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REVIEW ARTICLE

A review of additively manufactured pentamode lattices: Theory, methods and applications

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Abstract - Pentamode metamaterials, characterized by their near-zero shear modulus and high bulk modulus, have attracted solid interest due to their unique ability to mimic fluid-like behaviour. Pentamode metamaterials have driven by improvements in computational design, material selection and advanced manufacturing techniques, especially additive manufacturing. As a comprehensive review article specificially addressing pentamode metamaterials produced by additive manufacturing, this review systematically represents the theoretical foundations, essential design concepts, and main parameters, including the high bulk-to-shear modulus ratio that presents fluidic behaviour. Recent advancements in manufacturing methods such as selective laser melting, direct laser writing, and hybrid fabrication processes are also discussed, along with their impact on achieving precise lattice geometries. Extensive analysis of acoustic and mechanical applications reveals pentamode metamaterials' versatility in acoustic cloaking, wave scattering reduction, vibration isolation, shock absorption and tunable stiffness devices. Despite intensive progress, challenges such as scalability, precision of manufacturing, and material optimization continue to emerge. This systematic review concludes by highlighting future research opportunities, emphasizing hybrid material usage, computational optimization, and real-world applications, enhancing the transformative potential of pentamode metamaterials in acoustic and mechanical applications.

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1. Introduction

Pentamode metamaterials, initially proposed by Milton et al. [1], are characterised by their near-zero shear modulus and extraordinary mechanical tunability. These metamaterials have gained significant attention in recent years. According to Scopus, pentamode studies over the years, with titles, abstracts, and keywords such as "pentamode", are depicted in Figure 1. The number of studies has tended to increase over the last 15 years. Pentamodes are one of the groups of metamaterials [2]. This trend is mainly driven by three factors. First, advances in additive manufacturing technologies, particularly high-resolution Stereolithography (SLA), Selective Laser Sintering (SLS) and Digital Light Processing (DLP) systems, now allow the fabrication of ultra-slender struts and complex diamond-type unit cells that were previously impossible to manufacture, as explained in the fabrication techniques section. Second, the potential industrial demand has increased due to the transition of lattice-based designs from purely academic interest to practical applications such as lightweight structural cores, acoustic cloaks, vibration-isolation components, and energy-absorbing sandwich panels, as explained in the applications section. Third, the field has seen a conceptual shift from theoretical pentamode designs to application-oriented optimisation studies, especially following the work of Milton et al. [1]. These factors collectively explain the rapid rise in publications, as also shown by the increasing number of experimental and computational papers reported in recent years. Pentamodes are categorised under two main subgroups: mechanical and acoustic. The majority of the studies are in the acoustic group; there are also significant studies in the mechanical group. Figure 1 shows an increase in pentamode-related publications over the last 15 years, with a marked acceleration after 2018. This growth coincides with advancements in additive manufacturing techniques, particularly SLA and SLS, which enabled the fabrication of ultra-slender double-cone geometries required for high B/G ratios. The rapid increase after 2020 also reflects the shift toward multifunctional applications such as acoustic band gap engineering, seismic protection, and tunable-stiffness devices. The trend suggests that fabrication feasibility and computational optimisation tools have been the main drivers behind the field expansion rather than purely theoretical developments.

This systematic review aims to provide a comprehensive overview of pentamode metamaterials, including their theoretical foundations, fabrication methods, and applications. A structured and transparent review method was followed in this study. The literature search was conducted in Web of Science, Scopus and Google Scholar using the keywords: "pentamode metamaterial", "pentamode lattice", "mechanical metamaterial", "additively manufactured pentamode", and "diamond-shaped pentamode". Studies published between 2008 and 2025 were considered. Inclusion criteria were: (i) articles specifically addressing pentamode metamaterials or pentamode-like lattice structures, (ii) peer-reviewed journal papers and (iii) studies reporting analytical, numerical, experimental or manufacturing-related findings. Exclusion criteria are non-pentamode metamaterials, lack of full-text access and insufficient methodological details. After removing duplicates, studies that met all eligibility criteria were included in this review. This approach ensures a systematic, reproducible literature selection process. All the studies are grouped under theory and principles, design concepts, key parameters, fabrication techniques, material selection, structure and geometry, and methods and applications. The main

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objective of this review is not only to summarise recent advances but also to synthesise the findings reported in the literature by identifying common trends, comparing methodological approaches, and evaluating the strengths and limitations of existing studies, while outlining meaningful research gaps and future directions.

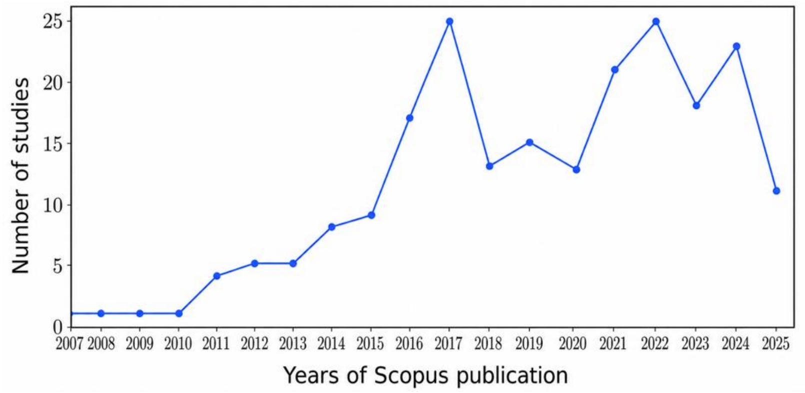

Figure 1. Pentamode studies by year according to Scopus (Source: https://www.scopus.com accessed in October 2025)

1.1 Fundamentals of Pentamode Lattices

The theoretical framework of pentamode metamaterials is introduced in the concept of achieving a high bulk modulus while maintaining an almost negligible shear modulus [1]. Pentamode materials are defined by their elasticity tensor, which is designed to minimise shear resistance while maximising compressive stiffness. A sample pentamode structure and its applications are depicted in Figure 2. The ideal pentamode elasticity tensor approaches the limits of mechanical anisotropy, making these materials unique for applications in acoustics and mechanical wave control [2]–[3]. A critical parameter for pentamode materials is the bulk-to-shear modulus ratio (*B/G*). Theoretically, this ratio can be infinitely large, but practical realisations aim to maximise it while maintaining material stability [4]– [5]. The relationship between lattice geometry and elastic moduli is essential, with designs such as double-cone or needle-like structures enabling the desired mechanical properties [6] – [7]. Further advancements in computational modelling and materials science continue to refine these principles, enabling tailored and functional designs [8] – [9]. In order to provide a more meaningful interpretation of Figure 2, indicative quantitative performance metrics reported in the literature are incorporated into each application category. These include B/G ratios (10–10^6), acoustic band gap ranges (1–20 kHz), stiffness tunability (0.1–1.5 GPa) and shock/energy absorption performance (20–35%).

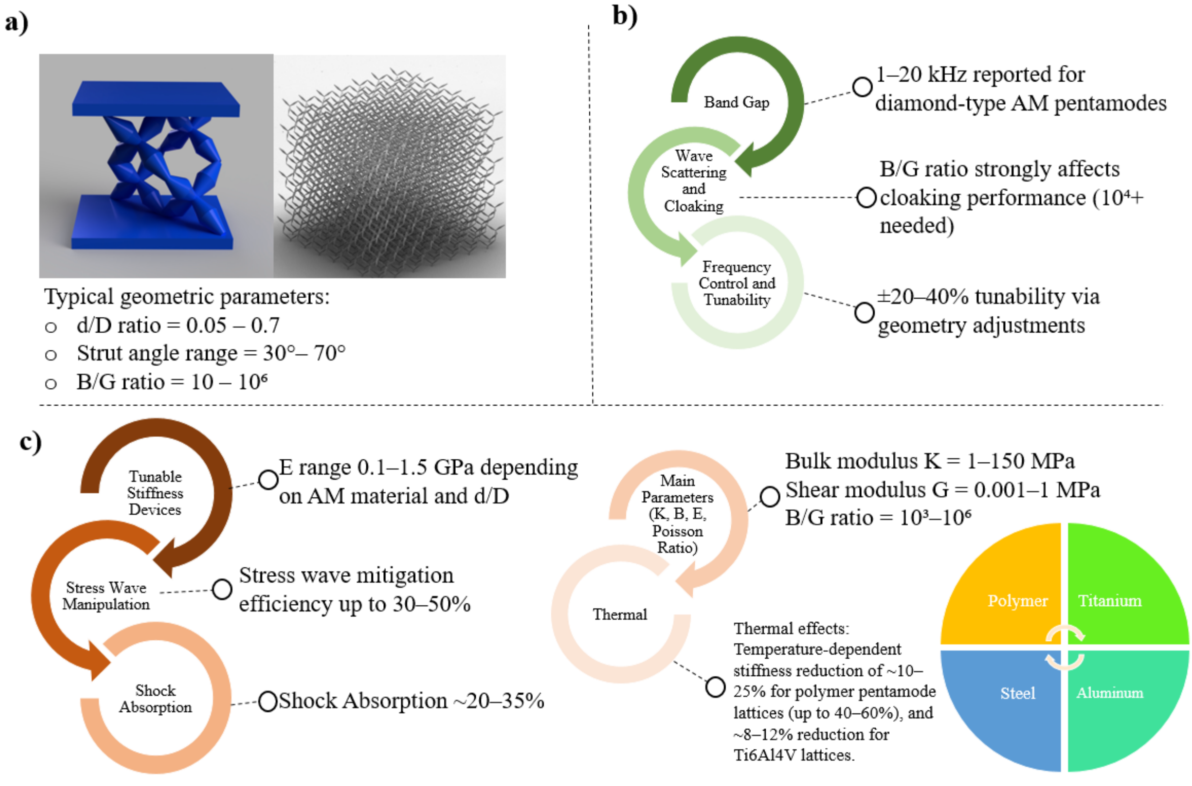

Figure 2. Pentamode structures: (a) Pentamode diamonds modelled on Autodesk Fusion and pentamode structure (Open source: CFN, KIT); (b) Acoustic applications of pentamodes; (c) Mechanical applications and used materials for pentamodes

1.2 Design Concepts

Designing pentamode metamaterials focuses on achieving a near-zero shear modulus while maintaining a high bulk modulus, thereby mimicking fluid-like behaviour. This property is realised through the careful geometric design of lattice structures, often characterised by needle-like or double-cone elements [5], [10]. Studies have shown that optimising the geometry of the unit cell, including the aspect ratio and joint configuration, is critical for achieving the desired mechanical properties [7], [11]. Needle-like lattice structures are particularly effective due to their ability to minimise shear resistance while maintaining structural stability under compression. Topology optimisation methods allow researchers to fine-tune the distribution of material within the lattice to achieve specific performance goals, such as maximising the bulk-to-shear modulus ratio or tailoring acoustic responses [3], [6]. Additionally, additive manufacturing methods enable precise control of the microstructure geometry of mechanical metamaterials [8], [12]. Additionally, additive manufacturing methods enable precise control of the microstructural geometry of mechanical metamaterials; however, printing pentamode-like structures still presents several limitations, including minimum printable strut diameter, over-curing or heat-affected zones that distort slender elements, reduced stability of ultra-thin connections and deviations between the designed and manufactured geometries [12].

1.3 Key Parameters

Performance evaluation of pentamode metamaterials relies on several critical parameters that define their mechanical and acoustic behaviour. These include anisotropy ratios, stiffness, acoustic impedance, and the bulk-to-shear modulus ratio [3], [11]. These metrics are vital for tailoring pentamode structures to specific applications. The bulk-to-shear modulus ratio (*B/G*) is a key indicator of the "fluid-like" nature of pentamode metamaterials. A higher B/G ratio yields enhanced performance in applications such as acoustic cloaking and stress-wave manipulation [5], [10]. Recent advancements have demonstrated the importance of precise geometric designs for achieving high B/G ratios, especially in needle-like lattice structures [12]–[13]. Anisotropy in pentamode metamaterials is another crucial parameter. By carefully designing unit cells, researchers have achieved significant directional control for mechanical properties, specifically for vibration damping and wave steering [3], [7]. Studies by Mejica and Landata [11] highlight the role of lattice geometry in tuning anisotropic behaviour. Acoustic impedance, a measure of resistance to acoustic wave propagation, is a main property for applications in soundproofing and wave manipulation. By optimisation of structural parameters such as cone dimensions and connection overlaps, researchers have expanded the utility of pentamode metamaterials in low-frequency acoustic applications [6], [12].

2. Fabrication Techniques

Additive manufacturing methods, such as two-photon polymerisation and 3D printing, are widely used to achieve the precise lattice structures required for pentamode properties. These methods enable the creation of micro- and nanoscale architectures with high accuracy. Material selection plays a critical role in ensuring the mechanical properties. Fabrication techniques used for pentamode metamaterials include conventional manufacturing, direct laser writing (DLW), and additive manufacturing. Additive manufacturing has the most studies among these manufacturing techniques. Additive manufacturing is a growing trend among manufacturing techniques, especially for mechanical metamaterials. Additive manufacturing has played a central role in enabling the fabrication of pentamode metamaterials (PMMs), but its performance must be evaluated through quantitative metrics rather than descriptive advantages alone. Current processes impose strict minimum feature-size limits—typically 20–40 µm for micro-SLA/DLP systems and 80–150 µm for SLM/SLS processes, which restrict the manufacturability of ultra-slender struts required for high B/G ratios. Geometric deviations between the designed and printed structures are also significant; studies have reported 5–20% strut-thickening, 3–12% node overgrowth, and tapering ratios up to 10%, all of which reduce the achievable B/G ratio by one to two orders of magnitude. Surface roughness further impacts performance, with SLM-fabricated metallic PMM lattices typically exhibiting Ra = 8–20 µm, while SLA/DLP polymer structures show Ra = 1–5 µm, affecting both stiffness and stability. Early DLW-based PMMs demonstrated micro-scale precision but suffered from limited structural integrity and poor scalability, restricting their application. Titanium-based PMMs fabricated via SLM (e.g., Ti-6Al-4V) have shown improved mechanical robustness, yet still exhibit 5–10% dimensional inaccuracy and heat-affected distortions. Recent hybrid approaches, such as SLM combined with electropolishing, have reduced strut diameter deviations to ~100 µm accuracy and decreased surface roughness by 30–50%, enabling more reliable PMM fabrication. These quantitative limitations highlight the importance of evaluating additive manufacturing not only by its design flexibility but also by its geometric accuracy, surface fidelity, and sensitivity to process parameters. [14] - [28]. Additive manufacturing also enables material customisation, allowing the use of lightweight polymers, metals, or composite materials with tailored stiffness. Early attempts to fabricate PMMs were limited by traditional methods such as electron discharge machining and DLW, which imposed constraints on scalability, precision, and materials [8]. DLW, for example, enabled the production of small-scale polymeric PMMs but lacked the versatility and structural integrity required for broader applications [29]. Early studies utilised SLM to produce titanium-based PMMs, such as Ti-6Al-4V lattices, thereby demonstrating the potential of AM to achieve complex geometries with improved mechanical performance [8], [29]. Advances in AM technologies led to more refined manufacturing approaches. Hybrid strategies, such as combining SLM with electropolishing, emerged to address issues related to surface roughness and dimensional accuracy. Jiang et al. [30] demonstrated how electropolishing could significantly enhance the precision of metallic PMMs, achieving strut diameters as small as 100 µm. In parallel, the use of topology optimisation became integral to AM-based design, enabling the customisation of lattice structures to maximise bulk-to-shear modulus ratios [31].

This optimisation, coupled with AM, facilitated the production of PMMs with tailored mechanical properties. For instance, variable cross-section beams were introduced to improve structural stability without compromising performance [32]. Hybrid anisotropic PMMs fabricated using advanced SLM techniques have shown promise in acoustic manipulation, underwater applications, and high-temperature applications [33] –[34]. Moreover, the integration of PMMs into functional systems, such as sodium/Inconel 718 heat pipes, highlights their potential in aerospace and thermal management applications [34]. Precision and scalability in AM techniques saw increasing advancements in 2022. Jiang et al. [30] introduced a hybrid method combining SLM and electropolishing, producing PMMs with strut diameters as small as 100 µm. Zhang et al. [35] explored sandwich structures for ultra-high compression-shear ratios, positioning PMMs as potential solutions for energy absorption and impact mitigation. Hedayati et al. [36] further advanced the understanding of PMMs by developing a comprehensive analytical framework to predict their mechanical properties under various loading conditions, enabling precise customisation to application-specific needs. In parallel, Mohammadi et al. [37] investigated the use of advanced AM techniques to fabricate PMMs. Their work demonstrated the ability to tune PMMs for sound manipulation and vibration control. The trends in AM for PMMs highlight a shift from concept studies to applications. Additive manufacturing contributions per studies on pentamodes are shown in Table 1. Table 1 is arranged according to materials used and specimen dimensions. Unit cells are mostly used for the studies rather than hybrid cells. Compression tests are carried out with a displacement/loading rate range from 2.5 mm/min to 10 mm/min. Displacement/loading rate is also used for controlled forces up to 100 kN.

Table 1. Additive manufacturing studies on pentamodes

Material	Specimen dimensions (mm^3)	Test type	Displacement/ loading rate	Observations	References
PLA	Unit cell 50 × 50 × 50	Compression	2.5 mm/min 20 kg and 100 kg	B/G ratio depend on d rather than D	[37]
PLA	Unit cell 50 × 50 × 50	Compression	5 mm/min 100 kN	Closed form analytical formula for E and v	[36]
PLA	Unit Cell 80 × 80 × 80	Compression	2 mm/min 100 kN	Semi analytical relationship for variable cross sections	[32]
PLA	Hybrid cell 40 × 40 × 40	Compression	5 mm/min	Hybrid design achieved anisotropic mechanical properties	[33]
Clear V4 Resin	Unit cell 30 × 30 × 30	Compression	10 mm/min	Tested till fracture, correlation btw. FEM and test till fracture	[38]
PA2200	Hybrid cell (not specified)	Compression and shear	5 N/min (Cyclically loaded)	Novel sandwich PM with ultrahigh compression-shear ratio proposed	[39]
Nylon	Hybrid cell (not specified)	Compression	Not specified	Truncated octahedron topology proposed as anisotropic tunable material	[40]
Ti-6Al-4V	Unit cell (dimension not specified)	Compression	50 N/min (Cyclically loaded)	New additive subtractive hybrid method proposed for quality. 100 microns processable size achieved	[30]
Titanium	Unit cell 98.9× 98.7 × 87.2	Acoustic test	-	Anisotropic PM yielded in better acoustic cloaking	[41]
Ti-6Al-4V	Unit cell 20×20×20 Unit cell 20×20×40	Compression	0.8 mm/s 2.4 mm/s	Confined by stiffening plates yielded tuneable stiffness	[42]
Ti-6Al-4V	Hybrid cell 305.66×264.71×50	Compression	0.4 mm/min	As strut widths increase, deformation transitions from plastic to brittle, with a critical width of 0.12 mm identified	[29]
Steel	Hybrid cell (not specified)	Hammer percussion	Time dependent impact force	Test model proposed for pentamode lattice ring structure	[35]
Aluminium	Hybrid cell 40×40×40	-	-	Topology optimization achieved near 0.5 Poisson's ratio	[31]
Sodium/ Inconel 718	Unit cell 0.7×0.7×0.7	Thermal test	-	PM reinforced Sodium/Inconel yielded better thermal performance	[34]

Conventional manufacturing methods for pentamode metamaterials, such as casting or moulding, are limited by their inability to produce the intricate structures. These methods often struggle with creating the high stiffness-to-compliance ratio. In contrast, direct laser writing, a form of two-photon polymerisation, offers unparalleled precision. However, conventional methods and DLW usage are limited. Researchers explored metal-based pentamodes using techniques like wire electrical discharge machining and precision milling, achieving enhanced mechanical stability in titanium and steel structures [7], [43]. The introduction of hybrid manufacturing strategies allowed for improved control over structural fidelity and performance [44]. Recent studies have focused on integrating new materials, including functionally graded metals and composites, to expand the capabilities of conventionally fabricated pentamodes [45].

3. Material Selection

Material selection plays a pivotal role in the fabrication and performance of pentamode metamaterials. The choice of material influences mechanical properties such as stiffness, strength, and acoustic impedance, as well as practical considerations such as manufacturability and cost [10], [15]. Recent studies have proposed a variety of materials tailored for specific applications, ranging from lightweight polymers to high-strength metals. Main contributions for each material are shown in Table 2. To enable a meaningful comparison between different material classes, Table 2 has been expanded to include key performance metrics that directly affect pentamode behaviour, such as achievable B/G ratio, shear modulus (G), elastic modulus (E) and material-specific manufacturing limitations. These parameters explain why certain materials (e.g., polymers) achieve high B/G ratios, whereas others (e.g., metals) limit pentamode performance due to their high shear modulus.

Table 2. Materials used on pentamodes

Material	Contribution	Manufacturing	Mechanical properties (*B/G*, *G* and *E*)	References
Polymer	The groundwork for the use of polymers	DLW	*B/G* = 1000-10000 *G* = 0.5 – 50 MPa *E* = 1-3 GPa Limitation: Thermal softening, creep, low long-term stability	[10]
	Polymer mechanical cloaks	DLW		[4]
	Semi-analytical relationships for mechanical prop.	AM		[32]
	Hybrid structures with low shear modulus	AM		[33]
	Precise tuning of bulk to shear modulus	AM		[40]
	Precise tuning of bulk to shear modulus	AM		[36]
	Multiphase hybrid honeycomb configuration	AM		[46]
	Ultra-high compression/shear ratio	AM		[39]
	Polymer with additive manufacturing	AM		[38]
	Cube, truncated cube, diamond, and truncated octahedron for B/G	AM		[37]
Steel and Metal	PM into thermal stress application	AM	*B/G*= 1-10 *G* = 80 - 100 GPa *E* = 200 GPa Limitation: High rigidity	[34]
	PM into hammer percussion tests	AM		[35]
	Hybrid metallic lattices polymeric interconnections mimics water up to 25 kHz	Conventional		[46]
Titanium	PM confined by stiffening plates for energy dissipation	AM	*B/G* = 100 – 1000 *G* = 40-45 GPa *E* = 110 GPa Limitation: Harder to achieve ultra-high B/G due to high G	[42]
	SLM to achieve high Bulk/Shear Ratio	AM		[29]
	Structural parameter effects on mechanical prop.	Conventional		[43]
	SLM on the anisotropy for acoustic cloaking	AM		[41]
	SLM and ECP combination for 100-micron dimension	AM		[30]
Aluminium	Validated aluminium-based elliptical cloaks achieving ν near 0.5	Conventional	*B/G* < 1000 *G* = 25-28 GPa *E*= 65–75 GPa Limitation: Limited *B/G*	[47]
	SLM and topology optimization	AM		[31]
	Anisotropic-density acoustic waveguides	Conventional		[44]
Rubber and Silicon	Silicon rubber and titanium hybrid	Theoretical	*B/G* < 100 *G* = 0.001-0.01 GPa *E* = 0.01 – 0.1 GPa Limitation: Cannot achieve stable pentamode unit cell due to low stiffness	[48]
	Topological concepts for waveguiding	Theoretical		[3]
	Silicon rubber PBG control over 52-548 Hz	Theoretical		[49]

The distribution of material usage in pentamode studies is depicted in Figure 3. Polymers have been widely used in pentamode metamaterials due to their lightweight nature and ease of fabrication [10]. Steel and other metals have been explored for pentamode metamaterials to improve mechanical strength [50]. Recent research demonstrated that steel-based pentamode structures can withstand high mechanical loads while maintaining tunable stiffness [51]. The integration of selective laser melting (SLM) and electropolishing techniques has improved the precision [30]. These advances suggest that steel-based pentamode metamaterials (PMs) can play a crucial role in structural reinforcement applications. Lymperopoulos et al. [52] demonstrated how steel-based pentamodes fabricated using these advanced techniques exhibit superior compressive resistance. Titanium and its alloys, particularly Ti-6Al-4V, have gained interest for pentamode metamaterials due to their high strength-to-weight ratio and corrosion resistance [7]. Material usage percentages in pentamodes are as follows: polymer (44%), titanium (38%), aluminium (6%), metal (6%) and steel (6%). Early experimental work demonstrated the feasibility of titanium-based pentamode lattices for impact resistance [42]. Later studies used additive manufacturing techniques, such as electron beam melting (EBM) and selective laser melting (SLM), to fabricate high-precision titanium pentamodes with optimised bulk-to-shear modulus ratios [29]. Recent research has focused on integrating titanium PMs into high-performance acoustic and mechanical systems [36]. Aluminium has been used in pentamodes due to its lightweight and ease of fabrication [53]. Early research explored its use in acoustic cloaking and waveguide applications [31]. Advances in additive manufacturing have enabled the creation of complex aluminium-based PM structures with improved stiffness and tunability [54]. Recent studies have refined aluminium-based designs for underwater acoustic applications, leveraging their anisotropic density distributions to achieve enhanced broadband performance [44]. These findings suggest that aluminium PMs are particularly suited for applications in acoustics and noise reduction. Rubber and silicon have been investigated for pentamode metamaterials. Recent work has explored tunable phononic band gaps in locally resonant pentamode structures, with rubber and polymer composites for improved acoustic wave [44]. These materials continue to be explored for their potential in pentamodes.

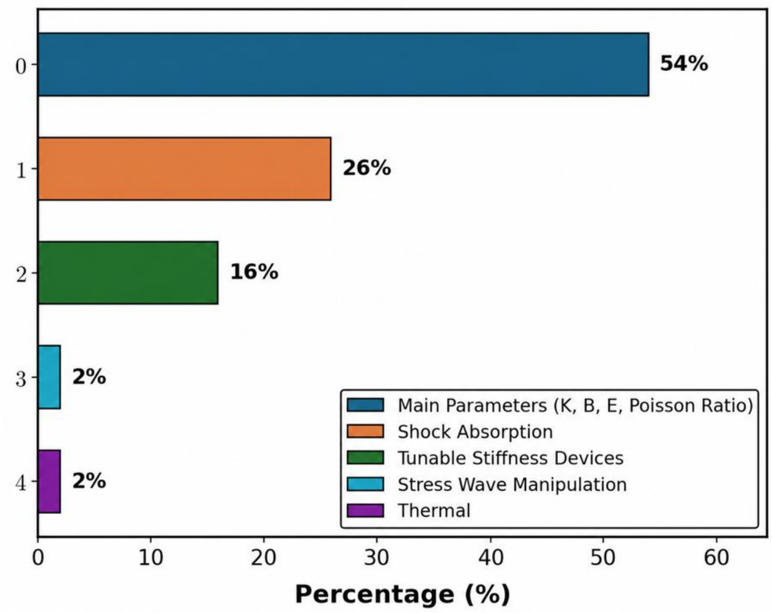

Figure 3. The distribution of the studies in the scope of mechanical applications of pentamodes

4. Computational and Experimental Methods

There are many computational and experimental methods used for pentamodes. The distribution of computational methods and tests in pentamode structures is depicted in Figure 4. Most of the methods are grouped under numerical methods; some tests are also carried out to verify the analyses. Recent studies have incorporated topology optimisation for designing structures with superior mechanical and acoustic performance [40], [55]. Methods used in pentamodes are analytical methods, numerical methods and tests (mechanical and acoustic tests). These computational approaches bridge the gap between theoretical predictions and experimental realisations. Advanced simulations have been employed to optimise designs for structural applications [55]. Recent studies integrate multi-scale FEA to improve performance predictions and enhance fabrication feasibility [36], [40]. Also, Chen et al. [56] demonstrated the tunability of PM structures for specific frequency ranges. The distribution of methods used in pentamodes is as follows: numerical methods (66%), numerical methods and tests (%15), analytical methods (7%), numerical and analytical methods (7%), numerical, analytical methods and tests (2%), tests (3%). A wide range of analytical, numerical and experimental methods has been applied to pentamode metamaterials; however, their capabilities and limitations differ substantially, which affects the reliability of design and validation. Numerical approaches dominate the field (66%), yet most rely on linear elastic FEA that does not fully capture geometric imperfections, slender-strut instability or post-yield behaviour—factors that strongly influence the achievable B/G ratios. Only a limited portion of studies combine numerical analysis with experimental validation (15%), and even these validations are often restricted to quasi-static compression, providing incomplete insight into shear-dominated deformation modes that define pentamode behaviour. Analytical models remain scarce (7%) and typically rely on idealised assumptions, such as perfect joints and negligible shear stiffness, which deviate significantly from printed geometries. Multi-scale FEA has recently improved prediction accuracy, but its adoption is still limited and computationally demanding. Experimental testing is the least represented methodology (3%) and current tests rarely quantify frequency-dependent or dynamic responses, despite their importance in acoustic applications. This imbalance across methods indicates that the field lacks a unified modelling–validation framework and highlights the need for systematic integration of analytical predictions, imperfection-sensitive numerical models and multi-mode experimental testing to accurately assess real-world pentamode performance.

5. Applications

5.1 Acoustic Applications

Pentamode metamaterials have significant potential in acoustic applications due to their ability to control and manipulate sound waves [57] –[70]. Their unique fluid-like mechanical properties make them ideal for designing acoustic cloaks. These materials are also used in noise reduction. Pentamode structures are employed in acoustic filters and waveguides, allowing precise control over sound propagation. Additionally, these materials are promising for underwater acoustics, where their tunable properties can optimise sound transmission. Acoustic applications over the years and their contributions are shown in Table 3. To enable technical assessment, Table 3 has been expanded to include key performance metrics, such as band-gap frequency ranges, which are the primary parameters governing each acoustic response. It is seen from the table that band gap frequency could reach up to 25 kHz by adjusting material and geometry. This table enables direct comparison across studies and clarifies how material selection and each study's contribution influence the achievable acoustic band gap frequency. Their anisotropic acoustic impedance and near-zero shear modulus make them ideal for controlling wave propagation, reducing scattering, and creating acoustic band gaps. Acoustic application subgroups are frequency control and tunability, band gaps, wave scattering and cloaking. Most studies focus on wave scattering, cloaking, and band gaps.

Table 3. Acoustic band gap studies on pentamodes

Contribution	Materials	Band gap frequency (kHz)	References
Introduced material damping significantly reduces shear resonance, improving broadband capabilities	Aluminium	5-12	[71]
Asymmetric double cone for ultrawide band gap	Polymer	8-20	[72]
Achieving low-frequency band gaps through composite material designs	Rubber	1-5	[49]
Composite pentamode for customized acoustic property	-	3-10	[73]
High-efficiency asymmetric wave propagation using PM meta surfaces	Titanium	10-25	[74]
2D composite rectangular unit cell for acoustic properties	Aluminium and Steel	5-18	[75]
A topology optimization strategy to create anisotropic PM units with customized parameters	-	4-15	[76]
Oblique, rectangular, and hexagonal shapes, influence the photonics band structures	-	3-12	[77]
Broadband and low-frequency band gap solutions, focusing on composite designs	-	2-8	[78]
Multi-layer cylindrical PMs for low-frequency applications	Aluminium	1-6	[44]
Hexagonal unit cells incorporating double-cone arms, focusing on band gaps at low frequencies	-	3-12	[77]
Adding finite spherical masses at the connection points of pentamode lattices on their photonics band structures	-	2-7	[79]
Elliptic pentamode metamaterials achieved dramatic increases in photonics band gap (PBG)	-	8-25	[80]
Increasing the asymmetry of double-cone elements and modulating material ratios can broaden the ABG	-	10-22	[81]
Increasing the double-cone array number improved the PBG	-	6-18	[82]
A novel pentamode metamaterial (PM) with dual functionalities—water-like properties and topological edge-state transmission	-	3-9	[83]
Pentamodes with larger beam diameters exhibit higher frequency band gaps, and the material's density contributes to the dynamic behaviour, with steel showing higher band gaps than polymer	Polymer and steel	4-12	[52]

5.1.1 Band gaps

The study of band gaps in acoustics, particularly in pentamode metamaterials, has seen significant advancements over the years. Band gaps, which suppress wave propagation in certain frequency ranges, are achieved in PMs through mechanisms like Bragg scattering and local resonance. Chen et al. [53] introduced latticed pentamode acoustic cloaks, highlighting the challenges of weak shear resistance and segmented frequency ranges. They demonstrated that material damping could improve broadband performance. Following this, Chu et al. [72] examined asymmetric double-cone structures, establishing their role in achieving ultrawide band gaps and expanding the bandwidth by reducing structural symmetry.

Around the same time, researchers investigated practical applications of PMs in underwater acoustics. Chen et al. [84] proposed broadband underwater acoustic cloaks, demonstrating how PMs could restore distorted wavefronts with flexible designs. This work provided significant insights into impedance matching.

Advancements in PMs continued with Cai et al. [49], who explored the tuning of phononic band gaps in locally resonant PMs. Their study highlighted the relationship between structural parameters and band gap behaviour to achieve low-frequency band gaps. Similarly, Chu et al. [74] demonstrated high-efficiency asymmetric wave propagation using PM surfaces, showcasing their ability to manipulate acoustic waves. From 2021 onward, studies began to focus on the dynamic control and optimisation of band-gap properties. Liu et al. [85] explored the impact of temperature on band gap tuning, introducing a novel approach to adaptive acoustic devices. Concurrently, Chen et al. [44] investigated multi-layer cylindrical PMs for low-frequency applications, emphasising the role of structural modifications in enhancing acoustic performance. This period also saw the integration of hexagonal unit cells and curved geometries, as demonstrated by Li and Zhang [50], to improve band gap width and adaptability. Most recently, Chu et al. [81] and Cai et al. [78] have worked on broadband and low-frequency band gap solutions. The development of composite pentamode structures with tailored properties has pushed the boundaries of PM applications, enabling more efficient acoustic cloaking. The shift from theoretical designs to practical applications, the integration of dynamic tuning mechanisms, and the exploration of low-frequency band gaps have defined the trajectory for pentamode acoustic applications.

5.1.2 Wave scattering and cloaking

Wave scattering and cloaking in acoustics have been transformative research areas, particularly with the emergence of PMMs. This idea laid the foundation for subsequent advancements in cloaking technologies, where acoustic waves are redirected to minimise scattering, effectively rendering objects "invisible." The theoretical groundwork for acoustic cloaking was further developed through transformation acoustics. Building on this, Scandrett et al. [9] demonstrated how layered PM configurations could cloak spherical objects, achieving significant reductions in scattering. These studies marked a critical transition from theory to practical implementations. Experimental progress followed with Bückmann et al. [4], who developed core-shell cloaks using three-dimensional PMs fabricated via rapid dip-in laser lithography. Their work validated the theoretical principles of transformation acoustics, demonstrating effective cloaking for solid mechanics. Huang et al. [13] investigated the pentamodal properties and acoustic band gaps of PMs with various cross-sectional shapes to maximise bulk-to-shear modulus ratios. Guo et al. [3] extended this work by designing honeycomb-lattice PMs to minimise scattering in underwater environments, thereby demonstrating the practical viability of PMs. The versatility of PMs was further highlighted by Cai et al. [6], who introduced multi-phase PM designs combining materials with contrasting properties to enhance broadband cloaking. Méndez et al. [5] advanced computational methods by employing topology optimisation to tailor PM microstructures, ensuring that designs met the specific requirements of acoustic environments. Wave scattering and cloaking research evolved further with the study of dispersion control and band gap engineering. Chen et al. [84], demonstrated underwater acoustic bottle beams, showing how PMs could manipulate wavefronts to create cloaking effects. These advancements established PMs as essential tools for practical underwater applications, including communication systems. The connection between material design and wave suppression was strengthened by Chen et al. [55], who studied boundary effects in PM-based cloaks, emphasising the role of material imperfections in scattering suppression. Their findings informed later work by Li et al. [54], who developed elliptical acoustic cloaks optimised for non-spherical objects. The focus on practical implementation continued with Zhang et al. [35], who employed graded PM designs to achieve broadband cloaking. Recent studies have explored multifunctionality in PM-based cloaking systems. Wave scattering and cloaking research has developed into a cohesive field that integrates theoretical insights and computational modelling. As the field advances, future research is likely to explore adaptive materials and multifunctional systems to ensure that wave scattering and cloaking technologies remain transformative in acoustics.

5.1.3 Frequency control and tunability

Frequency control and tunability in acoustics have seen significant advancements through the PMMs. Sun et al. [86] and Huang et al. [45] offer complementary insights, combining practical applications with material innovation to address key challenges in acoustic frequency management. Sun et al. [86] designed a pentamode surface for underwater acoustic communication, emphasising its ability to multiplex and demultiplex signals through orbital angular momentum. The system demonstrates real-time data transfer and highlights the role of frequency-selective metasurfaces in improving underwater communication. Sun et al. [86] addressed practical challenges in acoustic communication systems, while Huang et al. [45] provided the underlying material advancements that enable precise control over acoustic properties. These approaches point out the growing importance of geometry and material science in driving acoustic innovations. Emerging trends in the field reflect a focus on structural innovation and application-specific tunability. The customisation of geometric features in PMMs and metasurfaces offers new pathways for controlling frequency ranges. Additionally, a shift toward low-frequency acoustic applications is significant, with core-shell PMMs enabling tunable solutions for such challenges.

5.2 Mechanical Applications

Pentamode metamaterials have found extensive use in mechanical applications due to their tunable stiffness, low shear modulus and high bulk modulus. These properties make them ideal for stress control, shock absorption, and vibration damping. Pentamode metamaterials are promising for mechanical applications due to their ability to mimic fluid-like properties while maintaining structural integrity. One key application is in vibration and shock isolation, where their near-zero shear modulus allows them to absorb mechanical energy effectively. They are also explored for use in seismic protection systems, where their unique properties can help mitigate the impact of earthquakes by redirecting stress waves. Additionally, they are being considered for lightweight load-bearing structures. The distribution of the studies in the scope of mechanical applications of pentamodes is shown in Figure 3. Most applications are based on the main parameters; other major studies focus on shock-absorption and tunable-stiffness devices.

5.2.1 Stress wave manipulation

Pentamode metamaterials excel in directing and controlling stress waves, enabling innovative applications in seismic protection and energy dissipation. Studies have demonstrated the use of alternating pentamode lattices and confinement plates to enhance the ability to dissipate energy under dynamic loads [7] [48]. Pentamode metamaterials exhibit unique properties that make them highly effective in manipulating stress waves. With a near-zero shear modulus and high stiffness, these materials can create band gaps that block stress waves, enabling precise control over propagation [87]. Finite element analysis conducted by Lymperopoulos et al. [87] underscores the importance of geometric optimisation, particularly the small-to-large diameter ratio in pentamodes. Material selection also plays a key role, with steel demonstrating superior performance under large displacements due to its higher shear strength. Recent work by Lymperopoulos et al. [52] examined tailored unit-cell designs that maximise stress redistribution, thereby improving resilience against mechanical shocks. While primarily studied for antiseismic applications, these findings hold significant potential for broader stress wave manipulation applications. Pentamode metamaterials excel in redirecting and suppressing stress waves due to their extreme bulk-dominated behaviour. Studies show that alternating pentamode layers can reduce transmitted dynamic stress by 20–40%, while optimised geometries achieve stress-redistribution gains of 30–50% compared with conventional lattices. Lymperopoulos et al. [87] demonstrated that tuned d/D ratios produce directional band gaps in the range of 2–10 kHz, effectively blocking shear-dominated propagation paths. Material effects are also quantifiable: steel-based PMs withstand 35–50% higher peak loads during seismic-like loading due to their increased shear strength.

5.2.2 Shock absorption

The study established the theoretical framework for their anisotropic properties by emphasising their potential for suppressing shear waves [88]. This study also introduced the concept of asymmetric double-cone elements to enhance the bulk-to-shear modulus ratio, increasing the figure of merit (FOM) by up to 30%. Amendola et al. [8] and Amendola et al. [42] shifted the focus to experimental validation, using Electron Beam Melting (EBM) to fabricate metallic PMMs. These studies demonstrated that confinement between stiffening plates enhances vertical stiffness while maintaining low shear rigidity. Simultaneously, Amendola et al. [42] investigated the effect of micro- and macro-scale aspect ratios on PMM performance. Their results confirmed that the geometric parameters of lattice structures, such as rod thickness and nodal junction sizes, play a critical role in shock absorption. Fabbrocino et al. [89] introduced a discrete-to-continuum modelling approach that enables accurate prediction of PMM performance under dynamic loads. Their work highlighted the similarities between confined PMMs and elastomeric bearings, showcasing PMMs' ability to dissipate energy effectively in seismic isolation systems. Skripnyak et al. [90] explored the performance of PMMs under dynamic loading, demonstrating that they dissipate energy 1.5 times more effectively than bulk alloys. Simultaneously, Zhang et al. [35] investigated PMM behaviour under collision impacts, confirming their adaptability to high strain rates. These studies highlighted the importance of advanced manufacturing techniques in achieving precise control over PMM properties. Sapountzaki et al. [91] provided insights into the role of PMMs in seismic isolation and collision mitigation. Their work emphasised the versatility of PMMs across diverse applications, from aerospace to civil engineering, and identified emerging trends such as photonics band-gap manipulation and acoustic metamaterials. Pentamode metamaterials exhibit measurable advantages in shock mitigation. Asymmetric double-cone geometries improve the B/G ratio by ~30%, directly enhancing shock absorption performance. EBM-fabricated metallic PMMs confined between stiffening plates show 25–45% improvement in vertical shock resistance while preserving low shear rigidity. Skripnyak et al. [90] reported that PMMs dissipate dynamic energy 1.5 times more effectively than bulk metal alloys, attributed to their anisotropic stress channels. Experimental studies confirm stress reductions of 20–35%, demonstrating the clear mechanical benefits of pentamode designs in impact and collision scenarios.

5.2.3 Tunable Stiffness Devices

Pentamode metamaterials have gained attention as promising candidates for tunable stiffness devices due to their ability to decouple bulk and shear moduli. Milton et al. [92] highlighted the use of near-optimal pentamodes for guiding stress while minimising compliance in 3D-printed materials. These structures provided solutions to the weak G-closure problem, enabling the realisation of materials with tailored stiffness properties. Bronder et al. [93] introduced machine learning and adaptive sampling to optimise pentamode structures for stiffness and damping properties. This approach not only reduced computational costs but also provided high-performance designs tailored for specific applications such as seismic isolation and mechanical cloaking. Mohammadi et al. [37] further investigated the influence of unit cell topology on tunable stiffness, showing that diamond lattice designs offer superior performance in achieving high bulk-to-shear modulus. Brambilla et al. [94] addressed the challenge of achieving high bulk modulus while maintaining low shear modulus, introducing a novel "metal water" pentamode design. Their work pushed the boundaries of tunable stiffness devices by providing fluid-like mechanical behaviour in solid-state systems. The progression of research on tunable stiffness devices demonstrates a clear trend toward integrating advanced computational tools and optimising geometric designs. The tuneable stiffness capability of pentamodes has been quantitatively demonstrated across several studies. Diamond-type PMs routinely achieve B/G ratios of 10^3–10^5, while optimised double-cone architectures reach up to 10^6, thereby enabling stiffness modulation of 20–60% in targeted directions. Bronder et al. [93] showed that machine-learning-assisted optimisation reduces stiffness prediction error by 40–70%. Brambilla et al. [94] achieved bulk modulus tunability by a factor of 2–5 in their 'metal water' PM design, confirming that PMMs offer mechanical performance adjustments not possible in conventional lattice systems.

Table 4. Studies of main parameters on pentamodes

Material	Parameter	References
PLA	*d* = 0.125 mm *D* = 0.425 mm *l*= 3.37 mm *B/G* = 200	[36]
Clear Resin V4	*D* = 4 mm, *d* =1.43 mm, Effective Elastic Modulus= 0.4394 MPa; *D* = 4 mm, *d* =1.6 mm, Effective Elastic Modulus= 0.6057 MPa; *D* = 4 mm, *d* =1.8 mm, Effective Elastic Modulus= 0.8457 MPa; *D* = 4 mm, *d* =2 mm, Effective Elastic Modulus= 1.1365 MPa; *D* = 4 mm, *d* =2.2 mm, Effective Elastic Modulus= 1.4805 MPa; *D* = 4 mm, *d* =2.4 mm, Effective Elastic Modulus= 1.8796 MPa	[38]
-	*l* = 3.333 mm *d* = 0.111 mm *D/d* = 8.2 ; for 3 Joints as : 200 GPa *B/G* = 337.7 ; 200 GPa *B/G* = 696 ; 4.2 GPa *B/G*= 2021	[99]
PA2200	*D* = 2.5 mm *d* = 0.8 mm *E/G* for 3 different configuration *E/G* for S-FCC-PM1 *E/G*= 4.68 ; for S-SC-PM2 *E/G* =53.51; for S-SC-PM3 *E/G* =216.76	[39]
Steel	*E* and *G*	[100]
Ti-6AI-4V	*d* = 0.125 mm *l* = 3.37 mm *B/G* = 200 ; *d* = 0.7 mm *l* = 3.37 mm *B/G* = 50	[36]
Ti-6AI-4V	0.1 mm processable size; *D/d* = 52	[30]
Ti-6AI-4V	*G/K* = 0.006 *d/D* = 0.8 *d* = 1.04 mm	[101]
Ti-6AI-4V	Ground structure method for design, *d* = 0.02 mm *D* = 0.06 mm	[50]
Aluminium	*d* = 0.1 mm *B/G* = 212 v=0.498; d=0.2 mm *B/G* = 84 *v* = 0.494; *d* = 0.3 mm *B/G* = 43 *v* = 0.488; *d* = 0.4 mm *B/G* = 25 *v* = 0.480	[31]
Ti-170-500	Force/displacement curves, critical crack length for nonlinearity	[98]
Ti-6AI-4V	Poisson's ratio and Elastic Modulus for different thickness of 2D pentamodes based on honeycomb	[43]
Polymer	*d* = 0.3 mm *D* = 1.1 mm, anisotropic distribution with the same *d* and *D*. Positioning as %10, 15, 20, 25, 30, 35, 42. Max. B/G is 50 obtained for % 25 positioning.	[33]
Ti-6AI-4V	Critical width of strut for buckling found as 0.12 mm	[29]
Polymer	*d* = 0.0015 mm *D* = 0.002 mm *a* = 0.0373 mm *B/G* is 300	[97]
PLA	Effects of strut shapes (*R/r* or *d/D*) on stress-strain curve	[32]
-	Confined pentamode lattice in bending and stretch regime	[96]
-	Effects of confinement plates on lateral load versus lateral displacement	[48]
-	*d* = 0.55 mm fixed; when *D* = 9.66 mm *B/G* is around 425. Effects of *D* on *B/G* is studied	[12]
-	*D/d* = 5 *B/G* > 10^5 Inspired by Bravais Lattice Structures (tetragonal simple)	[11]
Polymer	*d* = 0.00055 mm, *D* = 0.001 mm *B/G* > 10^3 ; *d* = 0.00055 mm is achievable with state of art laser lithography	[10]
-	Pentamode proposed for the first time in literature	[1]

5.2.4 Main parameters (poisson's ratio, shear, bulk, elastic modulus)

PMs were conceptualised as diamond-lattice structures composed of double-cone elements that mimic fluid-like mechanical behaviour. Méjica and Lantada [11] expanded the scope by simulating Bravais lattice-inspired geometries, showcasing how lattice configuration could fine-tune mechanical properties to decouple compression and shear responses. Amendola et al. [95] introduced PMs confined between stiffening plates, thereby uncovering nonlinear elastic responses that are ideal for seismic isolation. Their following work on multilayered PMs with rigid and hinged nodes revealed high compressive stiffness and low shear rigidity, emphasising the role of layered configurations in dynamic applications [96]. Huang et al. [97] proposed centro-symmetrical PM designs that improved structural stability while maintaining high anisotropy. Lymperopoulos et al. [98] examined crack propagation in PMs, highlighting the importance of connection geometry in enhancing structural durability. Closed-form analytical formulas for PM mechanical properties, such as Poisson's ratio and bulk modulus, have provided accurate predictive tools [99]. Automated design frameworks for composite mechanical metamaterials have enabled optimisation of 2D PM structures for seismic and acoustic applications

[99]. Research on PMs has advanced significantly over the decades, progressing from theoretical concepts to practical applications. Initial studies focused on validating the high B/G ratio and Poisson's ratio close to 0.5. The studies and related main parameters are shown in Table 4. The correlations between *d* and *B/G,* and *between d/D* and *B/G, are* derived and depicted in Figure 4. It is observed that increasing d or d/D decreases *B/G*. In contrast, when d or d/D decreased, B/G tends to increase, as shown in Figure 4. It is important to note that since previous studies have employed varying standards for compression testing and different specimen sizes, making a direct correlation between *d, d/D* and *B/G* is infeasible. However, comparing studies that utilise similar specimen sizes remains valuable for identifying general trends in the relationships among d, d/D, and B/G that correspond to pentamodal characteristics when the number is high.

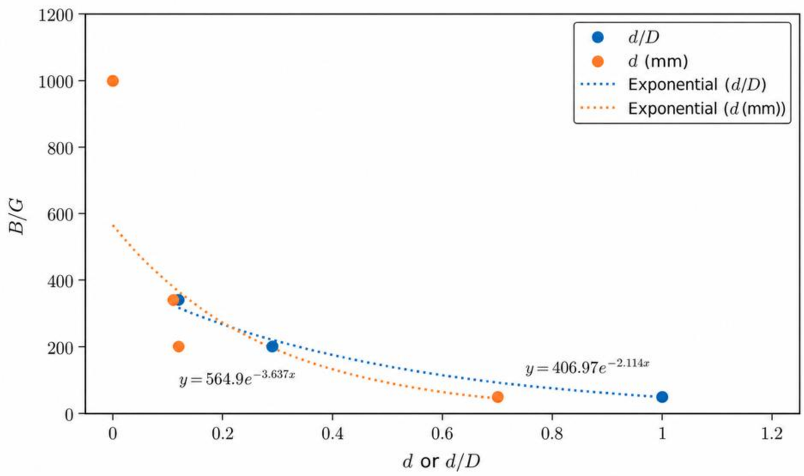

Figure 4. Correlation between *d* and *d/D* with *B/G*

6. Conclusions

Pentamode metamaterials have evolved from theoretical concepts to experimentally validated materials with significant potential across engineering fields. This systematic review highlights the progression of pentamode metamaterials from theoretical studies to experimental validations. Early studies provided foundational principles, followed by the first experimental realisations. Advances in computational modelling and mechanical testing further refined their properties. Recent developments focus on large-scale fabrication, dynamic tunability, and multifunctional integration for practical applications. From the literature review, the key parameters examined were band gaps, acoustic cloaking, acoustic scattering, and bulk-to-shear ratios. Studies in recent years have focused on acoustic cloaking, tunable stiffness, and the key parameters of pentamode applications. These findings emphasise the importance of continued interdisciplinary research. It was observed that tuning the stiffness of pentamodes has potential for both acoustic and mechanical applications. Rather than providing a descriptive summary, this review synthesises current knowledge by comparing findings in the literature. The aim is to integrate these results into a coherent understanding of how geometry, manufacturing constraints and mechanical behaviour are interconnected in pentamode metamaterials. Research tends to address scalability and the integration of hybrid materials to unlock the full potential of pentamode metamaterials for acoustic and mechanical applications in the future. To obtain thin pentamode structures with B/G as high as possible, studies on fabrication techniques are also increasing. As fabrication technologies improve, pentamode metamaterials will continue to expand their applications in the acoustic and mechanical domains. Pentamode metamaterials have progressed significantly from theoretical constructs to experimentally fabricated lattice systems with promising acoustic and mechanical applications. However, despite the rapid growth in publications, several critical challenges remain unresolved. There exists a gap between idealised analytical models and experimentally fabricated structures.

Most analytical formulations assume perfect double-cone geometries, whereas additive manufacturing introduces geometric deviations that can reduce the achievable B/G ratio by one to two orders of magnitude. This modelling–fabrication inconsistency remains insufficiently addressed. Second, mechanical validation methods are not standardised. Most studies rely on quasi-static compression testing, while shear-dominated or dynamic testing—essential for assessing true pentamode behaviour—remain limited. A unified modelling–validation framework integrating analytical, numerical and experimental methods is still lacking. Third, scalability remains a bottleneck. While micro-scale pentamodes achieve high B/G ratios (10^5–10^6), large-scale implementations often compromise shear suppression due to manufacturing constraints. Fourth, in acoustic applications, achieving broadband and low-frequency band gaps simultaneously remains challenging, particularly in load-bearing configurations. Therefore, future research should be directed towards imperfection-sensitive modelling frameworks, standardised dynamic and shear testing protocols, multi-material and hybrid pentamode designs, scalable fabrication strategies suitable for macro-scale applications, and integrated optimisation that combines topology optimisation with manufacturing constraints. Instead of solely expanding the scope of these technologies' applications, future efforts ought to focus on the previously discussed aspects. This strategy will aid in transitioning pentamodes from solely experimental laboratory demonstrations to dependable, real-world engineering solutions.

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B. Ocak: Investigation; Data curation; Writing & editing - orginal draft
Z. Evis: Investigation; Review & editing; Supervision

Availability of Data and Materials
The data supporting this study's findings are available on request from the corresponding author.

Ethics Declarations
This study did not involve human participants or animals. Ethical approval was therefore not required.

Generative Artificial Intelligence Declarations
The authors claim that artificially intelligent-assisted technologies, such as generative AI, were not used to generate content, ideas, or theories. We have just utilised AI to enhance readability and refine the language. This was used with extreme human control and oversight. The authors take full responsibility for reviewing and approving the content.

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