

RESEARCH ARTICLE

MHD dissipative Casson fluid flow through a Forchheimer pervious medium with Soret-Dufour and higher-order convection effects

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Abstract - This study investigates magnetohydrodynamic (MHD) dissipative Casson fluid flow through a Forchheimer pervious medium, incorporating Soret-Dufour cross-diffusion and higher-order convection effects to capture the coupled non-Newtonian, electromagnetic, inertial-porous, and cross-transport phenomena governing thermal management, species distribution, and flow stability in advanced porous-media engineering systems. New insights into the combined influence of these effects were explored through a detailed parametric analysis. The governing dimensional equations describing the flow were transformed into coupled nonlinear dimensionless forms using appropriate similarity variables. The collocation method based on Legendre polynomials of the first kind was employed to obtain approximate solutions for the flow characteristics. The numerical results are presented in tables and graphs to illustrate the influence of the key parameters. The findings revealed that increasing the linear and nonlinear convection parameters, Dufour number, and Eckert number enhanced the velocity while reducing the temperature profile. Furthermore, higher values of the stretching sheet index parameter decrease both the temperature and wall shear stress but increase the chemical concentration near the surface. The model is applicable to a wide range of real-world systems, and its importance lies in improving the design, optimisation, and control of engineering processes, biomedical applications, and environmental systems, where advanced heat and mass transfer mechanisms play a crucial role.

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1. Introduction

The importance of studying electrically conducting fluids in motion in the presence of magnetic fields, known as magnetohydrodynamics (MHD), is more visible in the sciences and engineering fields. Blood flow simulation in magnetic fields, MHD generators, nuclear reactor cooling, plasma confinement, and electromagnetic pumps are some of its applications, particularly when the fluid departs from Newtonian behaviour. In line with this importance, Makinde [20] investigated the Soret–Dufour effects on MHD convective flow in a penetrable medium through a vertical surface. He reported that the heat and mass movement rates were strongly influenced by cross-diffusion. Oyekunle et al. [29] analysed the stagnation point flow of MHD Nanofluid through a porous shrinking sheet with a buoyancy effect. An increase in the momentum field and a reduction in the temperature were observed. Reddy et al. [33] studied the MHD convective flow at a stagnation point. Bhuvaneswari et al. [18] studied the flow of an electrically conducting Casson Nano fluid on a slopply surface with Soret-Dufour effects. Kumar and Sood [16] investigated the combined effects of nonlinear convection and magnetic field past a shrinking surface. Prand et al. [30] studied the magnetohydrodynamic flow between parallel disks with a heat source. Other studies with different geometries include Shankar et al. [9], Abro et al. [1], Shateyi and Muzara [22], Oyekunle et al. [26] just to mention but a few.

In modelling and industrial fluids, materials such as blood, polymer solutions, slurries, and molten chocolate are considered useful because the Casson fluid model provides a reliable description by incorporating a finite yield stress and shear-thinning characteristics. The effects of Darcy's law, which is moderate for low Reynolds number regimes and high velocity viscous diffusion, are incorporated into the non-Darcy model, which accounts for inertial resistance and viscous shear in the penetrable medium with magnetic field impact, rheological resistance, permeable drag, and interaction between Lorentz forces, yielding rich and complex flow behaviour. Chamkha and Khaled [8] examined the effects of cross-diffusion on free convection on an upright plate in a previous medium. Postelnicu [31] investigated the Soret-Dufour influences on magnetohydrodynamic (MHD) free convective flow through a vertical surface in a permeable medium. Olajuwon and Oahimire [24] examined Hall currents with radiation impacts on the flow of MHD viscoelastic micropolar fluids through a penetrable medium. Khanafer and Vafai [15] examined the Nanofluid application of porous medium while Banerjee and Paul [6] stated various challenges of geometries and their contributions in the development of porous media and Nano particle phenomena. Butt et al. [7], Mahesh et al. [19], Idowu et al. [13], Raju et al. [32] and Akbar et al. [3] are among the studies in the literature that have considered porous plates or medium using different geometries.

The heat and mass transfer in these systems are complicated by cross-diffusion phenomena and dissipative effects. The species mass flux and heat flux caused by the temperature and concentration gradients describe the Soret and Dufour effects, respectively. They are important in double-diffusive convection and reactive flows, where the thermal and solutal fields are highly coupled. In addition, boundary layer structures can be significantly altered by the convection of higher

orders, such as radiative heat transfer, nonlinear convection, and variable coefficients, particularly in high-temperature or chemically reactive systems. Subsets of this problem have been explored in many studies, such as Hayat et al. [12], who examined the MHD boundary layer flow of a Casson fluid with chemical and radiation effects through a slopy stretching sheet, revealing a complex coupling between non-Newtonian rheology and thermal transport. The influence of cross-diffusion on the flow of unsteady MHD free convection through an upright plate was studied by Chamkha and El-Kabeir [11]. Narayana et al. [23] analysed the effects of chemical reactions and heat sources on the oscillatory flow of MHD through a corrugated channel. They revealed that the chemical reaction significantly influenced the flow distribution. Chamkha and Khaled [7] studied the effects of Dufour and Soret in a penetrable medium on free convection through a vertical plate using a finite difference scheme. Shaw et al. [36] investigated the double-diffusive effect of radiation on Marangoni convection on a stretching surface. They discovered that the boundary conditions determined the stability of the flow with variations in the cross-diffusion parameters. Shateyi and Muzara [22] examined the dissipative and Soret-Dufour effects on unsteady magnetohydrodynamic (MHD) boundary layer flow. They emphasised that nonlinear parameters play a significant role in heat and mass transfer. The influence of double-diffusive convection on the flow of a Casson fluid through a stretching surface was analysed by Kumar et al. [17]. Ajithkumar and Lakshminaravana [2] used long-wavelength approximations to investigate the peristaltic transport of MHD fluid flow within a permeable channel with cross-diffusion effects. Sun et al. [4] explored non-Darcy nanofluid flow on a turning disk with dissipation and radiation effects. Analytical solutions for the flow of MHD Casson fluid within parallel pervious plates were analysed by Abro et al. [1].

Other recent contributions include Rajput et al. [21] and, El Hama [10], who studied the influence of double-diffusive with thermal and solutal convection on Casson Nanofluid flow through an upright plate in a non-Darcy permeable medium while Gupta et al. [14] examined slip radiation with Soret and Dufour influences in the flow of MHD Casson fluid. Shankar et al. [9] explored Soret-Dufour with heat generation/absorption effects on the flow of MHD convective Casson fluid on an upright surface. Sekhal et al. [34] scrutinised the combined impacts of radiation, heat generation, Soret and Dufour on the flow of MHD Casson Nanofluid boundary layer over an extending slopy surface while Bejawada et al. [35] presented the impacts of heat source, chemical reaction and thermal radiation on the flow of MHD Casson fluid in a forchheimer penetrable medium. They observed a reduction in heat energy with an increase in the radiation and Forchheimer parameters. Oyekunle et al. [26] analysed the effects of dissipation and radiation on the flow of unsteady free magnetohydrodynamic (MHD) convection within irregular channels. The n th-order chemical reaction and Soret-Dufour influences on nonlinear thermosolutal convective Casson Nanofluid flow were analysed by Oyekunle et al. [28]. Many studies have analysed the impact of MHD forces, porous resistance, the Soret and Dufour effects, Casson rheology, and n th-order convection individually or in partial combination. To the best of our knowledge, these features have not been addressed simultaneously in a unified, unsteady, non-Darcy framework. Particularly, the combined effects of MHD, Casson fluid, cross-diffusion, Viscous dissipation, nonlinear convection, Joule heating, higher order chemical reaction, Darcy and Forchheimer parameters on flow fields remain under explored. This study introduces a novel unified model that couples MHD dissipative Casson flow with Forchheimer inertial-porous effects, Soret-Dufour cross-diffusion, and higher-order convection to enable improved design and optimisation of electrohydrodynamic thermal-management, separation, and porous-reactor systems.

2. Methodology

A comprehensive model of MHD-dissipative Casson fluid flow in a non-Darcy pervious medium was developed. The steady, electrically conducting, incompressible, laminar, two-dimensional flow with higher-order convection of Casson fluid in a permeable medium on an inclined plane was considered. Figure 1 shows the geometry of the flow, the x -axis is parallel to the stretching surface and y -axis and external transverse magnetic field $B(x) = B_0 x^{\frac{r-1}{2}}$ are normal to the surface of the substrate. The induced magnetic field is assumed to be very small and therefore neglected. The temperature of the plate surface and free stream are respectively T_f and T_∞ where C_f and C_∞ are the nanoparticle fractions. It is assumed that the stretching velocity $U_f(x) = bx^r$ and free stream velocity $U_\infty(x) = 0$, where b is a constant. The rheological Casson fluid equation, following Shankar et al. [32] and Vijaya et al. [38], is

$$\tau_{ij} = \left\{ \left(\mu_B + \frac{p_y}{\sqrt{2\pi}} \right) 2e_{ij}, \pi > \pi_c \right\}, \text{ where, } P_y = \frac{\mu_B \sqrt{2\pi}}{\gamma}. \quad (1)$$

The dynamic viscosity of Casson is defined as $\mu_f = \mu_B + \frac{p_y}{\sqrt{2\pi}}$, on substitution we have $\nu_f = \frac{\mu_B}{\rho_f} \left(1 + \frac{1}{\gamma} \right)$.

In view of the above assumptions and following the Boussinesq approximation theory, the governed equations of flow are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_f \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u}{\partial y^2} - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{k_p} \right) u - \frac{c_f}{\sqrt{k_p}} u^2 + g_x \cos \beta [\beta_0 (T - T_\infty) + \beta_1 (T - T_\infty)^2 + \beta_2 (C - C_\infty) + \beta_3 (C - C_\infty)^2] \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \frac{v_f}{c_p} \left(1 + \frac{1}{\gamma}\right) \left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_p} (T - T_\infty) \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2} + \frac{v_f}{c_p} \left(1 + \frac{1}{\gamma}\right) \left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{Q_0}{\rho c_p} (T - T_\infty) \quad (5)$$

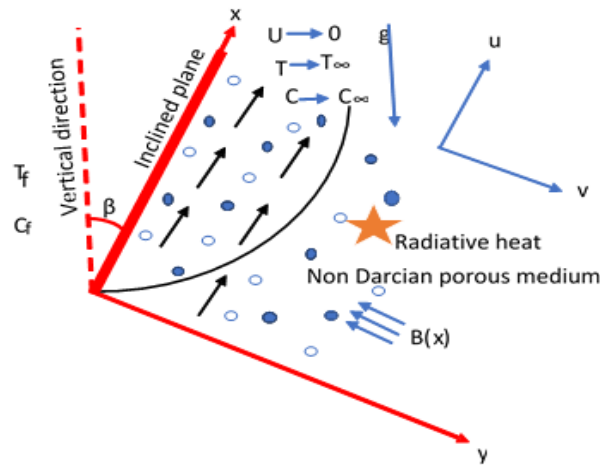


Figure 1. Casson fluid geometry with non-Darcian porous medium

The model boundary conditions are as follows:

$$\left. \begin{aligned} u = U(x) = u_f + u_s + bx^r + N\mu \frac{\partial u}{\partial y}, \quad v = v_f = -V(x) = -v_0 x^{\frac{r-1}{2}} \\ T(x) = T_f + T_0 \frac{\partial T}{\partial y}, \quad C(x) = C_f + C_0 \frac{\partial C}{\partial y} \end{aligned} \right\} \text{ at } y = 0 \quad (6)$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty$$

where $u_s = N\mu \frac{\partial u}{\partial y} = N \left(\mu_B + \frac{P_y}{\sqrt{\pi}} \right) \frac{\partial u}{\partial y}$, μ is the dynamic viscosity taken as $\left(\mu_B + \frac{P_y}{\sqrt{\pi}} \right)$, v_f is the mass transfer rate, $v_f < 0$ means mass suction while $v_f > 0$ denotes mass injection.

Using Roseland's approximation theory [39], the radiative heat flux q_r is taken as $q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$ where k^* is the average absorption coefficient, σ^* is the Stefan Boltzmann constant, T^4 is a linear function of temperature and is sufficiently small within the flow. Hence using the Taylor series expansion to expand T^4 and ignoring higher order terms gives $T^4 = 4T_\infty^3 - 3T_\infty^4$.

Following Sekhar et al. [34], the stream function and similarity variables are expressed as

$$\xi = \sqrt{\frac{b(r+1)}{2v}} x^{\frac{r-1}{2}}, \quad \psi = \sqrt{\frac{2bv}{r+1}} x^{\frac{r+1}{2}} f(\xi), \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad u = bx^r f'(\xi), \quad (7)$$

$$v = -\sqrt{\frac{bv(r+1)}{2}} x^{\frac{r-1}{2}} \left[f(\xi) + \frac{r-1}{r+1} \xi f'(\xi) \right], \quad \theta(\xi) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi(\xi) = \frac{C - C_\infty}{C_f - C_\infty}.$$

Implementing equation (7) on equations (2)- (6), yields

$$\left(1 + \frac{1}{\gamma}\right) f'''' + f f'' - \frac{2r}{r+1} f'^2 - \frac{2}{r+1} [(M+B)f' + f_c f'^2] + \frac{2}{r+1} \cos \beta [\gamma_1 (1 + \lambda_1 \theta) \theta + \gamma_2 (1 + \lambda_2 \phi) \phi] = 0, \quad (8)$$

$$\left(1 + \frac{4}{3} R\right) \theta'' + \text{Pr} f \theta' + \text{Pr} D_f \phi'' + \text{Pr} Ec \left(1 + \frac{1}{\gamma}\right) f'^2 + \frac{2}{r+1} \text{Pr} Q_0 \theta = 0 \quad (9)$$

$$\phi'' + Sc f \phi' + Sc Sr \theta - \frac{2}{r+1} Sc G \phi^n \quad (10)$$

with the boundary conditions

$$\left. \begin{aligned} f'(\xi) = 1 + \lambda_u f''(0), \quad f(\xi) = S, \quad \theta(\xi) = 1 + \lambda_t \theta'(0), \quad \phi(\xi) = 1 + \lambda_c \phi'(0) \text{ at } \xi = 0, \\ f(\xi) \rightarrow 0, \quad \theta(\xi) \rightarrow 0, \quad \phi(\xi) \rightarrow 0 \text{ as } \xi \rightarrow \infty, \end{aligned} \right\} \quad (11)$$

where

$$\begin{aligned}
 M &= \frac{\sigma B_0^2}{b\rho}, \quad B = \frac{\nu}{bk_p x^{r-1}}, \quad f_c = \frac{c_f x}{\sqrt{k_p}}, \quad \gamma_1 = \frac{G_T}{\text{Re}_t^2}, \quad \gamma_2 = \frac{G_c}{\text{Re}_c^2}, \quad G_T = \frac{g_x \beta_0 (T_f - T_\infty) x^3}{\nu^2}, \\
 G_c &= \frac{g_x \beta_1 (C_f - C_\infty) x^3}{\nu^2}, \quad \text{Re}_x = \frac{U_f x}{\nu}, \quad Pr = \frac{\nu}{\alpha}, \quad D_f = \frac{D_m K_T (C_f - C_\infty)}{C_s C_p (T_f - T_\infty)}, \quad Ec = \frac{(bx^r)^2}{C_p (T_f - T_\infty)}, \\
 R &= \frac{4\sigma * T_\infty^3}{\kappa * K}, \quad Q = \frac{Q_0}{\rho C_p b x^{r-1}}, \quad Sc = \frac{\nu}{D_B}, \quad Sr = \frac{D_m K_T (T_f - T_\infty)}{T_m \nu (C_f - C_\infty)}, \quad K_1 = \frac{(C_f - C_\infty)^{n-1}}{bx^{r-1}}, \\
 \lambda_u &= N \left(\mu_B + \frac{P_y}{\sqrt{2\pi}} \right) r x^{\frac{r-3}{2}} \sqrt{\frac{b(r+1)}{2\nu}}, \quad S = V_0 \sqrt{\frac{2}{bv(r+1)}}, \quad \lambda_t = T_0 \sqrt{\frac{b(r+1)}{2\nu}} x^{\frac{r-1}{2}}, \\
 \lambda_c &= C_0 \sqrt{\frac{b(r+1)}{2\nu}} x^{\frac{r-1}{2}}, \quad \lambda_1 = \frac{\beta_1}{\beta_0} (T_f - T_\infty), \quad \lambda_2 = \frac{\beta_3}{\beta_2} (C_f - C_\infty)
 \end{aligned} \tag{12}$$

The flow characteristics are the quantities of interest because of their applications in engineering and science. These includes the Cf_x (local Skin friction), Nu_x (local Nusselt number) and Sh_x (local Sherwood number), defined as

$$Cf_x = \left(\mu_B + \frac{P_y}{\sqrt{2\pi}} \right) \frac{\left(\frac{\partial u}{\partial y} \right)_{y=0}}{\rho_f u_f^2}, \quad Nu_x = -x \frac{\left(\frac{\partial T}{\partial y} \right)_{y=0}}{(T_f - T_\infty)}, \quad Sh_x = -x \frac{\left(\frac{\partial C}{\partial y} \right)_{y=0}}{(C_w - C_\infty)} \tag{13}$$

Introducing (6) into (12), gives the reduction form of Cf_x , Nu_x and Sh_x respectively, and $\text{Re}_x = \frac{u_f x}{\nu}$ is the Reynold number;

$$Cf_x \sqrt{\frac{2}{1+r}} \text{Re}_x = \left(1 + \frac{1}{\gamma} \right) f' '(0), \quad \frac{Nu_x}{\sqrt{\frac{1+r}{2}} \text{Re}_x} = -\theta' (0), \quad \frac{Sh_x}{\sqrt{\frac{1+r}{2}} \text{Re}_x} = -\phi' (0) \tag{14}$$

2.1 Method of Solution

The solutions of Eqs. (8)-(11) are obtained using the collocation method with MATHEMATICA 11.0 software. Through the mapping: $\eta = \frac{2\xi}{L} - 1$. The interval $[0, \infty]$ truncated by $[0, L]$ is transformed to the interval $[-1, 1]$, which is the Legendre polynomial definition (Olagunju *et al.* [25], Aysun and Salih [5]). The approximate solution of $f(\xi)$, $\theta(\xi)$ and $\phi(\xi)$ is given as;

$$\begin{aligned}
 f(\xi) &\approx f_N(\eta) = \sum_{j=0}^N a_j p_j(\eta), \quad \theta(\xi) \approx \theta_N(\eta) = \sum_{j=0}^N b_j p_j(\eta), \\
 \phi(\xi) &\approx \phi_N(\eta) = \sum_{j=0}^N c_j p_j(\eta), \quad \text{for } j = 0, 1, \dots, N,
 \end{aligned} \tag{15}$$

where $p_j(j)$ is a finite series sum and a_j , b_j , and c_j are determined using the generated equations and the solution for the flow fields and characteristics is obtained. The respective trial solutions in equation (14) is then substituted into the governing ordinary differential equations (8)-(10) to produce residue R_f , R_θ , and R_ϕ . The respective residue is then minimised to close to zero. Employing the collocation method, the respective unknown functions a_j , b_j , and c_j are then obtained and substituted into the trial function as the required solution for the respective profile. The polynomials are orthogonal, providing good stability and spectral accuracy for smooth problems with fewer basis functions.

3. Results and Discussion

This study focuses on establishing the influence of cross-diffusion, stretching sheet index and nth-order chemical reactions on the flow of MHD-dissipative Casson fluid through a Forchheimer permeable medium with quadratic convection over an inclined surface. The key parameters that influence the numerical solutions for the flow distribution were thoroughly explored. Table 1 presents a comparative analysis of different values of the stretching sheet index parameter (r), on the skin friction and Nusselt number when other parameters are set as $M = Ec = B = Fc = \lambda u = \lambda t = \lambda c = Df = Sc = \beta = Sr = Q = \gamma_1 = \gamma_2 = \lambda_1 = \lambda_2 = G = S = R = 0$, $Pr = n = 1$, and $\gamma = \infty$. The results obtained show good agreement with the earlier works of Ullah *et al.* [37] and Shankar *et al.* [32], thereby reinforcing the motivation for the objectives of this study. To obtain the numerical results for flow distributions and characteristics, these

parameter values are used: $M = Ec = B = Fc = \lambda u = \gamma_1 = \gamma_2 = S = 0.5$, $Pr = 0.71$, $r = Fc = Sr = \lambda t = \lambda c = R = \lambda_1 = \lambda_2 = \alpha = n = G = 1.0$, $Df = 0.1$, $Sc = 0.6$, $Q = 0.3$ and $\beta = \frac{\pi}{6}$.

Table 2 presents the influence of the chosen parameters on the flow characteristics for $Pr = 1$. Skin friction increased with Pr , r , B , Fc , β , M , and Sr , whereas it decreased with higher values of Df , Ec , and n . Lower wall temperatures were observed as M , r , Ec , B , Fc , Df , Sr , and β increased, whereas higher wall temperatures were associated with increasing Pr and n . The behaviour of the Sherwood number generally contrasts that of the Nusselt number, except in the case of Pr , where both show similar trends.

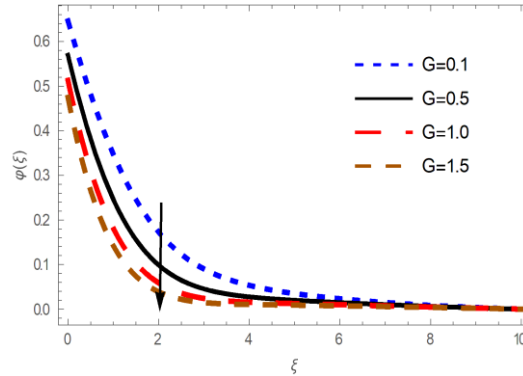


Figure 2. G impact on concentration

The influence of chemical reaction (G) is illustrated in Figure 2. The concentration profiles decrease as the chemical reaction rate increases. Physically, this results in a thinner solutal boundary layer owing to corrosive effects, leading to an elevated mass-transfer rate. Figure 3 shows the impact of the Dufour number (Df) on the flow fields. The velocity and temperature accelerated with Df and reduced the concentration profile. This is attributed to the Dufour effect, which enhances the thermal energy and aids fluid motion. The impact of the stretching sheet index (r) on the flow distribution is shown in Figure 4. An increase in r resulted in a decrease in the velocity and temperature, whereas the concentration increased. Physically, the momentum boundary layer compresses with increasing r . Figure 5 depicts the effect of the Eckert number (Ec) on the flow fields. As Ec increased, both the velocity and temperature increased owing to viscous dissipation, which converted kinetic energy into thermal energy, whereas the concentration decreased. The influence of the Forchheimer parameter (Fc) is shown in Figure 6. The velocity slowdown and temperature profile increased with increasing Fc , indicating the resistive effects of the previous medium.

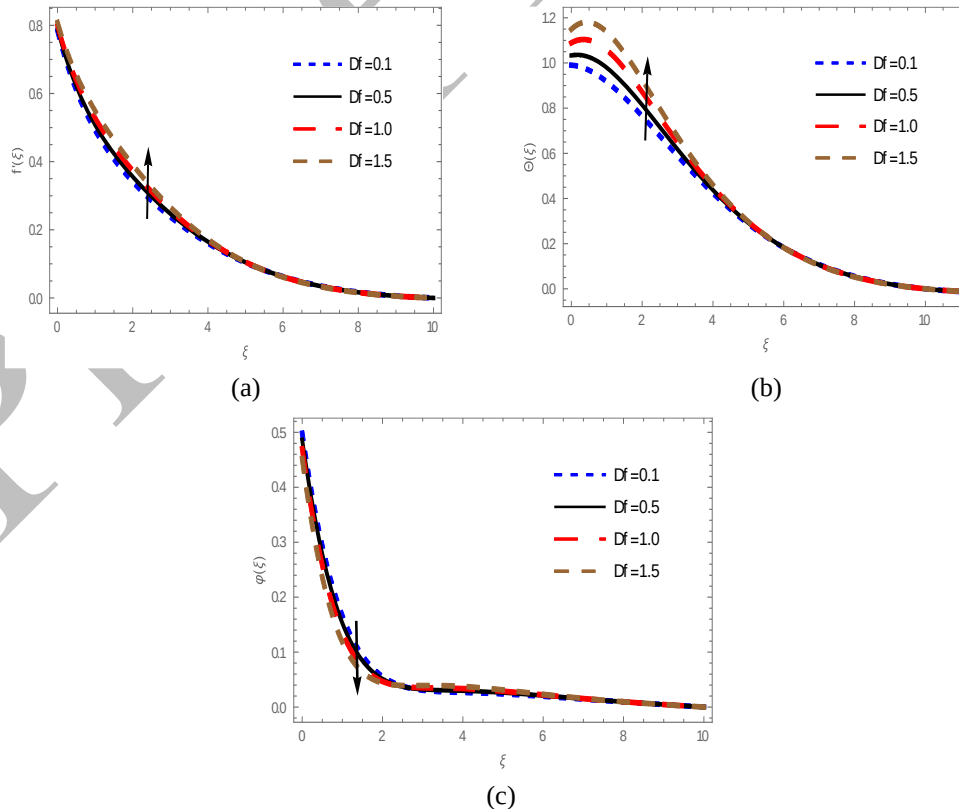


Figure 3. Df impact on a) velocity; (b) temperature; (c) concentration

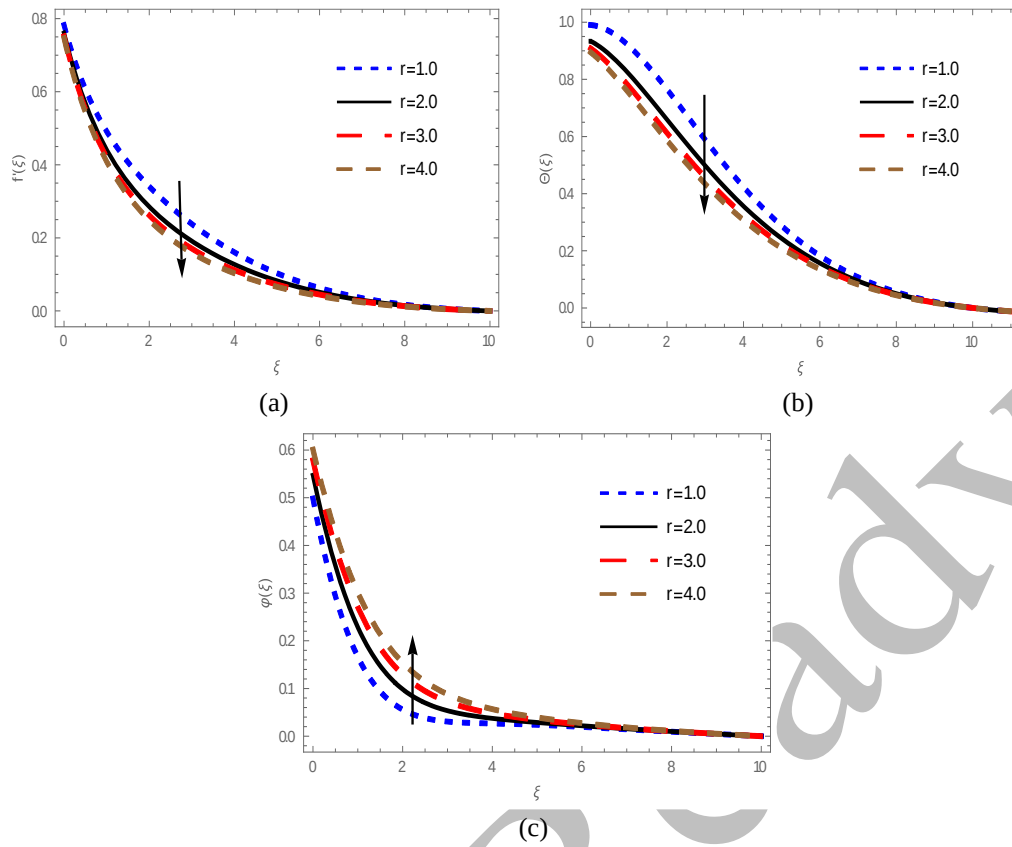


Figure 4. r impact on (a) velocity; (b) temperature; (c) concentrations

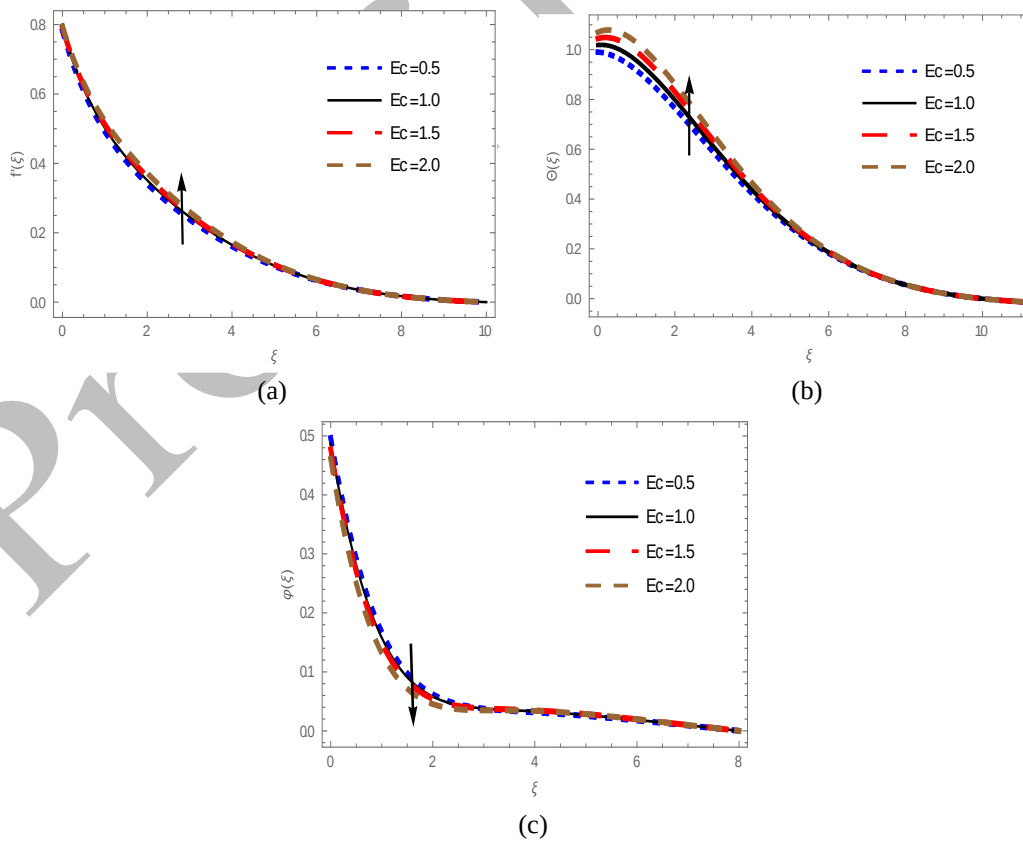


Figure 5. Ec impact on (a) velocity; (b) temperature; (c) concentrations

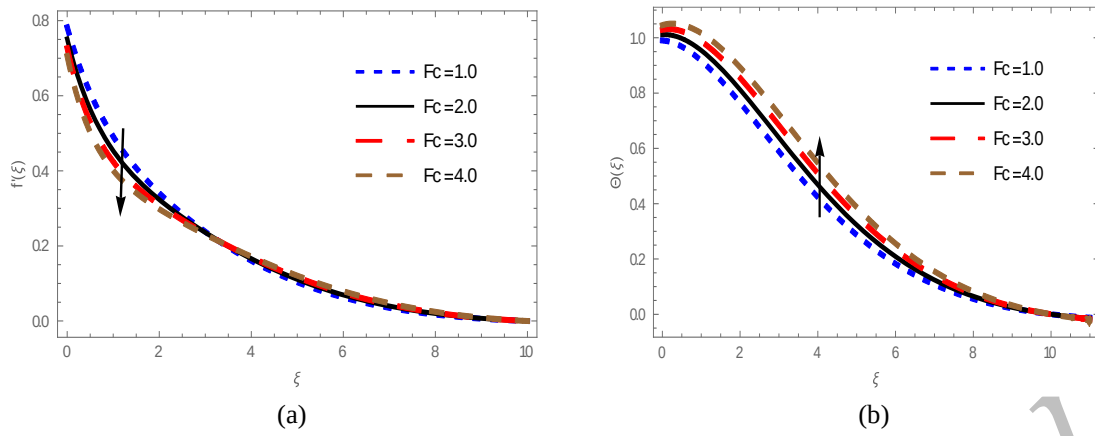


Figure 6. F_c impact on (a) velocity; (b) temperature

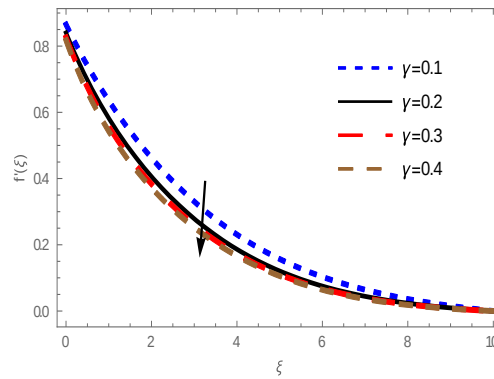


Figure 7. Impact on velocity

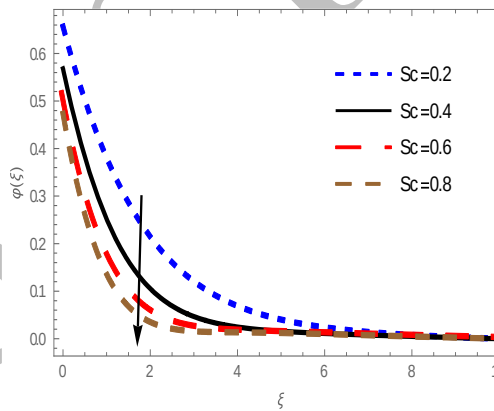


Figure 8. Sc impact on concentration

Figure 7 shows the effect of Casson (γ) on the velocity. Higher values of γ reduce the fluid speed owing to a decrease in the yield stress, which compresses the momentum boundary layer. The Schmidt number (Sc) reaction on the concentration is shown in Figure 8. The concentration decreased with increasing Sc , implying a reduction in the mass diffusivity. Figure 9 illustrates the effect of inclination angle (β) on the velocity and temperature. The velocity decreases as β increases, while the temperature increases, likely due to the enhanced influence of gravity and magnetic field alignment. The magnetic (M) influence is shown in Figure 10. The velocity slowdown and temperature rise with increasing M . Physically, a larger Forchheimer parameter amplifies inertial drag in the porous medium (slowing the flow and converting more mechanical energy into heat), higher Casson parameter values (γ) alter the fluid's yield/viscous resistance and compress the momentum boundary layer (reducing velocity), an increased Schmidt number lowers mass diffusivity and thins the concentration boundary layer (decreasing concentration), a greater inclination angle raises the effective body-force/magnetic alignment that suppresses motion while enhancing thermal stratification (velocity decreases, temperature increases), and a stronger magnetic parameter strengthens the Lorentz force and Joule dissipation which damps the flow and heats the fluid.

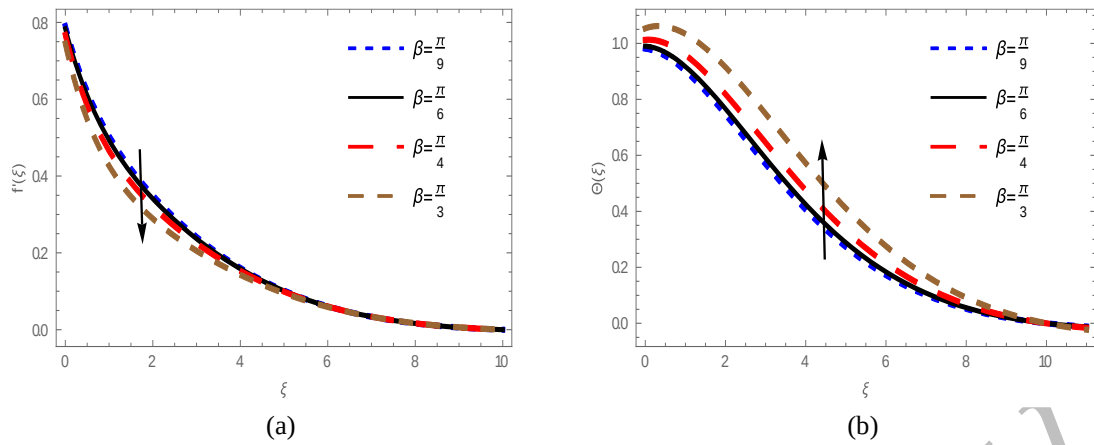
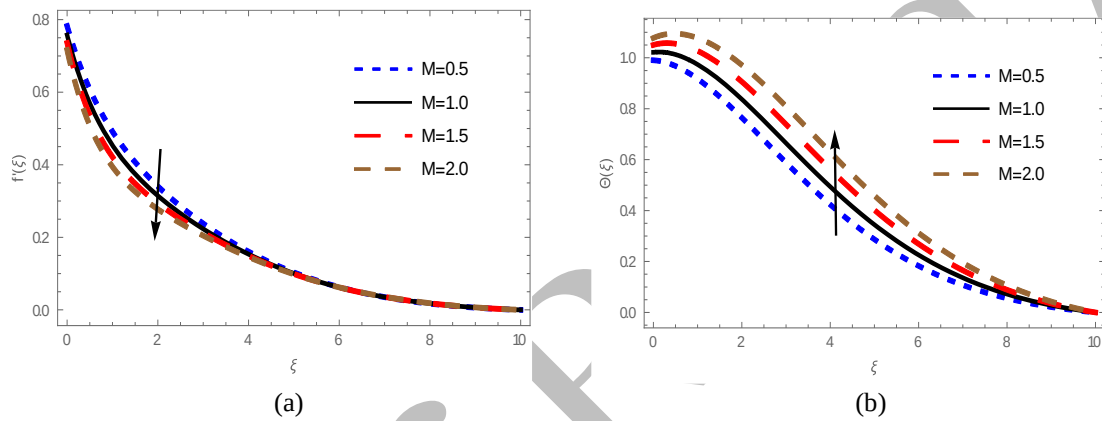

Figure 9. β impact on (a) velocity; (b) temperature

Figure 10. M impact on (a) velocity; (b) temperature

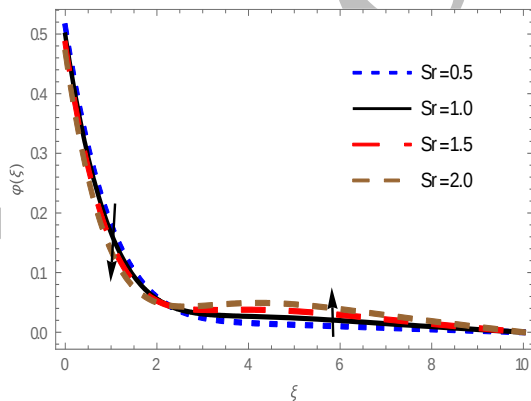
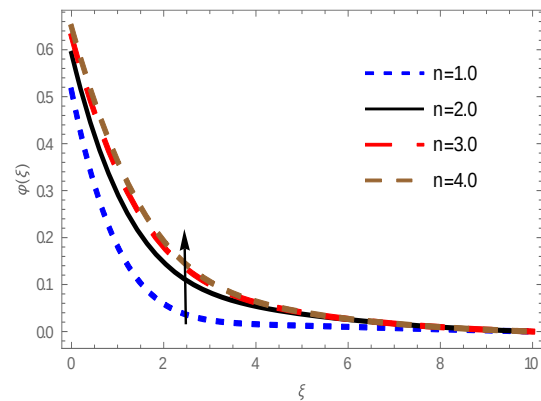
Figure 11 illustrates the reaction of the Soret number (Sr) with concentration. A decrease in concentration was observed up to a specific point ($\xi=2$), after which it increased, indicating a nonlinear response owing to thermal diffusion. The impact of the order (n) of the chemical reaction on the concentration is shown in Figure 12. It was found that the concentration increased with n . The effects of the thermal linear parameter (γ_1) on the velocity and temperature are shown in Figure 13. An increase in γ_1 enhances the velocity but reduces the temperature, whereas the solutal linear parameter (γ_2) shows a contrast, as depicted in Figure 14. Finally, Figures 15 shows the effect of the thermal quadratic convection (λ_1) on the velocity and temperature. An increase in the value of λ_1 enhanced the velocity profile and reduced the temperature profile, demonstrating the dual role of enhanced convective heating and flow acceleration. Physically, increasing the Soret number initially drives species away from the wall (reducing the concentration up to $\xi \approx 2$) before nonlinear thermal diffusion and convection redistribute it and cause a rise; a higher reaction order n weakens net depletion in low-concentration regions, increasing the overall concentration; the thermal linear parameter (γ_1) strengthens thermal buoyancy, accelerating the flow but thinning the thermal boundary layer (lower temperature), while the solutal linear parameter (γ_2) induces the opposite behaviour; and stronger thermal quadratic convection (λ_1) enhances nonlinear advection, boosting the velocity while carrying heat away more efficiently and thus lowering the temperature.

Table 1. Results comparison for $-f'(0)$ and $-\theta'(0)$ with previous studies

r	Ullah et al. [37]		Bejawada et al. [35]		Present work (Analysis)	
	$-f'(0)$	$-\theta'(0)$	$-f'(0)$	$-\theta'(0)$	$-f'(0)$	$-\theta'(0)$
0.0	0.6272	—	0.627556	—	0.627631	—
0.2	0.7668	0.6102	0.766838	0.610203	0.766999	0.610375
0.5	0.5896	0.5949	0.589545	0.595204	0.589611	0.595383
1.0	1.0	—	1.000001	—	1.000060	0.582228
3.0	1.1486	0.5647	1.148594	0.564670	1.148650	0.565034
10	1.2349	0.5549	1.234876	0.554890	1.234980	0.555339
100	1.2768	—	1.276775	—	1.276820	0.550710

Table 2. Computational results of the flow characteristics when $Pr = 1$, for key parameters

Parameter	Value	$-\left(1 + \frac{1}{\gamma}\right)f'(0)$	$-\theta'(0)$	$-\phi'(0)$
M	0.5	0.523710	0.106536	0.529307
	1.0	0.573080	0.089894	0.532190
	1.5	0.616531	0.063214	0.534888
Pr	0.3	0.506830	0.092868	0.524570
	0.5	0.513861	0.095869	0.527942
	0.7	0.523710	0.106536	0.529870
Ec	0.5	0.523710	0.106528	0.529307
	1.0	0.511733	0.066170	0.531729
	1.5	0.500009	0.028066	0.532262
r	1.0	0.523710	0.106536	0.529307
	2.0	0.554764	0.154467	0.483911
	3.0	0.569901	0.175319	0.455589
B	0.5	0.523710	0.106536	0.529307
	1.0	0.573088	0.089894	0.532190
	1.5	0.616531	0.063214	0.534888
f_c	1.0	0.523710	0.106536	0.529307
	2.0	0.576525	0.093653	0.530480
	3.0	0.619849	0.082815	0.531582
D_f	0.1	0.523710	0.106536	0.529307
	0.5	0.507860	0.053746	0.544697
	1.0	0.485714	0.015786	0.565660
B	$\pi/6$	0.523710	0.106536	0.529307
	$\pi/4$	0.553274	0.094567	0.531296
	$\pi/3$	0.594678	0.066397	0.534785
S_r	0.1	0.522395	0.104891	0.515833
	0.5	0.522495	0.105669	0.521855
	1.0	0.523710	0.106536	0.529307
n	1.0	0.523710	0.106536	0.529307
	3.0	0.503420	0.121583	0.410667
	5.0	0.499140	0.123762	0.385404


Figure 11. Effect of Sr on concentration

Figure 12. n impact on concentration

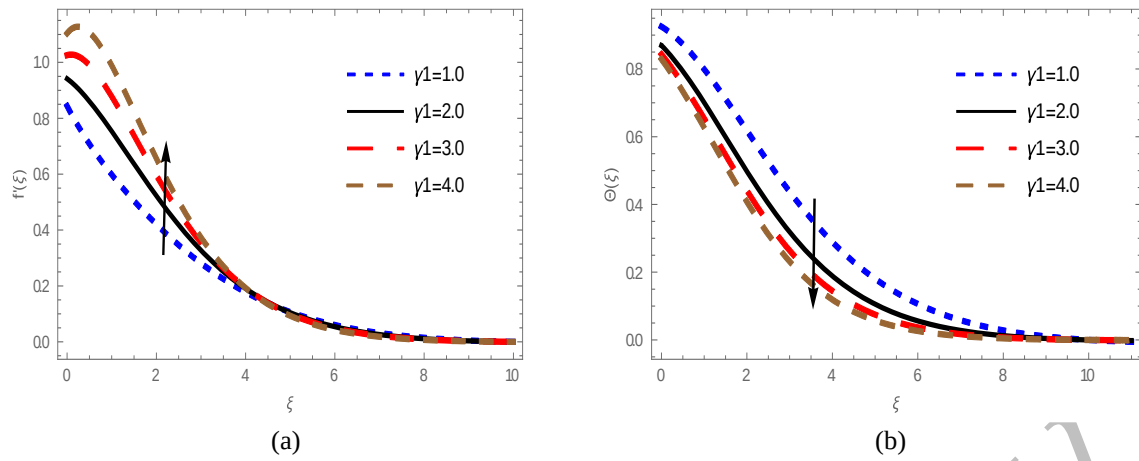


Figure 13. γ_1 impact on (a) velocity; (b) temperature

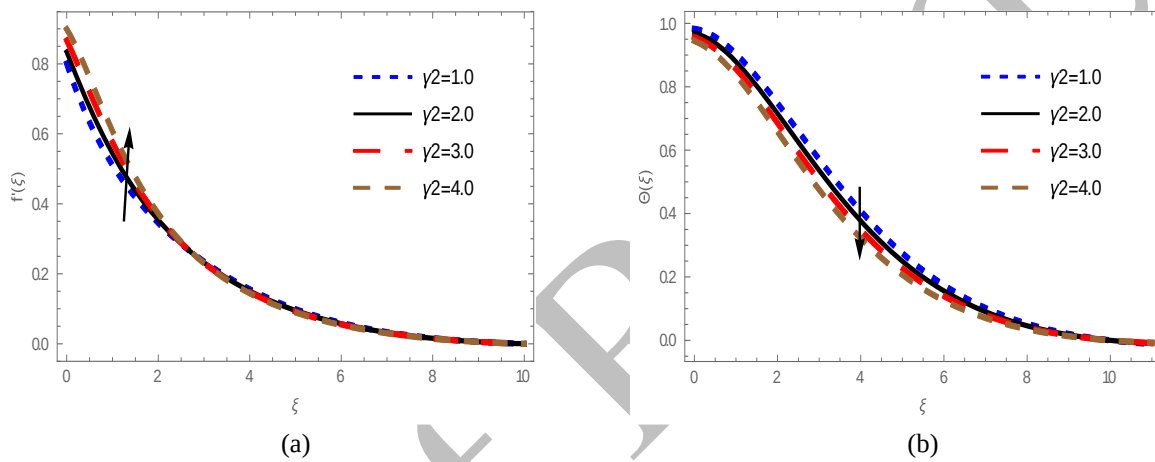


Figure 14. γ_2 impact on (a) velocity; (b) temperature

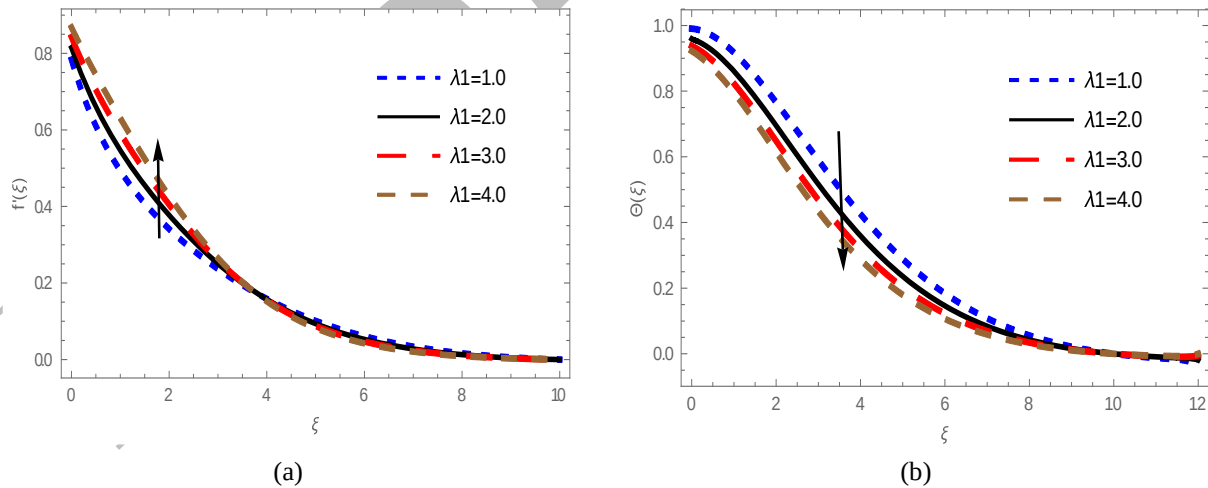


Figure 15. λ_1 impact on (a) velocity; (b) temperature

4. Conclusions

This study developed a unified mathematical framework to investigate the magnetohydrodynamic flow of a Casson fluid through a nonlinear porous medium in the presence of cross-diffusion (Soret-Dufour), quadratic convection, and higher-order chemical reactions. Using similarity transformations, the governing equations were reduced to a system of coupled ordinary differential equations and solved numerically using the Legendre collocation method. The velocity field increased with stronger thermal diffusion and viscous dissipation effects (D_f and E_c), indicating that internal heat generation promotes fluid mobility. However, velocity decreases under stronger porous drag, magnetic damping, and non-Newtonian resistance (higher r , F_c , γ , M), confirming that Lorentz forces and nonlinear porous inertia significantly

suppress momentum transport. This implies that magnetic and porous effects can be used as effective flow control mechanisms for conducting non-Newtonian fluids. The temperature field is strongly influenced by the energy coupling mechanisms. Parameters associated with internal heating and thermal diffusion (D_f , E_c) elevate the temperature, whereas enhanced reaction rates and porous resistance reduce it. This demonstrates a competing interaction between the thermal generation and thermal dissipation mechanisms, which is crucial for optimising thermal regulation in reactive porous systems and energy devices. The concentration transport is primarily governed by mass thermal coupling. The enhancement of the concentration with an increasing Soret number confirms the importance of thermodiffusion in the accumulation of species near the wall. Conversely, a higher Schmidt number and chemical generation reduce the concentration owing to stronger molecular diffusion resistance and reaction consumption. This provides insights into species control in catalytic and chemically reactive flows. Increasing the nonlinear porous resistance parameter reduces the wall shear stress and surface temperature, while elevating the near-wall concentration levels. This suggests that porous media can simultaneously reduce the thermal load and enhance reactant accumulation, which is advantageous in applications such as catalytic reactors, thermal insulation layers, and chemically active coatings. Overall, the present findings provide meaningful physical insights into the coupled interaction of magnetic forces, non-Newtonian rheology, and cross-diffusion effects in reactive porous flows. The results offer useful design guidance for thermal management systems, porous heat exchangers, electroconductive cooling devices, and chemically reactive transport processes, where simultaneous control of momentum, heat, and mass transfer is required.

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Declaration of Competing Interest

The authors declare no conflicts of interest.

CRedit Authorship Contribution Statement

Jos U. Abubakar: Conceptualization; Analysis; Visualization; Supervision; Writing – review & revise
 Timothy L. Oyekunle: Conceptualization; Analysis; Visualization; Supervision; Writing – review & revise
 Mojeed T. Akolade: Methodology; Data curation; Writing - original draft; Resources
 Samson A. Agunbiade: Methodology; Data curation; Writing - original draft; Resources
 Blessing R. Oladeji: Validation; Writing – review & revise; Resources
 Thomas O. Olotu: Validation; Writing – review & revise; Resources

Availability of the Data and Materials

The data used to support the findings of this study are included in this article.

Ethical Declarations

No artificial intelligence tools were used to prepare this manuscript. All the content was developed manually by the authors. This study did not involve human participants or animal subjects. Therefore, ethical approval was not required.

Generative Artificial Intelligence Declarations

The authors claim that artificial intelligence-assisted technologies, such as generative AI, were not used to generate content, ideas, or theories. We have used AI to enhance readability and refine the language. This was used with extreme human control and supervision. The authors take full responsibility for reviewing and approving the content.

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