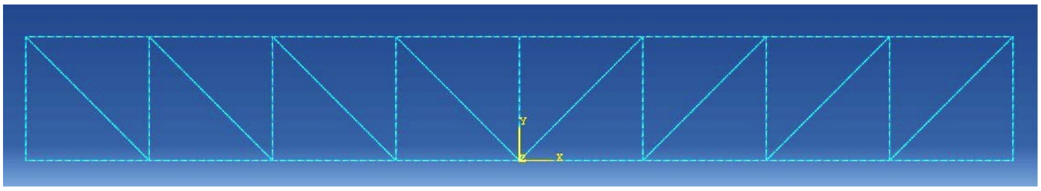

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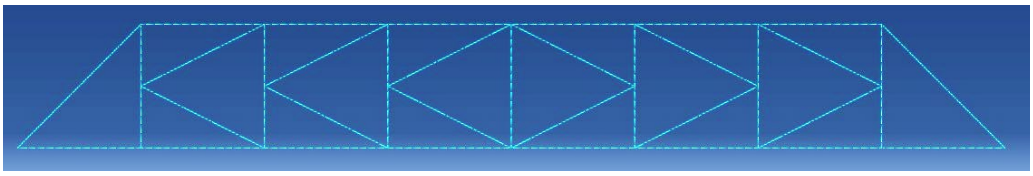


(d)

Figure 4. Meshed model: (a) V-structure, (b) V-structure with vertical members, (c) N-structure and (d) K-structure

2.7 Mesh

The mesh for the model was created by applying a seed to the part instance with an approximate global element size of 0.01, ensuring uniformity and consistency across all truss configurations (V-structure, V-structure with vertical members, N-structure, and K-structure). This fine mesh size was selected to achieve a high level of accuracy in the finite element analysis, as finer meshes generally lead to more precise results. A mesh sensitivity analysis was performed to ensure the selected mesh size was adequate. This study tested different mesh sizes from 0.005 to 0.02. A global element size of 0.01 provided consistent stress and displacement results with less than 5% deviation from finer meshes with efficient computational resources. This indicates that the chosen mesh size is adequate to obtain detailed distributions of stress and displacement fields, especially in critical areas like joints and regions subjected to high-load gradients. Reseeding the part instance after the first mesh generation was performed to check for consistency throughout the entire model mesh. All truss configurations were subjected to the same mesh size to enable comparable results. The meshed models of the truss configurations are shown in Figure 4.

3. RESULTS AND DISCUSSION

This section presents the finite element analysis results using ABAQUS on the four truss beams. The modeled trusses: V-structure, V-structure with vertical members, N-structure, and K-structure, were all loaded at the exact joint locations with vertical loads of 200 kN, 500 kN, and 200 kN at each end. The performance parameters analyzed included stress, displacement, tension and compression force, and compression buckling within the truss beams. Both results from the simulations and manual calculations are thoroughly discussed to validate the two-dimensionality and reliability of the models employed.

3.1 Stress Analysis

The mechanical analysis was carried out using the ABAQUS/standard solver, and the resulting stress distribution is presented in Figure 5. As indicated in Figure 5, the stress concentration region for all four trusses was observed in the middle span on the top chord. The maximum stress values for the V-structure, V-structure combined with vertical members, N-structure, and K-structure are approximately 196.7 MPa, 205.2 MPa, 275.8 MPa, and 207.4 MPa, respectively. Consequently, the N-structure exhibits the highest stress, while the V-structure has the lowest stress value. The V-structure demonstrates the lowest stress because its diagonal members are directly aligned to effectively transmit loads from the top chord to the supports, minimizing stress concentrations on individual members. This design ensures that the load distribution is more balanced, reducing the overall stress in the structure. Trusses with direct load paths, such as the V-structure, typically exhibit reduced stress levels due to the efficient transfer of axial forces. Furthermore, the V-structure's symmetry and minimal complexity contribute to lower secondary stress effects, often in trusses with additional members or intricate designs like the N-structure.

The stress distribution at 18.75 kN loading results of finite element analysis for all four truss structures using ABAQUS shows that the stress concentration area occurs mainly in the middle span on the top chord for the four truss structures. This is due to how the load is distributed through the truss, where the top chord is usually placed under compressive stress and the bottom chord under tensile stress. The central span, the longest part not supported by vertical members, experiences the largest compressive force, resulting in stress concentrations. The maximum stress differs between all four structures and can be related to the geometry and how the load is distributed. In a V-structure, the sloping members disperse the load on multiple points, preventing any single point from bearing too much stress. This aligns with the concept that stress is alleviated when loads are applied over a broader area or multiple pathways, found in [21]. Stress concentrations arise when the load is exerted at a single point, or there are sharp geometric discontinuities; this is precisely why the inclined members of the V-structure prove advantageous, as they diffuse the forces and thus help hold down peak stress values. Added vertical members in the V-structure with vertical members increase stiffness and do not significantly increase stress to 205.2 MPa. Stiffness increases (local/global stiffness ratio) can not only restrict the deformation of the structure. Still, they may also increase the localized forces on the load transfer points, especially in the case of stiff vertical members [22]. In the N-structure, the diagonal and horizontal members configuration results in less efficient load transfer, translating to greater stress concentrations at member intersections. In trusses, horizontal members, such as the top chord, experience compressive forces. When diagonal members cross these structure members up formation angles, the angles created in the horizontal members impede the transfer of forces, causing localized stress peaks at the crossing points. This observation is analogous to findings in research on truss mechanics, where sharp angles and sudden changes in member orientation exacerbate stress concentrations [23]. The K-structure uses both vertical and diagonal members, but this system may, at the same time, provide balance and distribute forces efficiently while keeping stiffness at a decent level. The downward point loads are taken directly by vertical members of a truss. In contrast, diagonal members help transfer loads across the truss instead of directly vertically, resulting in less stress concentration than the N-structure. Though the K-structure (207.4 MPa) experiences more stress than the V-structure, it has less involved stress due to more complexity and extra members than the N-structure. Stress distribution and maximal values depend on how load transfer functions for the individual structure. In contrast, force transfer is better managed in efficient structures, such as the V-structure, resulting in the spread of localized stress concentrations. In contrast, poor designs, such as the N-structure, show higher stress concentrations due to abrupt geometric transition.

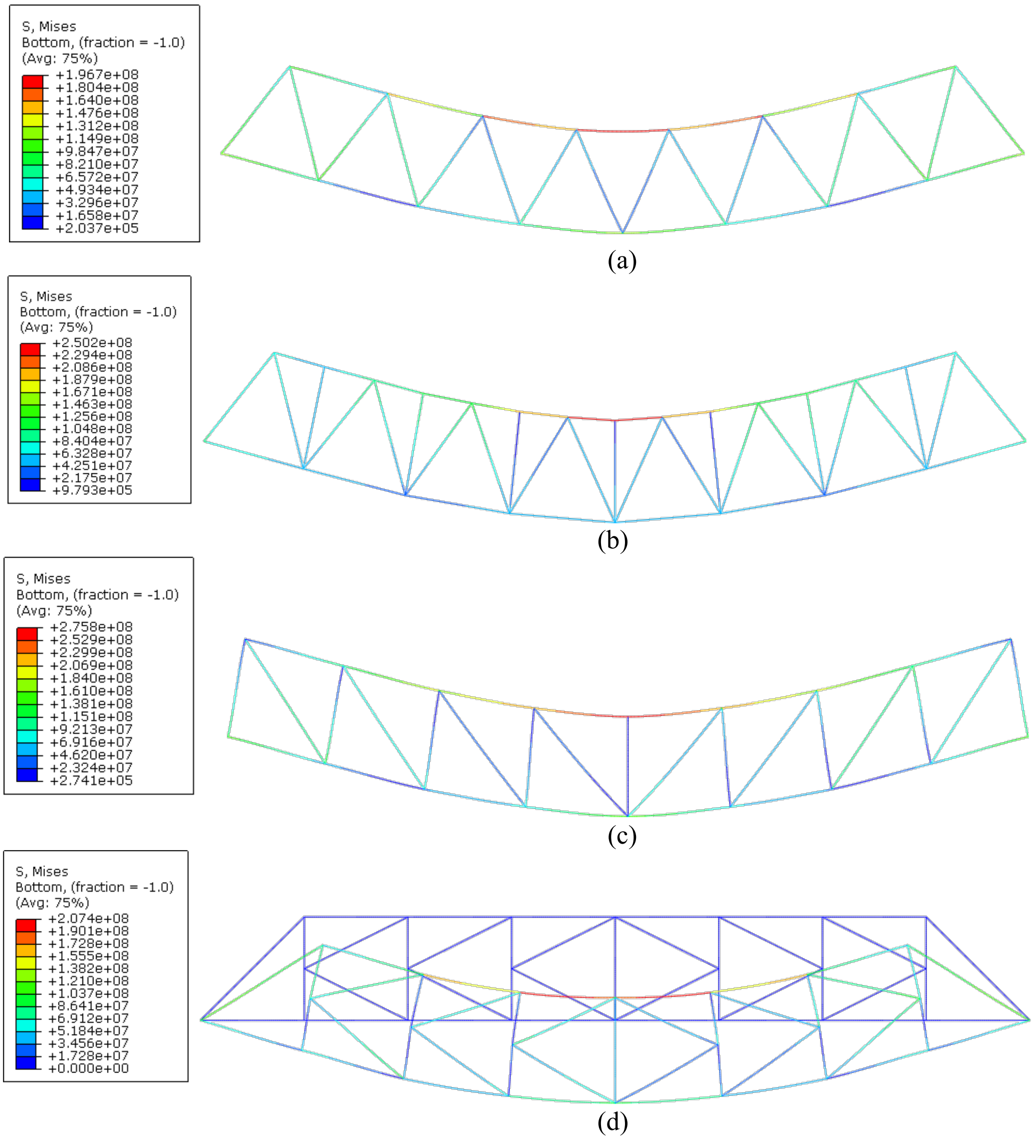


Figure 5. Stress distribution for (a) V-structure, (b) V-structure with vertical bars, (c) N-structure and (d) K-structure

3.2 Displacement Analysis

The deflection of each truss type, including the V-structure, V-structure with vertical members, N-structure, and K-structure, was determined in response to the applied loading. The ABAQUS simulation results show the displacement distribution for all four trusses, as depicted in Figure 6. As shown in Figure 6, displacement increases as it approaches the mid-span of the truss. The maximum displacement values recorded are -15.896 mm for the V-structure, -15.789 mm for the V-structure with vertical members, -20.210 mm for the N-structure, and -12.277 mm for the K-structure. The K-structure exhibits the smallest deflection, followed by the V-structure with vertical members. The results indicate that the displacement increases as the distance from the supports increases toward the truss's mid-span. This behavior is typical for truss structures under load, as the mid-span is often where bending moments are greatest, leading to pronounced deflection. This occurs because the truss members' internal forces (axial and shear) combine at the mid-span, creating a maximum bending moment. Additionally, the lack of intermediate vertical support at the mid-span allows for greater deflection compared to the regions closer to the supports. According to Gere and Timoshenko [21], displacement in structural elements is proportional to the bending moment and inversely proportional to the stiffness. The mid-span experiences the largest bending moments, and because the stiffness of a truss structure depends on its geometry and material properties, this explains the observed displacement pattern.

Table 5 displays the results of the FEA of displacement for the V-structure, V-structure with vertical members, N-structure, and K-structure under the applied loading. Nodes A and I for these four trusses show 0 mm displacement because they act as pinned supports, allowing the truss to rotate while restricting horizontal and vertical translation [24]. The maximum displacement results for the V-structure, V-structure with vertical members, N-structure, and K-structure are 15.90 mm, 15.79 mm, 20.21 mm, and 12.28 mm, respectively. The K-structure exhibits the smallest deflection, followed by the V-structure with vertical members and the V-structure, while the N-structure shows the largest deflection. On the other hand, analytical results are considered valid as they are obtained through proven mathematical manipulation. Hence, it is essential to compare the simulation results with theoretical data.

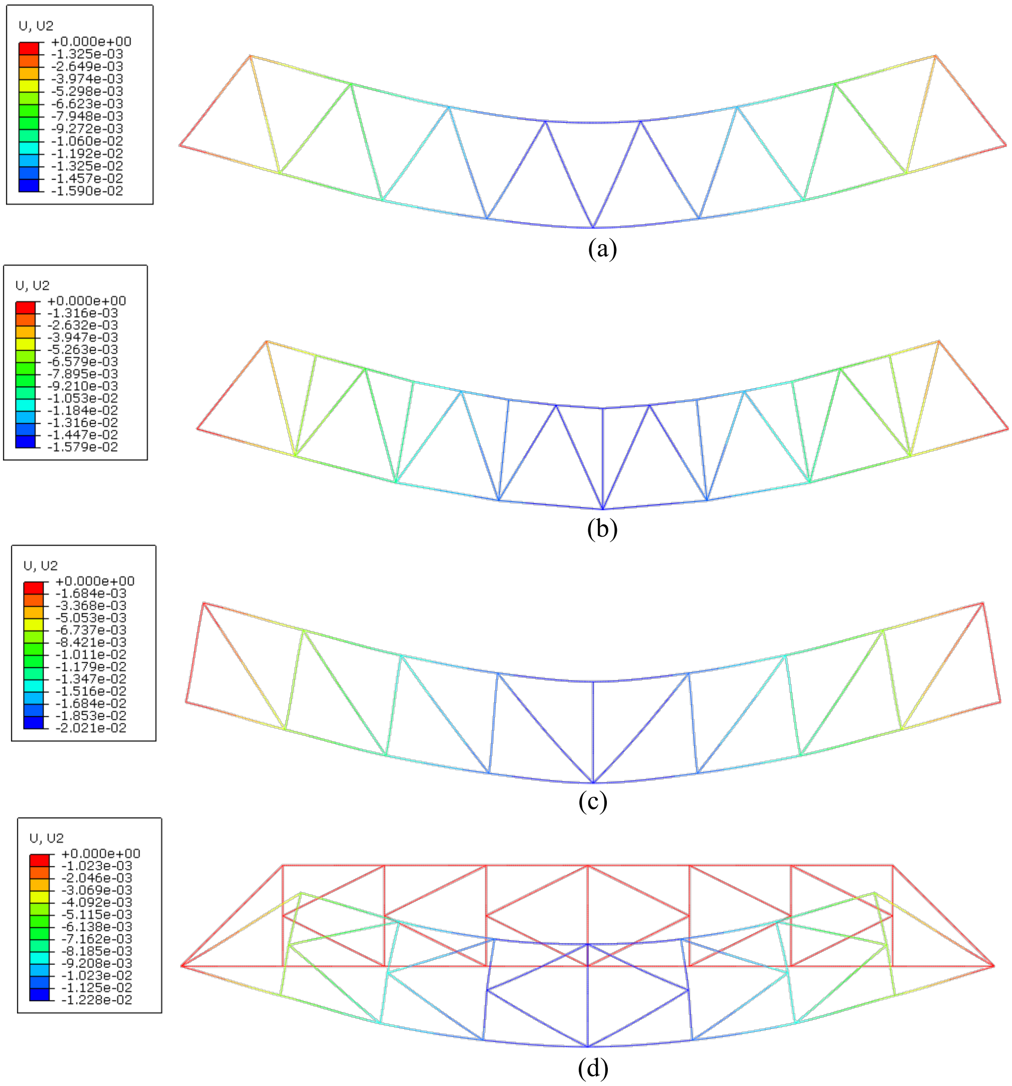


Figure 6. Displacement distribution for (a) V-structure, (b) V-structure with vertical bars, (c) N-structure, and (d) K-structure

Table 5. FEA displacement of the V-structure, V-structure with vertical members, N-structure and K-structure

V-Structure displacement (mm)				V-structure displacement with vertical members			
Node	FEA	Theoretical	Error (%)	Node	FEA	Theoretical	Error (%)
A	0.00	0.00	0.00	A	0.00	0.00	0.00
B	5.36	5.27	1.66	B	5.33	5.27	1.12
C	10.56	10.43	1.25	C	10.50	10.43	0.65
D	14.09	13.92	1.24	D	14.02	13.92	0.70
E	15.90	15.73	1.05	E	15.79	15.73	0.38
F	14.09	13.92	1.24	F	14.02	13.92	0.70
G	10.56	10.43	1.25	G	10.50	10.43	0.65
H	5.36	5.27	1.66	H	5.33	5.27	1.12
I	0.00	0.00	0.00	I	0.00	0.00	0.00
N-structure Displacement				K-Structure displacement			
A	0.00	0.00	0.00	A	0.00	0.00	0.00
B	6.82	6.94	1.80	B	4.45	4.44	0.11
C	13.32	13.58	1.93	C	8.68	8.81	1.53
D	17.76	18.05	1.62	D	11.24	11.34	0.90
E	20.21	20.67	2.23	E	12.28	12.32	0.35
F	17.76	18.05	1.62	F	11.24	11.34	0.90
G	13.32	13.58	1.93	G	8.68	8.81	1.53
H	6.82	6.94	1.76	H	4.45	4.44	0.11
I	0.00	0.00	0.00	I	0.00	0.00	0.00

Tables 5 presents the percentage error when comparing the simulation results with the theoretical results for all trusses. Manual calculations validated the finite element analysis results and assessed the model's accuracy. All simulations were performed using linear elastic analysis in ABAQUS, which assumes that the material behavior is in the elastic range and follows Hooke's Law. The lower the percentage error, the more accurate the FE model. The percentage of errors in results produced by ABAQUS software compared to theoretical results, according to Nurhaniza [25], is generally around 5% and not too high. This research shows that the maximum percentage of errors ranges from 0% to 2.23%, which can be considered a reasonable percentage range. This shows that all theoretical predictions highly agree with the simulations, indicating the FEA model is validated for the V-structure (with or without the vertical members present), the N-structure, and the K-structure. Generally speaking, it can be concluded that all FEA models are valid since the level of agreement between theoretical and simulation results is acceptable.

3.3 Axial Stress Analysis

The results of the calculations were determined manually by the internal force of truss members, respectively. This enabled the identification of whether axial tension or compression was applied to each member, and the axial stress for each was extracted from the results. According to Figure 7 and Table 6, member MN, the most stressed V-structure, suffers an axial compression stress equal to 217.729 MPa. In the V-structure with vertical members, members NU and UM have the most stress, with axial compression stress of 212.425 MPa. The same is true for the least stressful members of the N-structure, MN and NO, which also underwent tensile stress of approximately 218.223 MPa. However, the most stressed members of the K-structure are MN and LM, encountering an axial compression stress of approximately 178.849 MPa. The axial compression stress on member MN is about 217.729 MPa, the most stressed element in the V-structure. This high stress suggests that member MN is significant in how the load is carried in the V-structure configuration. This indicates a substantial contribution to the truss's overall stability and load distribution, thus causing heightened stress in this member. Based on the V-Structure with vertical bars, members NU and UM have the highest axial compression stress of 212.425 MPa each. Adding vertical members redistributes the loads more evenly than in the original V-structure, although NU and UM still carry significant compressive forces. Overall truss performance and stability are affected by this redistribution, indicating that while vertical members assist in load sharing, significant stress concentrations in certain members cannot be completely relieved. In the case of the N-structure, it is notable that the axial compression stress states for members MN and NO are similar to the ones of the V-structure with an axial compression stress of 218.223 MPa. Due to the higher stress values as opposed to the V-structure, it can also be said that for the N-structure configuration, the loads are greater for certain members, impacting their performance under load. In the K structure, members MN and LM can withstand axial stresses of 178.849 MPa.

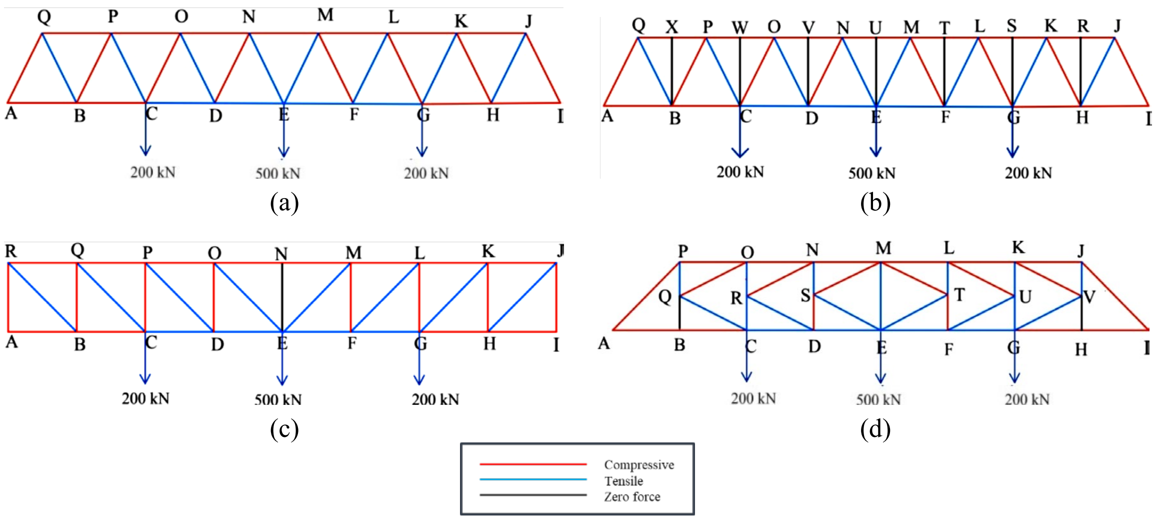


Figure 7. Most stressful member for: (a) V-structure, (b) V-structure with vertical bars, (c) N-structure and (d) K-structure

This lower stress level than other trusses suggests that the K-structure is more effective in distributing loads and reducing stress concentrations in its members. In truss systems, the term members refers to the individual components, such as chords and diagonals, which collectively transfer loads through axial forces. The geometry and arrangement of the members significantly affect how the truss's forces are distributed and where stress concentrations occur. Megson [24] states that how the truss members are aligned and connected affects how effectively loads are transferred; widely distributed loads reduce localized moderations and increase structural stability. The greater load-bearing efficiency that K-structure configuration design innovations have is due to the lower consumption of structural resources, resulting in overall structure efficiency. Structural efficiency aims to make the best product with the least amount of metal and the least amount of stress in the joints. K-structure solves this problem by adding diagonal and vertical members and distributing force over many paths to reduce the stress intensity on a member. This finding aligns with the work by Gere and Timoshenko [21], who explain that such optimized truss configurations can balance load-sharing and minimize critical stress, ultimately improving the structure's performance.

Table 6. Data of axial force

Shape	Member	Length (m)	Axial stress (MPa)	Condition
V-structure	MN	1.50	-217.729	Compressive
V-structure with vertical members	NU	0.75	-212.425	Compressive
	UM	0.75	-212.425	Compressive
N-structure	MN	1.50	-218.223	Compressive
	NO	1.50	-218.223	Compressive
K-structure	LM	1.50	-178.849	Compressive
	MN	1.50	-178.849	Compressive

3.4 Compression Buckling Check

Since steel members are subjected to axial loads, additional resistance checks are required. Table 7 compares critical compressive forces and compression buckling resistances for the V-structure, V-structure with vertical members, N-structure, and K-structure. The buckling resistance values, $N_{b,Rd}$ are calculated using Equation 1:

$$N_{Ed} \leq N_{b,Rd} = \frac{\chi_{min} \cdot A \cdot f_y}{\gamma_{M1}} \quad (1)$$

where N_{Ed} is the critical axial compression force, $N_{b,Rd}$ is the buckling resistance of the compression member, A is the cross-section area of the member, χ is the reduction factor, f_y is the yield strength, and γ_{M1} is equal to 1. Upon analysing Table 7, it is evident that the critical compressive forces, N_{Ed}, for all trusses are smaller than the buckling resistance values, $N_{b,Rd}$. As a result, no buckling occurs in any of the trusses. Therefore, the selected steel section, HEA 220, is considered adequate to withstand the applied loads and avoid buckling in all trusses.

Table 7. Comparison of critical buckling load and compression buckling resistance

Truss Structure	Critical compressive force, N_{Ed} (kN)	Buckling resistance, $N_{b,Rd}$ (kN)	$N_{Ed} < N_{b,Rd}$
V-structure	1400	1697.52	Yes
V-structure with vertical members	1320	1684.23	Yes
epN-structure	1470	1781.34	Yes
K-structure	1150	1677.61	Yes

3.5 Selection of Best Truss Shape

Overall, after considering various parameters discussed previously, the K-structure emerges as the best truss shape among all the models. While the V-structure has the lowest stress value of about 196.7 MPa, it also exhibits more deflection during displacement analysis than the K-structure, with a maximum displacement of -15.896 mm versus -12.277 mm for the K-structure. Additionally, the critical members of the K-structure experience the least stress among all truss members, with an axial compression stress of about 178.849 MPa for these critical members. Considering all these factors, the K-structure is identified as the optimal truss shape for the given conditions.

3.6 Size Optimization

The K-structure was chosen as the best truss shape for the sizing optimization process. In this process, the cross-section of the truss members, specifically the I-beam, was considered the design variable. The displacement and stress of the truss are reevaluated in the subsequent optimization process. This iterative process continues until the best shape of the truss and the size of the I-beam are determined, achieving an optimal design for the truss structure.

3.6.1 Stress analysis

Figure 8 displays the stress distribution for the K-structure trusses constructed using HEA 160, HEA 200, HEA 220, HEA 240, and HEA 500 steel sections. The stress concentration is observed in the middle on the top chord for all five trusses. The maximum stresses for the K-structures with HEA 160, HEA 200, HEA 220, HEA 240, and HEA 500 steel sections are 353.4 MPa, 256.8 MPa, 207.4 MPa, 179.4 MPa, and 49.9 MPa, respectively. Therefore, the K-structure with HEA 160 steel section exhibits the highest stress value, while the K-structure with HEA 500 steel section has the lowest stress value among the five trusses. The lower stress value in the K-structure with the HEA 500 steel section can be attributed to its larger cross-sectional area and greater inertia, significantly increasing the section's stiffness. According to Gere and Timoshenko [21], the stress in a structural member is inversely proportional to its cross-sectional area and stiffness, as larger sections reduce internal forces and deformations under applied loads. The HEA 500 steel section, with a cross-sectional area of 197.54 cm^2, distributes the applied load over a larger area, reducing the stress concentration in the critical top chord. Additionally, the increased stiffness minimizes deflections and secondary stresses, further contributing to the lower maximum stress values. This finding aligns with the principles of structural analysis, which state that sections with higher moments of inertia resist bending and axial forces more effectively, leading to improved structural performance and reduced stress concentrations [24].

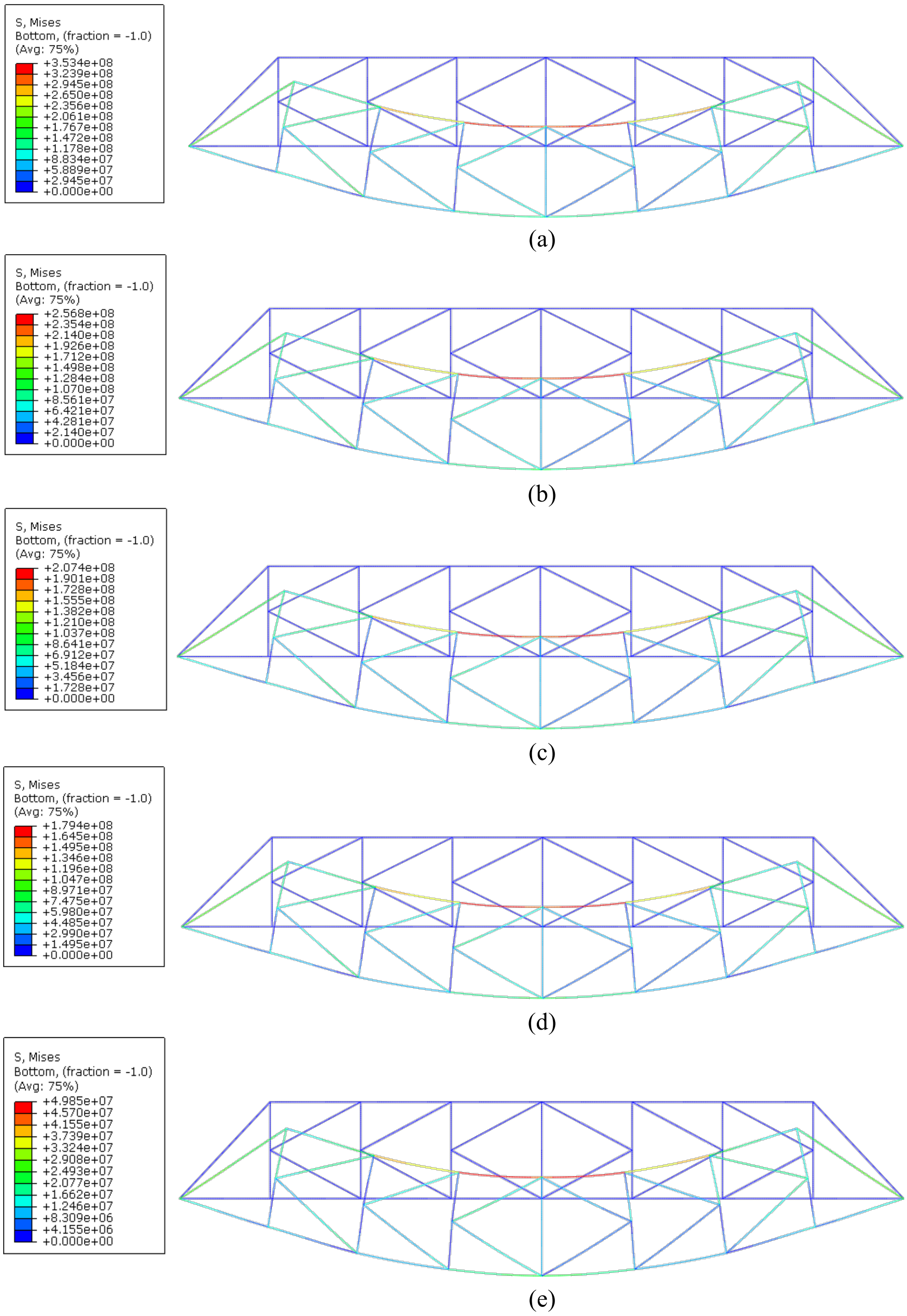
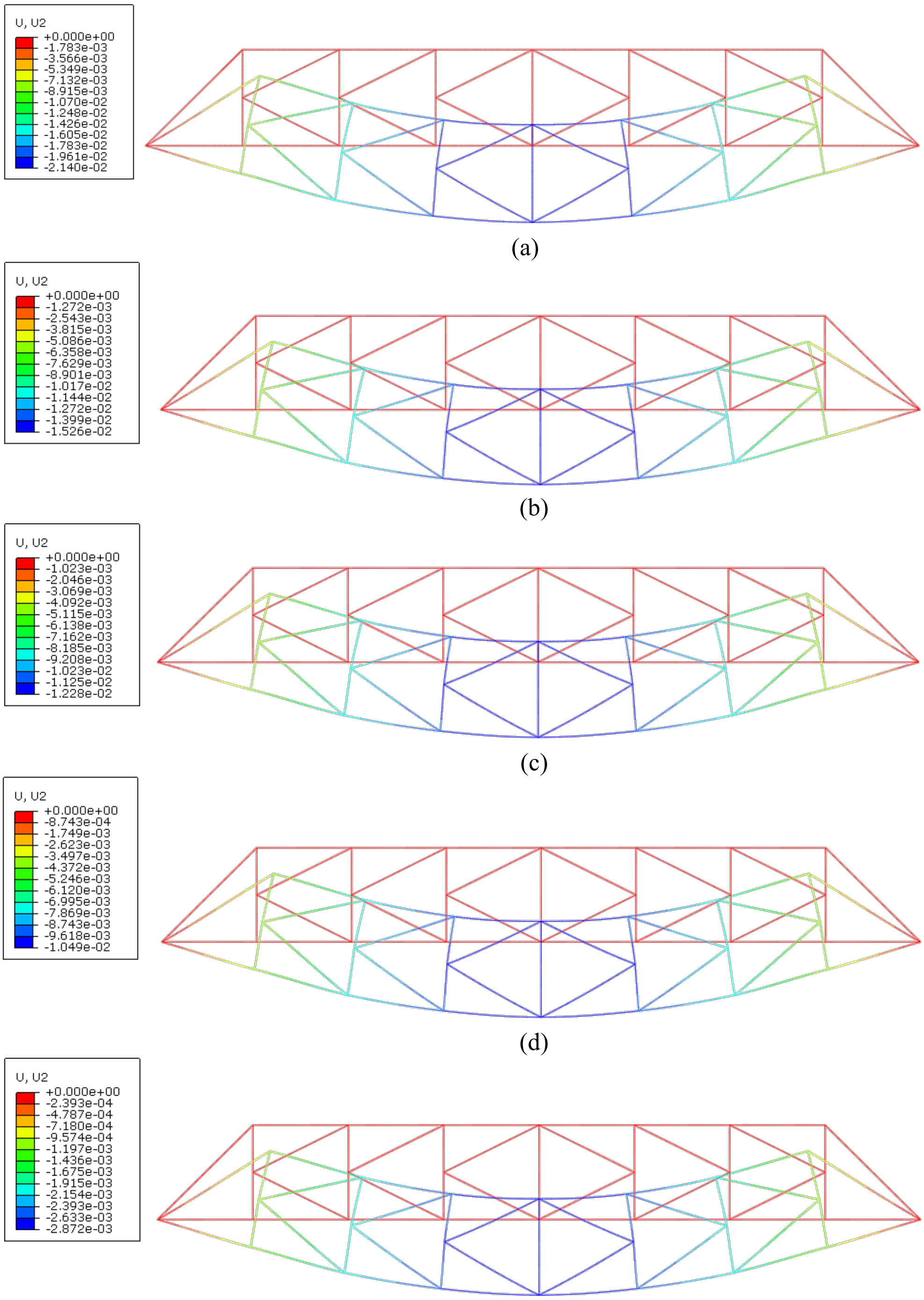


Figure 8. Stress distribution for K-structure with different HEA profiles: (a) 160 section, (b) 200 section, (c) 220 section, (d) 240 section and (e) 500 section

The K-structure with HEA 160 shows the highest stress concentration, with a maximum stress of approximately 353.4 MPa. This elevated stress level indicates that the HEA 160 section, while providing significant load-bearing capacity, also experiences substantial stress under applied loads. This higher stress can be attributed to this section's smaller cross-sectional area and increased load concentration. According to Gere and Timoshenko [21], stress in a structural member is inversely proportional to its cross-sectional area. A smaller cross-sectional area increases stress because the same force is distributed over a smaller area, increasing stress concentration. Additionally, a smaller area reduces the stiffness and moment of inertia, making the member less resistant to axial and bending forces, further exacerbating stress levels. Smaller sections tend to localize stress, especially in regions of high compressive forces, such as the top chord of a truss under applied loads. The maximum stress observed for the K-structure with HEA 200 is around 256.8 MPa. Although this stress is lower than that of HEA 160, it still represents a significant concentration in the top chord. The larger cross-sectional area compared to HEA 160 reduces the stress but does not eliminate it, reflecting an improved but still notable stress concentration. For the HEA 220 section, the maximum stress was reduced further to approximately 207.4 MPa. This reduction demonstrates the benefit of increasing the steel section size, which helps distribute the loads more effectively and reduces stress concentrations. The K-structure with HEA 240 exhibits a maximum stress of about 179.4 MPa. This section shows a further reduction in stress concentration compared to the smaller sections, highlighting the improved load distribution and structural efficiency achieved with larger steel sections. The K-structure with HEA 500 experiences the lowest maximum stress of approximately 49.9 MPa. This significant stress reduction indicates that the HEA 500 section effectively distributes loads and minimizes stress concentrations in the top chord, providing superior structural performance compared to the smaller sections. The trend observed from higher stress in HEA 160 to significantly lower stress in HEA 500 demonstrates that larger steel sections contribute to better load distribution and reduced stress concentrations.



(e)

Figure 9. Displacement distribution for K-structure with different HEA profiles: (a) 160 section, (b) 200 section, (c) 220 section, (d) 240 section and (e) 500 section

3.6.2 Displacement analysis

Figure 9 illustrates the displacement distribution obtained from the FEA results for the K-structure with HEA 160, HEA 200, HEA 220, HEA 240, and HEA 500 steel sections. The displacement values were determined for each truss in response to the applied loading. Table 8 presents the maximum displacement results for the K-structure with HEA 160, HEA 200, HEA 220, HEA 240, and HEA 500 steel sections, which are 21.39 mm, 15.25 mm, 12.27 mm, 10.49 mm, and 2.87 mm, respectively. The displacement decreases as the cross-section of the member increases. Based on these results, it can be concluded that the K-structure with the HEA 500 steel section experiences the least deflection compared to the other trusses with different HEA steel sections.

Table 8. Displacement results of nodes A, B, C, D, E, F, G, H, and I for size optimization

Optimization	HEA steel section	Area of cross-section (mm^2)	Displacement (mm)								
			Node								
			A	B	C	D	E	F	G	H	I
1	HEA160	3877	0.0	7.72	15.17	19.61	21.39	19.61	15.17	7.72	0.0
2	HEA200	5383	0.0	5.52	10.78	13.97	15.25	13.97	10.78	5.52	0.0
3	HEA220	6434	0.0	4.44	8.67	11.23	12.27	11.23	8.67	4.44	0.0
4	HEA240	7684	0.0	3.80	7.40	9.59	10.49	9.59	7.40	3.80	0.0
5	HEA500	19754	0.0	1.04	2.02	2.62	2.87	2.62	2.02	1.04	0.0

3.6.3 Compression buckling check

Table 9 provides the critical compressive force and compression buckling resistance for K structures constructed using HEA 160, HEA 200, HEA 220, HEA 240, and HEA 500 steel sections. Based on the table, it can be observed that the critical compressive forces, N_{Ed}, are smaller than the buckling resistance values, $N_{b,Rd}$, for K structures made of HEA 200, HEA 220, HEA 240, and HEA 500 steel sections. Hence, no buckling occurs in those four trusses, indicating that the steel sections HEA 200, HEA 220, HEA 240, and HEA 500 are adequate. However, for the K-structure with HEA 160 steel section, the critical compressive force, N_{Ed}, is greater than the buckling resistance value, $N_{b,Rd}$. Consequently, buckling occurs in the K-structure constructed with the HEA 160 steel section. Furthermore, the results demonstrate that increasing the cross-sectional area of a member enhances its compressive strength.

Table 9. Comparison of critical buckling load and compression buckling resistance in size optimization

Optimization	HEA steel section	Area of cross-section (mm^2)	Critical compressive force, N_{Ed} (kN)	Buckling resistance, $N_{b,Rd}$ (kN)	$N_{Ed} < N_{b,Rd}$
1	HEA160	3877		949	No
2	HEA 200	5383		1406	Yes
3	HEA220	6434	1150	1697	Yes
4	HEA240	7684		2071	Yes
5	HEA 500	19754		5378	Yes

3.6.4 Margin of safety for K-structure with different HEA profile

The safety factor used in this study is set at 2, based on the work of Reddy and Nagaraju [16], which also examines the structural optimization of various truss members. With a yield strength of 275 MPa and a safety factor 2, the allowable stress is calculated as 137.5 MPa. The results are shown in Table 10. These results indicate that, in the K-structure with HEA 240 and HEA 500 profiles, the margin of safety for all members is greater than 1, suggesting that these structures are stable and durable. In contrast, K-structures with HEA 200 and HEA 220 profiles have four members with a margin of safety of less than 1, and those with HEA 160 profiles have six such members. These findings suggest that K-structures with HEA 160, HEA 200, and HEA 220 profiles are likely to experience failure.

3.6.5 Selection of best cross-section

Table 11 shows that the K-structures made of HEA 500 steel section exhibit the lowest values of maximum displacement and maximum axial stress compared to other steel sections. Additionally, it has the highest value of minimum margin of safety, with a value of 2.36. However, despite these favorable properties, the HEA 500 steel section is not considered the best cross-section due to its significantly larger truss weight than other cross-sections. The weight of the truss plays a critical role in overall costs, and the designer's objective is to minimize the material quantity while maintaining the truss's stiffness as much as possible [14]. As a result, in this study, the HEA 240 steel section is identified as the best cross-section. It meets the requirement of having a minimum margin of safety not less than or equal to 1, as recommended by [15]. Furthermore, the K-structure with HEA 240 steel section achieved the second-lowest values of maximum displacement and maximum axial stress, indicating high stability. Moreover, the truss weight of the K-structure with the HEA 240 steel section is approximately 2.5 times smaller than the HEA 500 steel section, leading to significant cost savings.

Table 10. Margin of safety

	K structure									
	HEA160		HEA200		HEA220		HEA 240		HEA 500	
Member	Axial Stress (MPa)	Margin of Safety (M.S)	Axial Stress (MPa)	Margin of Safety (M.S)	Axial Stress (MPa)	Margin of Safety (M.S)	Axial Stress (MPa)	Margin of Safety (M.S)	Axial Stress (MPa)	Margin of Safety (M.S)
AB	-74.16	1.85	-53.41	2.57	-44.68	3.08	-37.42	3.67	-14.55	9.45
BC	-74.16	1.85	-53.41	2.57	-44.68	3.08	-37.42	3.67	-14.55	9.45
CD	41.91	3.28	30.19	4.55	25.26	5.44	21.15	6.50	8.23	16.71
DE	106.40	1.29	76.63	1.79	64.11	2.14	53.68	2.56	20.88	6.58
EF	106.40	1.29	76.63	1.79	64.11	2.14	53.68	2.56	20.88	6.58
FG	41.91	3.28	30.19	4.55	25.26	5.44	21.15	6.50	8.23	16.71
GH	-74.16	1.85	-53.41	2.57	-44.68	3.08	-37.42	3.67	-14.55	9.45
HI	-74.16	1.85	-53.41	2.57	-44.68	3.08	-37.42	3.67	-14.55	9.45
IJ	-164.15	0.84	-118.22	1.16	-98.91	1.39	-82.82	1.66	-32.22	4.27
JK	-116.07	1.18	-83.60	1.64	-69.94	1.97	-52.06	2.64	-22.78	6.04
LK	-232.14	0.59	-167.19	0.82	-139.88	0.98	-104.11	1.32	-45.56	3.02
LM	-296.62	0.46	-213.64	0.64	-178.74	0.77	-123.63	1.11	-58.22	2.36
MN	-296.62	0.46	-213.64	0.64	-178.74	0.77	-123.63	1.11	-58.22	2.36
NO	-232.14	0.59	-167.19	0.82	-139.88	0.98	-104.11	1.32	-45.56	3.02
OP	-116.07	1.18	-83.60	1.64	-69.94	1.97	-52.06	2.64	-22.78	6.04
PA	-164.15	0.84	-118.22	1.16	-98.91	1.39	-82.82	1.66	-32.22	4.27
PQ	116.07	1.18	83.60	1.64	69.94	1.97	58.56	2.35	22.78	6.04
QB	0.00	-	0.00	-	0.00	-	0.00	-	0.00	-
OQ	-129.77	1.06	-93.46	1.47	-78.19	1.76	-65.47	2.10	-25.47	5.40
QC	129.77	1.06	93.46	1.47	78.19	1.76	65.47	2.10	25.47	5.40

Table 11. Comparison of critical buckling load and compression buckling resistance in size optimization

HEA steel section	Area of cross-section (mm^2)	Truss Weight (kg)	Maximum Axial Stress (N/mm^2)	Maximum Displacement (mm)	Minimum Margin safety (M.S)
HEA 160	3877	1860.30	296.62	21.39	0.46
HEA 200	5383	2588.51	213.64	15.25	0.64
HEA 220	6434	3090.30	178.74	12.27	0.77
HEA 240	7684	3690.00	149.66	10.49	1.11
HEA 500	19754	9485.07	58.22	2.87	2.36

4. CONCLUSIONS

This study comprehensively analyzed four truss configurations: V-structure, V-structure with vertical members, N-structure, and K-structure through finite element analysis using ABAQUS. The findings identify the K-structure as the most efficient and stable truss design among the configurations analyzed, particularly when paired with the HEA 240 steel section. The following impactful findings support this conclusion:

i) The K-structure demonstrated the smallest maximum displacement (-12.277 mm), outperforming the V-structure (-15.896 mm), V-structure with vertical members (-15.789 mm), and N-structure (-20.210 mm). While the V-structure exhibited the lowest stress value of 196.7 MPa among all configurations, the K-structure proved more efficient in displacement control and stress distribution.

ii) The K-structure with the HEA 240 steel section achieved the second lowest displacement (-10.492 mm) and axial stress while satisfying the buckling resistance check. HEA 500 exhibited the lowest stress concentration (49.9 MPa) and displacement (-2.872 mm) among all steel sections, but its larger cross-section makes it less material-efficient compared to HEA 240. HEA 160 was deemed inadequate due to its critical compressive force exceeding buckling resistance, highlighting the importance of selecting appropriate cross-sections for stability.

iii) The FEA results were successfully validated with manual calculations, yielding percentage errors ranging from 0% to 2.23%, which is within an acceptable range and demonstrates the reliability of the simulation models.

iv) Critical members in each truss configuration were identified. For the K-structure, members MN and LM experienced the lowest axial compression stress (-178.849 MPa) compared to other trusses, emphasizing its efficiency in load distribution.

These findings highlight the potential of the K-structure with HEA 240 steel section for applications requiring optimized load distribution, structural stability, and cost-efficiency. The results contribute to the field by providing a systematic approach for optimizing truss configurations and size selection, offering practical insights for engineers in designing stable and efficient structures for roofs, bridges, and other load-bearing applications. This study used a linear elastic analysis approach, assuming material behavior remains within the elastic range. This analysis did not account for real-world conditions, such as material imperfections, dynamic loads, and non-linear behavior. Additionally, all trusses were modeled as 2D line structures, which limits the scope for 3D stress distribution or torsional analysis.

Future research explores several avenues to enhance the understanding and application of truss structures. As loads in real-world environments can often be dynamic or demand consideration for seismic loading, one potential avenue of further research is investigating the behavior of particular truss configurations under these conditions. A new paradigm might be to optimize hybrid truss systems by analysing combinations of materials or cross-sections to attain superior structural performance and value. Going forward, it could be worth using proven non-linear material models instead of linear elastic types, as this would certainly make realistic tests much closer to capturing actual structural and material behavior in practical applications, at least in extreme load or boundary conditions. In addition, including 3D modeling in the analysis can help solve torsional effects and stress concentrations present in 3-dimensional truss systems, giving a deeper understanding of the behavior of such structures. Such directions cover the limitations of this particular study but also allow progress in terms of truss design and optimization approaches in complex engineering problems.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest to report regarding the present study.

AUTHORS CONTRIBUTION

N. L. Rahim (Conceptualization; Methodology; Validation; Formal analysis; Writing - original draft; Supervision; Project administration, Writing - review & editing)

J. Ang (Conceptualization; Methodology; Data curation; Software; Writing - original draft; Visualisation)

M. Z. A. M. Zahid (Formal analysis; Validation; Resources; Writing - review & editing)

T. Imjai (Methodology; Investigation; Writing - review & editing)

S. Salehuddin (Data curation; Investigation; Writing - review & editing)

N. M. Ibrahim (Resources; Formal analysis; Writing - review & editing)

A. R. A. Karim (Resources; Validation; Writing - review & editing)

M. A. Rahim (Software; Data curation; Visualisation)

L. A. Sofri (Data curation; Resources; Writing - review & editing)

AVAILABILITY OF DATA AND MATERIALS

The data supporting this study's findings are available on request from the corresponding author.

ETHICS STATEMENT

Not applicable

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