DATA ANALYTICS AND APPLIED MATHEMATICS
E-ISSN: 2773-4854
VOLUME 5, ISSUE 2, 2024, 1 – 16
DOI: https://doi.org/10.15282/daam.v5i2.11030

RESEARCH ARTICLE

Analytical Hierarchy Process based on intuitionistic interval approximation of trapezoidal intuitionistic fuzzy numbers

Nik Muhammad Farhan Hakim Nik Badrul Alam[1,*], Ku Muhammad Naim Ku Khalif[2,3] and Nor Izzati Jaini[2]

[1]College of Computing, Informatics and Mathematics, Universiti Teknologi MARA Pahang, 26400 Bandar Tun Abdul Razak Jengka, Pahang, Malaysia
[2]Centre for Mathematical Sciences, Universiti Malaysia Pahang Al-Sultan Abdullah, Lebuh Persiaran Tun Khalil Yaakob, 26300 Kuantan, Pahang, Malaysia
[3]Centre of Excellence for Artificial Intelligence and Data Science, Universiti Malaysia Pahang Al-Sultan Abdullah, Gambang, Malaysia

ABSTRACT - The fuzzy Analytical Hierarchy Process (AHP) is a method in fuzzy decision-making, which helps decision makers make the best choice of alternatives when dealing with uncertainty. The objective of this paper is to propose a conversion method of trapezoidal intuitionistic fuzzy numbers (TrIFN) into interval-valued intuitionistic fuzzy numbers (IVIFN), followed by a new fuzzy AHP based on the proposed approximation. The linguistic judgments of decision makers are converted into TrIFN before being transformed into IVIFN using the nearest weighted intuitionistic interval approximation (NWIIA). The proposed model is implemented in supplier selection problems by taking into account five criteria comprising cost, quality, service, cycle time, and reputation. The results showed that the proposed model simplifies the fuzzy AHP using TrIFN without affecting the consistency of the solutions from the existing method, thus supporting the fact that TrIFN has a better capability of processing ill-defined quantities. This is because TrIFN entails a generalization of the triangular intuitionistic fuzzy number (TIFN), making it a better form of representation of linguistic variables for reducing the inexactness.

ARTICLE HISTORY
Received : 17th July 2024
Revised : 23rd Aug 2024
Accepted : 15th Sept 2024
Published : 30th Sept 2024

KEYWORDS
Trapezoidal intuitionistic fuzzy numbers
Analytical Hierarchy Process
Supplier selection
Interval-valued intuitionistic fuzzy numbers
Intuitionistic interval approximation

1. INTRODUCTION

Making the best decision is rather difficult when there is a variety of alternatives available. Decision makers may be influenced by the surrounding factors, which may lead to minor errors when selecting the best alternative, and this issue is very common in multi-criteria decision-making problems. Besides, imperfect information may lead to imprecise and inaccurate judgment among decision-makers due to their restricted competence, intricate alternatives, and psychological biases [1].

A pairwise comparison matrix is a cooperative tool for decision-makers to provide a measure of the relative importance of one element to the other [2]. The Analytical Hierarchy Process (AHP) was developed by Saaty [3] as a decision-making method in which pairwise comparisons are summarized in the form of a judgment matrix using additive weightage. Decision makers then use the matrix to make judgments on the level of importance of one criterion to the other. There are three main parts involved in the hierarchical structure of the AHP: the goal, the criteria, and the alternatives.

However, despite the applicability, easiness, and consistency of AHP, the use of crisp numbers in the traditional AHP is insufficient for dealing with real-world problems [4-7]. van Laarhoven and Pedrycz [8] modified Saaty's AHP method by using triangular fuzzy numbers in the comparison matrices instead of crisp numbers. The logarithmic regression method was employed to obtain fuzzy weights via their method. Since experts' perceptions are the main input for the AHP and the accuracy of data may be interfered with due to uncertainty and doubt when making the assessment, the use of the fuzzy AHP is more helpful than the classical AHP [9]. The use of fuzzy numbers in the method has also made it able to cater to the uncertainty and vagueness in the decision-making process, thus highlighting its ability to better express human perception and subjectivity [10].

Buckley [11] utilized the geometric mean method to derive the fuzzy weights of criteria in his fuzzy AHP model. He illustrated the implementation of the model using triangular fuzzy numbers. Later, the extent analysis was employed by Chang [12] to calculate the weights of criteria in the fuzzy AHP. However, it should be noted that there is a possibility of obtaining zero weight when two fuzzy numbers are not intersecting, which may lead to improper derivation of criteria weights [13]. In addition, the concept of intuitionistic fuzzy set (IFS), which was introduced by Atanassov [14] is a generalization of a fuzzy set. Generally, the IFS incorporates both membership and non-membership grades to indicate the degree of belongingness and non-belongingness, respectively, of the elements of the set. Likewise, the IFS has been widely implemented in decision-making, image processing, clustering, and time series forecasting, in which its advantages of handling uncertainty over classical fuzzy sets were highlighted.

***CORRESPONDING AUTHOR | Nik Muhammad Farhan Hakim Nik Badrul Alam** | ✉ farhanhakim@uitm.edu.my

Xu and Liao [15] proposed an intuitionistic fuzzy AHP in which the decision makers' preferences constitute intuitionistic fuzzy values in the form of symmetric triangular intuitionistic fuzzy numbers. In the literature, there are many practitioners who developed the intuitionistic fuzzy AHP using intuitionistic fuzzy values, with different applications in drug risk assessment [16], global supplier selection for manufacturing firms [17], and the best college selection [18].

By generalizing the IFS, intuitionistic fuzzy numbers are better capable of defining uncertain human knowledge. Similar to classical fuzzy numbers, some popular shapes of intuitionistic fuzzy numbers are triangular and trapezoidal, and they have been used to develop many fuzzy decision-making methods. Triangular intuitionistic fuzzy numbers (TIFN) have also been used to develop the intuitionistic fuzzy AHP. The developed methods were then applied to the environmental decision-making problem [19], sustainable energy planning [20], evaluation of hospital web services [21], and reverse logistics outsourcing [22], among others.

In addition to intuitionistic fuzzy numbers and intuitionistic fuzzy values, some other studies have also used interval-valued intuitionistic fuzzy numbers (IVIFN) due to their ability to model and solve complex problems [23] and yield more precise results than IFS [24]. IVIFN is an extension of the interval-valued intuitionistic fuzzy set (IVIFS) introduced by Atanassov and Gargov [25]. Some IVIFN applications in the AHP method include the safety warning rating of hot and humid environments [26], corridor selection for locating autonomous vehicles [27], and assessment of digital service quality [28]. Verma and Chandra [29] have also implemented IVIFN-based AHP to assess the security of the quantitative parameters of Fog-Internet of Things (IoT).

Using triangular fuzzy numbers, Kaur [30] improved the fuzzy AHP method to solve the supplier selection problem. Nirmala and Uthra [31] then modified Kaur's work by using the Nearest Weighted Intuitionistic Interval Approximation (NWIIA) approach to convert TIFN into IVIFN. The pairwise comparisons in the form of TIFN were converted into IVIFN using the NWIIA function, which simplified Kaur's AHP method. Since it is a known fact that trapezoidal fuzzy numbers are the generalization of triangular fuzzy numbers, this study aims to extend the conversion method of trapezoidal intuitionistic fuzzy numbers (TrIFN) into IVIFN. Hence, the research objectives are as follows:

(a) to derive the NWIIA function based on the α- and β-cuts for converting TrIFN into IVIFN. This function allows for the presentation of fuzzy numbers into an interval form, which will ease the calculation and exhibit a better capability of handling linguistic uncertainty under a complex environment [32].
(b) to develop a new fuzzy AHP model using the conversion of TrIFN into IVIFN. The model employs the NWIIA function for the conversion purpose. Furthermore, the score functions are calculated to defuzzify the IVIFN and, finally, the priority weights can be calculated.
(c) to validate the proposed AHP model by solving the supplier selection problem adopted from [30]. Later, the final ranking of alternatives is compared to the results obtained in [31] in order to highlight the validity and practicality of the proposed fuzzy AHP model.

The organization of this research paper is based on several sections pertinent to this study. Section 1 provides the introduction, while Section 2 reviews some preliminaries related to the method used and Section 3 proposes a new AHP method based on TrIFN. Subsequently, Section 4 illustrates the proposed model using the supplier selection problem and Section 5 presents the results and discussion. Finally, Section 6 concludes this paper.

2. PRELIMINARIES

In this section, some preliminaries related to the interval-valued intuitionistic fuzzy set (IVIFS), interval-valued intuitionistic fuzzy number (IVIFN), and the score function are reviewed. IVIFS denotes a generalization of an intuitionistic fuzzy set [25]. The definition of IVIFS is given below.

Definition 1 [25]
Let X be an ordinary finite non-empty set. An IVIFS $\tilde{\alpha}$ on X is given by

$$\tilde{\alpha}=\left\{\left\langle x,\mu_{\tilde{\alpha}}(x),\nu_{\tilde{\alpha}}(x)\right\rangle \mid x\in X\right\}, \tag{1}$$

where $\mu_{\tilde{\alpha}}(x)\subset[0,1]$ and $\nu_{\tilde{\alpha}}(x)\subset[0,1]$ are intervals satisfying the condition

$$0\leq \sup\mu_{\tilde{\alpha}}(x)+\sup\nu_{\tilde{\alpha}}(x)\leq 1 \tag{2}$$

for all $x\in X$.

From the above definition, the concept is extended to IVIFN, denoted by a pair $\left(\mu_{\tilde{\alpha}}(x),\nu_{\tilde{\alpha}}(x)\right)$. If $\mu_{\tilde{\alpha}}(x)=\left[\tilde{\rho}_{\tilde{\alpha}}^{-}(x),\tilde{\rho}_{\tilde{\alpha}}^{+}(x)\right]=\left[\tilde{\rho}_{\tilde{\alpha}}^{-},\tilde{\rho}_{\tilde{\alpha}}^{+}\right]$ and $\nu_{\tilde{\alpha}}(x)=\left[\tilde{\kappa}_{\tilde{\alpha}}^{-}(x),\tilde{\kappa}_{\tilde{\alpha}}^{+}(x)\right]=\left[\tilde{\kappa}_{\tilde{\alpha}}^{-},\tilde{\kappa}_{\tilde{\alpha}}^{+}\right]$, then IVIFN can be conveniently denoted as [26] and [27]. The $\tilde{\rho}_{\tilde{\alpha}}^{-}$ and $\tilde{\rho}_{\tilde{\alpha}}^{+}$ are the lower and upper limit of the membership function of the IVIFN, respectively, meanwhile $\tilde{\kappa}_{\tilde{\alpha}}^{-}$ and $\tilde{\kappa}_{\tilde{\alpha}}^{+}$ are the lower and upper limit of the non-membership function of the IVIFN, respectively.

$$\tilde{\alpha}=\left(\left[\tilde{\rho}_{\tilde{\alpha}}^{-},\tilde{\rho}_{\tilde{\alpha}}^{+}\right],\left[\tilde{\kappa}_{\tilde{\alpha}}^{-},\tilde{\kappa}_{\tilde{\alpha}}^{+}\right]\right). \tag{3}$$

The definition of an interval-valued multiplicative intuitionistic fuzzy set is defined below.

Definition 2 [31]
Let X be a fixed set. An interval-valued multiplicative intuitionistic fuzzy set is defined as

$$D=\left\{\left\langle x,\tilde{\rho}_D(x),\tilde{\kappa}_D(x)\right\rangle \mid x\in X\right\}, \tag{4}$$

where $\tilde{\rho}_D(x)=\left[\tilde{\rho}_D{}^-(x),\tilde{\rho}_D{}^+(x)\right]$ and $\tilde{\kappa}_D(x)=\left[\tilde{\kappa}_D{}^-(x),\tilde{\kappa}_D{}^+(x)\right]$ are the membership degree interval and non-membership degree interval of each element x, with the conditions that $\frac{1}{9}\le\tilde{\rho}_D{}^-(x),\tilde{\rho}_D{}^+(x)\le 9$ and $\frac{1}{9}\le\tilde{\kappa}_D{}^-(x),\tilde{\kappa}_D{}^+(x)\le 9.$

Subsequently, the conversion procedure of triangular intuitionistic fuzzy numbers (TIFN) into IVIFN using the nearest weighted intuitionistic interval approximation (NWIIA) [31] is presented.

Definition 3 [31]
Let $\underline{f}:[0,1]\rightarrow\mathbb{R}$ and $\overline{f}:[0,1]\rightarrow\mathbb{R}$ where $\underline{f}$ and $\overline{f}$ are non-negative, uniformly increasing functions satisfying

$$\int_0^1\underline{f}(\alpha)d\alpha=\int_0^1\overline{f}(\beta)d\beta=1. \tag{5}$$

where $\alpha\in[0,1]$ and $\beta\in[0,1]$. The functions $\underline{f}$ and $\overline{f}$ may be selected according to decision-makers but should satisfy (5). If $A=\left\langle\left(a_1,a_2,a_3;a_1',a_2,a_3'\right)\right\rangle$ is a TIFN and $\underline{f}(\alpha)=n\alpha^{n-1}$ and $\overline{f}(\beta)=n\beta^{n-1}$ are weighting functions for $n\ge 0$, then the IVIFN using NWIIA is given by

$$\tilde{\alpha}=\left(\left[\frac{a_1+na_2}{n+1},\frac{a_3+na_2}{n+1}\right],\left[\frac{a_1'n+a_2}{n+1},\frac{a_3'n+a_2}{n+1}\right]\right). \tag{6}$$

Additionally, a score function is used to rank the fuzzy numbers. Many score functions of IVIFN were derived in the literature, and their advantages and disadvantages have been discussed. In this paper, two score functions of IVIFN are reviewed.

Definition 4 [31]
Let $\tilde{\alpha}=\left(\left[\tilde{\rho}^-,\tilde{\rho}^+\right],\left[\tilde{\kappa}^-,\tilde{\kappa}^+\right]\right)$ be the interval-valued multiplicative intuitionistic fuzzy number. The score function of $\tilde{\alpha}$ is defined as

$$S(\tilde{\alpha})=\sqrt{\frac{\tilde{\rho}^-\tilde{\rho}^+}{\tilde{\kappa}^-\tilde{\kappa}^+}}. \tag{7}$$

Definition 5 [33]
Let $\tilde{\alpha}=\left(\left[\tilde{\rho}^-,\tilde{\rho}^+\right],\left[\tilde{\kappa}^-,\tilde{\kappa}^+\right]\right)$ be the interval-valued intuitionistic fuzzy number. The score function of $\tilde{\alpha}$ is defined as

$$S(\tilde{\alpha})=\frac{\tilde{\kappa}^++\tilde{\kappa}^--\tilde{\rho}^+-\tilde{\rho}^-}{2}+\frac{\tilde{\rho}^-+\tilde{\rho}^++2(\tilde{\rho}^-\tilde{\rho}^+-\tilde{\kappa}^-\tilde{\kappa}^+)}{\tilde{\rho}^-+\tilde{\rho}^++\tilde{\kappa}^-+\tilde{\kappa}^+}. \tag{8}$$

Next, the definition of a fuzzy number is reviewed. The fuzzy number is typically an extension of a fuzzy set, which has the capability of describing human perception and subjectivity [10]. The most popular shapes of fuzzy numbers are triangular and trapezoidal. However, trapezoidal intuitionistic fuzzy numbers (TrIFN) are used in the present paper due to their better capability of handling ill-known quantities than triangular fuzzy numbers (TIFN). Accordingly, TrIFN is defined as follows:

Definition 6 [34]
A TrIFN, $\tilde{A}=\left\langle\left(a_1,a_2,a_3,a_4\right)\left(a_1',a_2,a_3,a_4'\right)\right\rangle$, is characterized by the membership function,

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1} & , \ a_1 \le x < a_2 \\ 1 & , \ a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & , \ a_3 < x \le a_4 \\ 0 & , \ \text{otherwise}\end{cases}, \tag{9}$$

and non-membership function,

$$\nu_{\tilde{A}}(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'} & , \ a_1' \le x < a_2 \\ 0 & , \ a_2 \le x \le a_3 \\ \dfrac{x-a_3}{a_4'-a_3} & , \ a_3 < x \le a_4' \\ 1 & , \ \text{otherwise.}\end{cases}. \tag{10}$$

The arithmetic properties of TrIFN are given in the following definition:

Definition 7 [35]
Let $\tilde{A}$ and $\tilde{B}$ denote two TrIFN, defined by $\tilde{A}=\left\langle (a_1,a_2,a_3,a_4)\left(a_1',a_2,a_3,a_4'\right)\right\rangle$ and $\tilde{B}=\left\langle (b_1,b_2,b_3,b_4)\left(b_1',b_2,b_3,b_4'\right)\right\rangle$.
Then

(i) $\tilde{A}+\tilde{B}=\left\langle (a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)\left(a_1'+b_1',a_2+b_2,a_3+b_3,a_4'+b_4'\right)\right\rangle$

(ii) $\tilde{A}-\tilde{B}=\left\langle (a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1)\left(a_1'-b_4',a_2-b_3,a_3-b_2,a_4'-b_1'\right)\right\rangle$

(iii) $\lambda\tilde{A}=\left\langle (\lambda a_1,\lambda a_2,\lambda a_3,\lambda a_4)\left(\lambda a_1',\lambda a_2,\lambda a_3,\lambda a_4'\right)\right\rangle$ if $\lambda>0$

(iv) $\tilde{A}\otimes\tilde{B}=\left\langle (a_1b_1,a_2b_2,a_3b_3,a_4b_4)\left(a_1'b_1',a_2b_2,a_3b_3,a_4'b_4'\right)\right\rangle$

(v) $\tilde{A}\div\tilde{B}=\left\langle \left(\dfrac{a_1}{b_4},\dfrac{a_2}{b_3},\dfrac{a_3}{b_2},\dfrac{a_4}{b_1}\right)\left(\dfrac{a_1'}{b_4'},\dfrac{a_2}{b_3},\dfrac{a_3}{b_2},\dfrac{a_4'}{b_1'}\right)\right\rangle$.

3. PROPOSED METHOD

In this section, the proposed new method of fuzzy Analytical Hierarchy Process (AHP) using trapezoidal intuitionistic fuzzy numbers (TrIFN) is presented. TrIFN was converted into interval-valued intuitionistic fuzzy numbers (IVIFN) before calculating the score function. The conversion method of TrIFN into IVIFN is first proposed, followed by the AHP procedure.

3.1 Proposed intuitionistic interval approximation

Consider the $\alpha,\beta-$cuts of a generalized trapezoidal intuitionistic fuzzy number (TrIFN) as shown in Figure 1. Resultantly, the cuts of the TrIFN are defined by

$$(A)_\alpha=\left[A_{\mu1}^{\alpha},A_{\mu2}^{\alpha}\right]=\left[a_1+\frac{(a_2-a_1)\alpha}{\delta},a_4-\frac{(a_4-a_3)\alpha}{\delta}\right] \tag{11}$$

and

$$(A)_\beta=\left[A_{\nu1}^{\beta},A_{\nu2}^{\beta}\right]=\left[\frac{a_2'-a_1'\eta+\beta\left(a_1'-a_2'\right)}{1-\eta},\frac{a_3'-a_4'\eta+\left(a_4'-a_3'\right)\beta}{1-\eta}\right], \tag{12}$$

where δ and η are the heights of the membership and non-membership functions, respectively.

The conversion of TrIFN into IVIFN $\tilde{\alpha} = \left(\left[\tilde{\rho}_{\tilde{\alpha}}^{-}, \tilde{\rho}_{\tilde{\alpha}}^{+}\right], \left[\tilde{\kappa}_{\tilde{\alpha}}^{-}, \tilde{\kappa}_{\tilde{\alpha}}^{+}\right]\right)$ is based on the cuts of TrIFN, which are defined as

$$\left[\tilde{\rho}^{-}, \tilde{\rho}^{+}\right] = \left[\int_0^1 A_{\mu 1}^{\alpha} f(\alpha)\, d\alpha, \int_0^1 A_{\mu 2}^{\alpha} f(\alpha)\, d\alpha\right] \tag{13}$$

and

$$\left[\tilde{\kappa}^{-}, \tilde{\kappa}^{+}\right] = \left[\int_0^1 A_{\nu 1}^{\beta} f(\beta)\, d\beta, \int_0^1 A_{\nu 2}^{\beta} f(\beta)\, d\beta\right], \tag{14}$$

where $f(\alpha)$ and $f(\beta)$ are chosen such that $\int_0^1 f(\alpha)\, d\alpha = \int_0^1 f(\beta)\, d\beta = 1$.

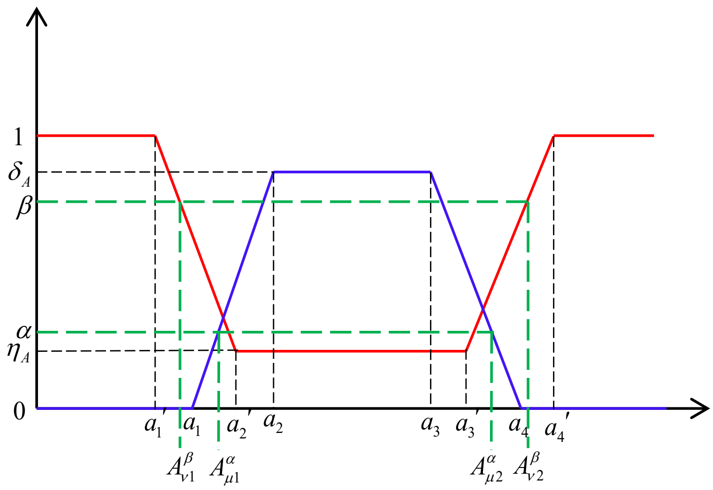

Figure 1. $\alpha, \beta -$cuts of a generalized TrIFN

Theorem 1

Let $f(\alpha) = n\alpha^{n-1}$ and $f(\beta) = n\beta^{n-1}$. For a generalized trapezoidal intuitionistic fuzzy number $A = \left\langle \left(a_1, a_2, a_3, a_4, \delta; a_1', a_2', a_3', a_4', \eta\right)\right\rangle$, the IVIFN obtained using the Nearest Weighted Intuitionistic Interval Approximation (NWIIA) is given by

$$\tilde{\alpha} = \left(\left[a_1 + \frac{(a_2 - a_1)n}{\delta(n+1)}, a_4 + \frac{(a_3 - a_4)n}{\delta(n+1)}\right], \left[\frac{\left(a_2' - a_1'\eta\right)}{1-\eta} + \frac{\left(a_1' - a_2'\right)n}{(n+1)(1-\eta)}, \frac{\left(a_3' - a_4'\eta\right)}{1-\eta} + \frac{\left(a_4' - a_3'\right)n}{(n+1)(1-\eta)}\right]\right). \tag{15}$$

Proof. Since $f(\alpha) = n\alpha^{n-1}$ and $f(\beta) = n\beta^{n-1}$ satisfy the equation $\int_0^1 f(\alpha)\, d\alpha = \int_0^1 f(\beta)\, d\beta = 1$, then

$$\int_0^1 A_{\mu 1}^{\alpha} f(\alpha)\, d\alpha = \int_0^1 \left(a_1 + \frac{(a_2 - a_1)\alpha}{\delta}\right) n\alpha^{n-1}\, d\alpha = a_1 + \frac{(a_2 - a_1)n}{\delta(n+1)} \tag{16}$$

and

$$\int_0^1 A_{\mu 2}^{\alpha} f(\alpha)\, d\alpha = \int_0^1 \left(a_4 - \frac{(a_4 - a_3)\alpha}{\delta}\right) n\alpha^{n-1}\, d\alpha = a_4 + \frac{(a_3 - a_4)n}{\delta(n+1)}. \tag{17}$$

Hence, from equations (16) and (17), $\left[\tilde{\rho}^{-},\tilde{\rho}^{+}\right]=\left[a_1+\frac{(a_2-a_1)n}{\delta(n+1)},a_4+\frac{(a_3-a_4)n}{\delta(n+1)}\right]$. Analogously,

$$\int_0^1 A_{v1}^{\beta} f(\beta)\,d\beta=\int_0^1 \frac{a_2'-a_1'\eta+\beta\left(a_1'-a_2'\right)}{1-\eta}n\beta^{n-1}d\beta=\frac{\left(a_2'-a_1'\eta\right)}{1-\eta}+\frac{\left(a_1'-a_2'\right)n}{(n+1)(1-\eta)} \tag{18}$$

and

$$\int_0^1 A_{v2}^{\beta} f(\beta)\,d\beta=\int_0^1 \frac{a_3'-a_4'\eta+\left(a_4'-a_3'\right)\beta}{1-\eta}n\beta^{n-1}d\beta=\frac{\left(a_3'-a_4'\eta\right)}{1-\eta}+\frac{\left(a_4'-a_3'\right)n}{(n+1)(1-\eta)}. \tag{19}$$

Hence, $\left[\tilde{\kappa}^{-},\tilde{\kappa}^{+}\right]=\left[\frac{\left(a_2'-a_1'\eta\right)}{1-\eta}+\frac{\left(a_1'-a_2'\right)n}{(n+1)(1-\eta)},\frac{\left(a_3'-a_4'\eta\right)}{1-\eta}+\frac{\left(a_4'-a_3'\right)n}{(n+1)(1-\eta)}\right]$.

□

Corollary 1
Let $f(\alpha)=n\alpha^{n-1}$ and $f(\beta)=n\beta^{n-1}$. For a normal trapezoidal intuitionistic fuzzy number, the Nearest Weighted Intuitionistic Interval Approximation (NWIIA) is given by

$$\tilde{\alpha}=\left(\left[\frac{a_1+na_2}{n+1},\frac{na_3+a_4}{n+1}\right],\left[\frac{na_1'+a_2'}{n+1},\frac{a_3'+na_4'}{n+1}\right]\right). \tag{20}$$

Proof. For a normal TrIFN, the heights of the membership and non-membership functions are given by $\delta=1$ and $\eta=0$, respectively. Hence, from equation (15), the following equation is obtained:

$$\tilde{\alpha}=\left(\left[a_1+\frac{(a_2-a_1)n}{n+1},a_4+\frac{(a_3-a_4)n}{n+1}\right],\left[a_2'+\frac{\left(a_1'-a_2'\right)n}{n+1},a_3'+\frac{\left(a_4'-a_3'\right)n}{n+1}\right]\right)$$
$$=\left(\left[\frac{a_1+na_2}{n+1},\frac{a_4+na_3}{n+1}\right],\left[\frac{a_2'+na_1'}{n+1},\frac{a_3'+na_4'}{n+1}\right]\right).$$

□

Remark 1. For a normal triangular intuitionistic fuzzy number $A=\left(a_1,a_2,a_4;a_1',a_2',a_4'\right)$, the Nearest Weighted Intuitionistic Interval Approximation (NWIIA) is given by

$$\tilde{\alpha}=\left(\left[\frac{a_1+na_2}{n+1},\frac{na_2+a_4}{n+1}\right],\left[\frac{na_1'+a_2'}{n+1},\frac{a_2'+na_4'}{n+1}\right]\right). \tag{21}$$

This result is obtained from equation (20), by assuming $a_2=a_3$ and $a_2'=a_3'$. Meanwhile, equation (21) is similar to the result obtained in [31].

3.2 Proposed trapezoidal intuitionistic fuzzy Analytical Hierarchy Process

A new intuitionistic fuzzy AHP is proposed based on the interval approximation of TrIFN. The proposed model can be divided into two parts: calculation of criteria weights and selection of the best alternative, as shown in Figure 2. In both parts, the crisp values in the judgment matrix are transformed into TrIFN before they are converted into IVIFN.

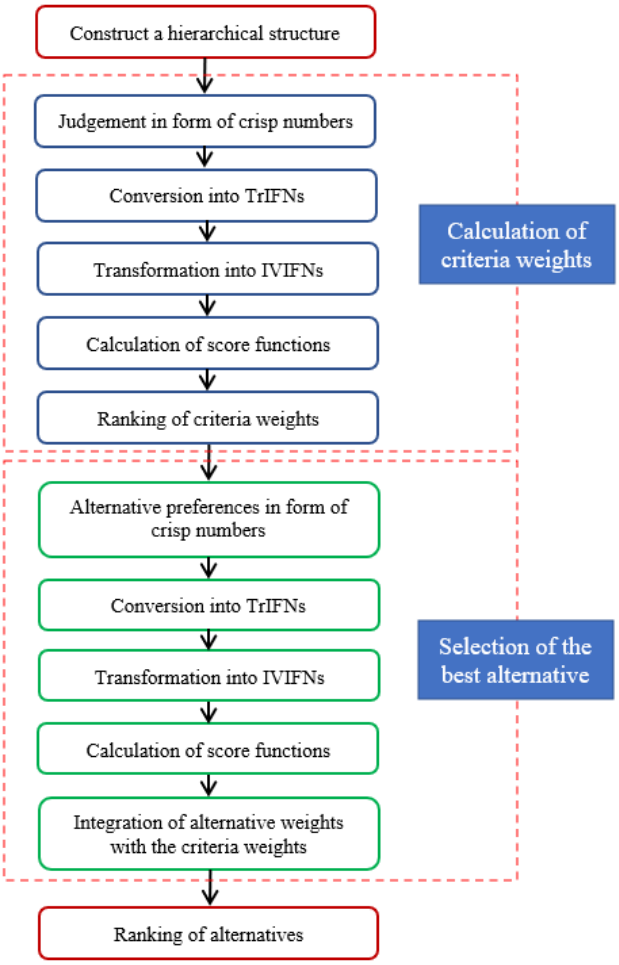

Figure 2. Framework of the proposed method

3.2.1 Calculation of criteria weights

The criteria weights are important to determine the most important criterion to be emphasized. The criteria weights may also affect the final decision on the best alternative. The calculation of the criteria weights of the proposed model is given in the steps below.

Step 1: Construct a hierarchical structure for the problem. A hierarchical structure consists of at least three layers: the goal, criteria, and alternatives.

Step 2: The decision maker gives his preference in the form of crisp numbers. The scale of preference is 1 to 7, representing the least important to the most important element, respectively. A pairwise comparison matrix is also used to ease the decision maker in giving the judgment on the relative importance of one element to the other. Hence, a decision matrix is obtained and presented in equation (22).

$$C = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mm} \end{pmatrix} \tag{22}$$

where m is the number of criteria involved in the decision-making process and c_{ij} represents the preference of criterion i as compared to criterion j.

Step 3: The judgment matrix $C = \{c_{ij}\}_{m \times m}$ is converted into TrIFN in correspondence with the crisp preference as shown in Table 1.

Table 1. TrIFN corresponding to crisp numbers

Crisp number	Linguistic term	TrIFN
1	Absolutely low	(0,0.5,1.5,2) (0,0.5,1.5,2)
2	Low	(1,1.5,2.5,3) (0.5,1.5,2.5,3.5)
3	Fairly low	(2,2.5,3.5,4) (1.5,2.5,3.5,4.5)
4	Medium	(3,3.5,4.5,5) (2.5,3.5,4.5,5.5)
5	Fairly high	(4,4.5,5.5,6) (3.5,4.5,5.5,6.5)
6	High	(5,5.5,6.5,7) (4.5,5.5,6.5,7.5)
7	Absolutely high	(6,6.5,7.5,8) (6,6.5,7.5,8)

Step 4: TrIFN is then transformed into IVIFN using NWIIA.

Example 1

Suppose $\tilde{5}=(4,4.5,5.5,6)\ (3.5,4.5,5.5,6.5)$, then its reciprocal is given by $\tilde{5}^{-1}=(0.17,0.18,0.22,0.25)\ (0.15,0.18,0.22,0.29)$, which is also a TrIFN. Transforming $\tilde{5}^{-1}$ into IVIFN using equation (20) with $f(\alpha)=2\alpha$ and $f(\beta)=2\beta$, thus NWIIA $\tilde{\alpha}=([0.177,0.231],[0.163,0.265])$.

Step 5: Evaluate the score function for each element in the comparison matrix. Next, determine the priority vectors using equation (23).

$$w_i=\frac{\sum_{j=1}^{n}\tilde{A}_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{A}_{ij}}. \tag{23}$$

Example 2

From the previous example, $\tilde{5}^{-1}$ is transformed into IVIFN and $([0.177,0.231],[0.163,0.265])$ is obtained. The score function for such IVIFN is calculated using equation (22), resulting in 0.4927 as its score value.

3.2.2 Selection of the best alternative

Once the criteria weights have been determined, the decision maker will make his or her judgment on the available alternatives. The procedure for selecting the best alternative is given below.

Step 1: The decision maker gives his preference for alternatives for each criterion. The preference made is in crisp values ranging from 1 to 7.

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \tag{24}$$

where n is the number of alternatives involved in the decision-making process and a_{ij} represents the preference of alternative i as compared to alternative j.

Step 2: The judgment matrix from equation (24) is converted into TrIFN based on Table 1.

Step 3: TrIFN is transformed into IVIFN using NWIIA as in equation (20).

Step 4: Evaluate the score function for each element in the comparison matrix. Subsequently, the priority weights are calculated using equation (23).

Step 5: Integrate the criteria weights with the alternative weights by multiplying each alternative weight with the respective criteria weight. Hence, the final preference for alternative is calculated using equation (23).

4. CASE STUDY: SELECTION OF VENDORS

Since this paper extends the triangular intuitionistic fuzzy AHP proposed by [31] in the vendor selection problem [30], the vendor selection problem is also considered in this paper. Firstly, the criteria weights were evaluated using the proposed model in the previous section. Next, the selection of the best vendor was carried out using trapezoidal intuitionistic fuzzy AHP.

4.1 Calculation of criteria weights

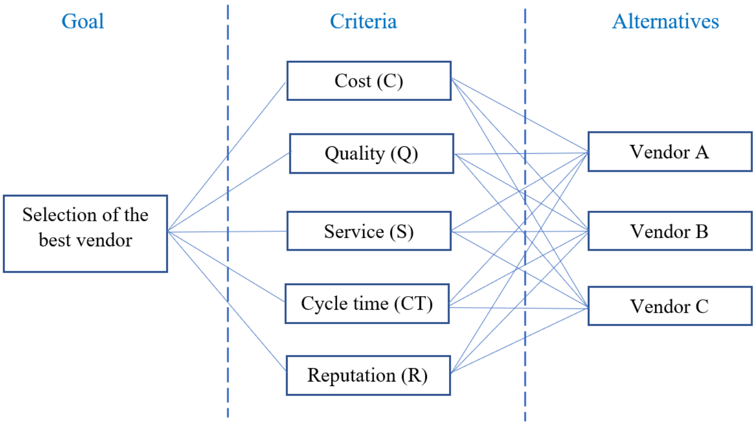

Figure 3. Hierarchical structure for the selection of the best vendor [30]

Step 1: The hierarchical structure is constructed, as shown in Figure 3.
Step 2: The decision maker gives preference to the criteria on a scale of 1 to 7, as shown in Table 2.

Table 2. Crisp data sets for the main comparison matrix [30]

Goal	C	Q	S	CT	R
C	1	1	1	4	1
Q	1	1	2	4	2
S	0.25	0.25	0.2	1	0.33
CT	1	0.5	1	5	3
R	1	1	0.33	3	1

Step 3: The judgment matrix is converted into trapezoidal intuitionistic fuzzy numbers (TrIFN). The pairwise matrix in the form of TrIFN is shown in Table 3.

Table 3. Judgment matrix in the form of TrIFN

	C	Q	
C	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$\cdots$
Q	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$\cdots$
S	$(0.2,0.22,0.29,0.33)$ $(0.18,0.22,0.29,0.4)$	$(0.2,0.22,0.29,0.33)$ $(0.18,0.22,0.29,0.4)$	$\cdots$
CT	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$(0.33,0.4,0.67,1)$ $(0.29,0.4,0.67,2)$	$\cdots$
R	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$\cdots$

Table 3. Continued

	S	CT	
C	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$(3,3.5,4.5,5)$ $(2.5,3.5,4.5,5.5)$	$\cdots$
Q	$(1,1.5,2.5,3)$ $(0.5,1.5,2.5,3.5)$	$(3,3.5,4.5,5)$ $(2.5,3.5,4.5,5.5)$	$\cdots$
S	$(0.17,0.18,0.22,0.25)$ $(0.15,0.18,0.22,0.29)$	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$\cdots$
CT	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$	$(4,4.5,5.5,6)$ $(3.5,4.5,5.5,6.5)$	$\cdots$
R	$(0.25,0.29,0.4,0.5)$ $(0.22,0.29,0.4,0.67)$	$(2,2.5,3.5,4)$ $(1.5,2.5,3.5,4.5)$	$\cdots$

Table 3. Continued

	R
C	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$
Q	$(1,1.5,2.5,3)$ $(0.5,1.5,2.5,3.5)$
S	$(0.25,0.29,0.4,0.5)$ $(0.22,0.29,0.4,0.67)$
CT	$(2,2.5,3.5,4)$ $(1.5,2.5,3.5,4.5)$
R	$(0,0.5,1.5,2)$ $(0,0.5,1.5,2)$

Step 4: TrIFN is transformed into IVIFN using NWIIA in equation (20). The approximated intuitionistic intervals are shown in Table 4.

Table 4. Judgment matrix in the form of IVIFN

Goal	C	Q	S	CT	R
C	[0.333,1.667]; [0.167,1.833]	[0.333,1.667]; [0.167,1.833]	[0.333,1.667]; [0.167,1.833]	[3.333,4.667]; [2.833,5.167]	[0.333,1.667]; [0.167,1.833]
Q	[0.333,1.667]; [0.167,1.833]	[0.333,1.667]; [0.167,1.833]	[1.333,2.667]; [0.833,3.167]	[3.333,4.667]; [2.833,5.167]	[1.333,2.667]; [0.833,3.167]
S	[0.215,0.302]; [0.195,0.362]	[0.215,0.302]; [0.195,0.362]	[0.177,0.231]; [0.163,0.265]	[0.333,1.667]; [0.167,1.833]	[0.274,0.433]; [0.243,0.578]
CT	[0.333,1.667]; [0.167,1.833]	[0.378,0.778]; [0.324,1.556]	[0.333,1.667]; [0.167,1.833]	[4.333,5.667]; [3.833,6.167]	[2.333,3.667]; [1.833,4.167]
R	[0.333,1.667]; [0.167,1.833]	[0.333,1.667]; [0.167,1.833]	[0.274,0.433]; [0.243,0.578]	[2.333,3.667]; [1.833,4.167]	[0.333,1.667]; [0.167,1.833]

Step 5: The score functions and criteria weights are calculated using equations (8) and (22), respectively. The criteria weights are shown in Table 5.

Table 5. Score function and criteria weight

Goal	C	Q	S	CT	R	TOTAL	WEIGHT
C	0.6250	0.6250	0.6250	0.6146	0.6250	3.1146	0.2057
Q	0.6250	0.6250	0.7292	0.6146	0.7292	3.3229	0.2194
S	0.4904	0.4904	0.4927	0.6250	0.4910	2.5895	0.1710
CT	0.6250	0.6044	0.6250	0.5917	0.6528	3.0988	0.2046
R	0.6250	0.6250	0.4910	0.6528	0.6250	3.0187	0.1993
					TOTAL	15.1445	1.0000

4.2 Vendor selection

Step 1: The decision maker gives preference on vendors for each criterion.

Table 6. Decision maker's preference of each criterion for three alternatives [30]

Criteria	C			Q			S			CT			R		
Vendor	A	B	C	A	B	C	A	B	C	A	B	C	A	B	C
A	1	0.25	0.5	1	0.25	0.2	1	0.33	5	1	3	3	1	1	7
B	4	1	3	4	1	0.5	3	1	7	0.33	1	1	1	1	7
C	2	0.33	1	5	2	1	0.2	0.14	1	0.33	1	1	0.14	0.14	1

Step 2: The comparison matrix is converted into trapezoidal intuitionistic fuzzy numbers as defined in Section 3.

Table 7. Pairwise matrix of alternatives for cost (C) in the form of TrIFN

	A	B	C
A	(0,0.5,1.5,2) (0,0.5,1.5,2)	(0.2,0.22,0.29,0.33) (0.18,0.22,0.29,0.4)	(0.33,0.4,0.67,1) (0.29,0.4,0.67,2)
B	(3,3.5,4.5,5) (2.5,3.5,4.5,5.5)	(0,0.5,1.5,2) (0,0.5,1.5,2)	(2,2.5,3.5,4) (1.5,2.5,3.5,4.5)
C	(1,1.5,2.5,3) (0.5,1.5,2.5,3.5)	(0.25,0.29,0.4,0.5) (0.22,0.29,0.4,0.67)	(0,0.5,1.5,2) (0,0.5,1.5,2)

Table 8. Pairwise matrix of alternatives for quality (Q) in the form of TrIFN

	A	B	C
A	(0,0.5,1.5,2) (0,0.5,1.5,2)	(0.2,0.22,0.29,0.33) (0.18,0.22,0.29,0.4)	(0.17,0.18,0.22,0.25) (0.15,0.18,0.22,0.29)
B	(3,3.5,4.5,5) (2.5,3.5,4.5,5.5)	(0,0.5,1.5,2) (0,0.5,1.5,2)	(0.33,0.4,0.67,1) (0.29,0.4,0.67,2)
C	(4,4.5,5.5,6) (3.5,4.5,5.5,6.5)	(1,1.5,2.5,3) (0.5,1.5,2.5,3.5)	(0,0.5,1.5,2) (0,0.5,1.5,2)

Table 9. Pairwise matrix of alternatives for service (S) in the form of TrIFN

	A	B	C
A	(0,0.5,1.5,2) (0,0.5,1.5,2)	(0.25,0.29,0.4,0.5) (0.22,0.29,0.4,0.67)	(4,4.5,5.5,6) (3.5,4.5,5.5,6.5)
B	(2,2.5,3.5,4) (1.5,2.5,3.5,4.5)	(0,0.5,1.5,2) (0,0.5,1.5,2)	(6,6.5,7.5,8) (6,6.5,7.5,8)
C	(0.17,0.18,0.22,0.25) (0.15,0.18,0.22,0.29)	(0.12,0.13,0.15,0.17) (0.12,0.13,0.15,0.17)	(0,0.5,1.5,2) (0,0.5,1.5,2)

Table 10. Pairwise matrix of alternatives for cycle time (CT) in the form of TrIFN

	A	B	C
A	(0,0.5,1.5,2) (0,0.5,1.5,2)	(2,2.5,3.5,4) (1.5,2.5,3.5,4.5)	(2,2.5,3.5,4) (1.5,2.5,3.5,4.5)
B	(0.25,0.29,0.4,0.5) (0.22,0.29,0.4,0.67)	(0,0.5,1.5,2) (0,0.5,1.5,2)	(0,0.5,1.5,2) (0,0.5,1.5,2)
C	(0.25,0.29,0.4,0.5) (0.22,0.29,0.4,0.67)	(0,0.5,1.5,2) (0,0.5,1.5,2)	(0,0.5,1.5,2) (0,0.5,1.5,2)

Table 11. Pairwise matrix of alternatives for reputation (R) in the form of TrIFN

	A	B	C
A	(0,0.5,1.5,2) (0,0.5,1.5,2)	(0,0.5,1.5,2) (0,0.5,1.5,2)	(6,6.5,7.5,8) (6,6.5,7.5,8)
B	(0,0.5,1.5,2) (0,0.5,1.5,2)	(0,0.5,1.5,2) (0,0.5,1.5,2)	(6,6.5,7.5,8) (6,6.5,7.5,8)
C	(0.12,0.13,0.15,0.17) (0.12,0.13,0.15,0.17)	(0.12,0.13,0.15,0.17) (0.12,0.13,0.15,0.17)	(0,0.5,1.5,2) (0,0.5,1.5,2)

Step 3: The judgment matrix for each criterion is approximated into IVIFN using NWIIA. The pairwise matrices in the form of IVIFN are shown in Tables 12-16.

Table 12. Pairwise matrix for cost (C) in the form of IVIFN

	A	B	C
A	[0.333,1.667];[0.167,1.833]	[0.215,0.302];[0.195,0.362]	[0.378,0.778];[0.324,1.556]
B	[3.333,4.667];[2.833,5.167]	[0.333,1.667];[0.167,1.833]	[2.333,3.667];[1.833,4.167]
C	[1.333,2.667];[0.833,3.167]	[0.274,0.433];[0.243,0.578]	[0.333,1.667];[0.167,1.833]

Table 13. Pairwise matrix for quality (Q) in the form of IVIFN

	A	B	C
A	[0.333,1.667];[0.167,1.833]	[0.215,0.302];[0.195,0.362]	[0.177,0.231];[0.163,0.265]
B	[3.333,4.667];[2.833,5.167]	[0.333,1.667];[0.167,1.833]	[0.378,0.778];[0.324,1.556]
C	[4.333,5.667];[3.833,6.167]	[1.333,2.667];[0.833,3.167]	[0.333,1.667];[0.167,1.833]

Table 14. Pairwise matrix for service (S) in the form of IVIFN

	A	B	C
A	[0.333,1.667];[0.167,1.833]	[0.274,0.433];[0.243,0.578]	[4.333,5.667];[3.833,6.167]
B	[2.333,3.667];[1.833,4.167]	[0.333,1.667];[0.167,1.833]	[6.333,7.667];[6.167,7.833]
C	[0.177,0.231];[0.163,0.265]	[0.131,0.158];[0.128,0.162]	[0.333,1.667];[0.167,1.833]

Table 15. Pairwise matrix for cycle time (CT) in the form of IVIFN

	A	B	C
A	[0.333,1.667];[0.167,1.833]	[2.333,3.667];[1.833,4.167]	[2.333,3.667];[1.833,4.167]
B	[0.274,0.433];[0.243,0.578]	[0.333,1.667];[0.167,1.833]	[0.333,1.667];[0.167,1.833]
C	[0.274,0.433];[0.243,0.578]	[0.333,1.667];[0.167,1.833]	[0.333,1.667];[0.167,1.833]

Table 16. Pairwise matrix for reputation (R) in the form of IVIFN

	A	B	C
A	[0.333,1.667];[0.167,1.833]	[0.333,1.667];[0.167,1.833]	[6.333,7.667];[6.167,7.833]
B	[0.333,1.667];[0.167,1.833]	[0.333,1.667];[0.167,1.833]	[6.333,7.667];[6.167,7.833]
C	[0.131,0.158];[0.128,0.162]	[0.131,0.158];[0.128,0.162]	[0.333,1.667];[0.167,1.833]

Step 4: The score functions of all alternatives for each criterion are calculated using equation (22) as shown in Table 17.

Table 17. Score functions of all alternatives

Criterion	Vendor	A	B	C	TOTAL
C	A	1.3484	0.9574	0.7638	3.0696
	B	1.0308	1.3484	1.0583	3.4375
	C	1.1608	0.9186	1.3484	3.4277
Q	A	1.3484	0.9574	0.9736	3.2794
	B	1.0308	1.3484	0.7638	3.1430
	C	1.0192	1.1608	1.3484	3.5284
S	A	1.3484	0.9186	1.0192	3.2862
	B	1.0583	1.3484	1.0026	3.4093
	C	0.9736	0.9974	1.3484	3.3194
CT	A	1.3484	1.0583	1.0583	3.4650
	B	0.9186	1.3484	1.3484	3.6154
	C	0.9186	1.3484	1.3484	3.6154
R	A	1.3484	1.3484	1.0026	3.6994
	B	1.3484	1.3484	1.0026	3.6994
	C	0.9974	0.9974	1.3484	3.3432

Step 5: The weights from Table 17 are integrated with the respective criteria weights obtained from Table 5. The priority weights of vendors obtained are shown in Table 18.

Table 18. Priority weights of vendors

Goal	C 0.2057	Q 0.2194	S 0.1710	CT 0.2046	R 0.1993	TOTAL	WEIGHT
A	0.0648	0.0647	0.0535	0.0724	0.0646	0.3199	0.3283
B	0.0713	0.0741	0.0563	0.0653	0.0646	0.3316	0.3402
C	0.0695	0.0782	0.0507	0.0653	0.0593	0.3230	0.3314
					TOTAL	0.9745	1.0000

5. RESULTS AND DISCUSSION

The vendors were ranked according to the priority weights obtained from the previous section. The weights were compared with the existing AHP methods as shown in Table 19. Based on Table 19, the ranking of vendors using the proposed method is $B \succ C \succ A$, which is consistent with the ranking obtained using the methods from [30], [31]. This indicates that the proposed trapezoidal intuitionistic fuzzy AHP model is relevant even with simplified steps introduced via intuitionistic interval approximation.

Compared to other fuzzy AHP models such as the geometric mean method [11], the proposed model converted TrIFN into IVIFN using the derived NWIIA function. The score values of IVIFN were also calculated before obtaining the priority weights. On the other hand, Buckley's [11] geometric mean method suggests that the geometric mean combining all criteria is evaluated first prior to the summation of the geometric means for all criteria. To obtain the fuzzy weight of each criterion, its geometric mean will be multiplied by the inverse of the summation of the geometric means of all criteria. Nonetheless, the fuzzy weights must still be defuzzified to obtain the final priority weights.

Table 19. Comparison of priority weights of vendors

Alternative	Proposed	[31]	[30]
Vendor A	0.3283	0.330	0.2914
Vendor B	0.3402	0.338	0.4096
Vendor C	0.3314	0.332	0.2987

Table 20. Comparison of the ranking of criteria

Method	Proposed		[31]		[30]	
Criterion	Weight	Ranking	Weight	Ranking	Weight	Ranking
C	0.2057	2	0.202	3	0.2057	2
Q	0.2194	1	0.206	1	0.2194	1
S	0.1710	5	0.191	5	0.1710	5
CT	0.2046	3	0.199	4	0.2046	3
R	0.1993	4	0.203	2	0.1993	4

As can be seen in Table 20, the ranking of criteria obtained using the proposed method is $Q \succ C \succ CT \succ R \succ S$, which is similar to that of [30]. On the contrary, the ranking of criteria obtained by [31] is $Q \succ R \succ C \succ CT \succ S$, which is different from [30]. Furthermore, the sensitivity analysis was performed to observe the effect of changing the criteria weights on the final ranking of alternatives for the proposed method as well as the fuzzy AHP model by [31]. Table 21 and Table 22 present the sensitivity analysis results when the criteria weights were increased by 30% for the proposed model and the model by [31], respectively.

Table 21. Sensitivity analysis of the proposed fuzzy AHP model

Alternative	C	Q	S	CT	R
Vendor A	0.3273	0.3257	0.3286	0.3305	0.3294
Vendor B	0.3408	0.3404	0.3409	0.3388	0.3404
Vendor C	0.3320	0.3339	0.3305	0.3307	0.3302
Ranking	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$
Consistency	Consistent	Consistent	Consistent	Consistent	Consistent

Table 22. Sensitivity analysis of the proposed fuzzy AHP model [31]

Alternative	C	Q	S	CT	R
Vendor A	0.3301	0.3308	0.3315	0.3320	0.3318
Vendor B	0.3376	0.3369	0.3375	0.3366	0.3373
Vendor C	0.3323	0.3323	0.3310	0.3314	0.3309
Ranking	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ A \succ C$	$B \succ A \succ C$	$B \succ A \succ C$
Consistency	Consistent	Consistent	Inconsistent	Inconsistent	Inconsistent

In reference to Table 21, the ranking of alternatives is maintained when the weights of all criteria are increased by 30%. This shows that the proposed model using the conversion of TrIFN into IVIFN is stable and suitable for modeling the uncertainty. On the other hand, the fuzzy AHP model by [31] shows changes in the ranking of alternatives when the weights of three criteria (S, CT, and R) are changed. Hence, there is inconsistency in the results, which can be overcome by replacing TIFN with TrIFN.

In fact, if the criteria weights are increased gradually by 10%, 20% and 30%, the trend shows that the proposed method maintains its consistency, meanwhile the inconsistency of the alternative ranking drops when using the AHP model by [31]. These results are summarised in appendix.

This finding also shows that the use of TrIFN instead of TIFN is more precise since TrIFN is a generalization of TIFN. Moreover, since linguistic variables are used in the fuzzy AHP such as low, medium, and high, they are likewise associated with inexactness due to ill-defined terms [36]. Hence, using TrIFN, which has a better representation of fuzzy numbers than TIFN, enables the fuzzy AHP to better process the ill-known quantities [37-39]. Besides, the score function [33] used to calculate the priority weights notably plays an important role in the selection problem. In this regard, the advantages and special features of the proposed trapezoidal intuitionistic fuzzy AHP model can be discussed as follows:

1) The use of trapezoidal intuitionistic fuzzy numbers (TrIFN) has generalized the triangular intuitionistic fuzzy numbers (TIFN). The generalization results in the model having a better capability of describing human knowledge, especially the decision maker's judgment on the criteria and alternatives. TrIFN has also been proven to be able to describe ill-known quantities, which cannot be done by TIFN.
2) The proposed model has converted TrIFN into interval-valued intuitionistic fuzzy numbers (IVIFN). In the method, a new conversion method of TrIFN into IVIFN was presented using the nearest weighted intuitionistic interval approximation (NWIIA). Resultantly, IVIFN is able to model and solve complex problems other than giving more precise results than the intuitionistic fuzzy set.
3) The ranking of priority weights using the score function has simplified the procedure. The proposed model is contingent on the score function of IVIFN defined by [33], which is based on the idea of the total probability rule. Additionally, the ranking has resulted in more precise results.

However, it should be noted that this study only illustrates the implementation of TrIFN, which is then converted into IVIFN to solve the supplier selection problem. Since the proposed model has been generalized as a trapezoidal intuitionistic fuzzy AHP, its implementation in solving other decision-making problems is, therefore, possible with some modifications to the linguistic variables. For example, this model can be applied in an attempt to obtain the risk factor weights for quantifying hazards in the oil and gas industry as depicted in [40]. Since the model is simpler than other fuzzy AHP methods, it allows for a better understanding among decision-makers in terms of how the model quantifies the priority weights. When the decision-makers really understand the process, they are able to justify the results obtained using the model.

6. CONCLUSION

The present paper has extended the AHP method based on trapezoidal intuitionistic fuzzy numbers (TrIFN). The implemented results have shown that TrIFN is a generalization of a triangular intuitionistic fuzzy number (TIFN) with a better capability of handling ill-defined quantities. Besides, the use of interval-valued intuitionistic fuzzy numbers (IVIFN) has simplified the ranking of priority weights using the score function. The simplification makes the proposed

AHP method more efficient and reliable since its accuracy has been proven with the obtained results. The following describes the contributions of this study:

(a) The extension of IVIFN from TrIFN allows for the flexibility of the application of such fuzzy numbers under certain environments. The IVIFN from TrIFN can be reduced to TIFN since TrIFN is the generalization of TIFN, in which TrIFN exhibits a better capability of handling ill-defined quantities.

(b) The conversion of TrIFN into IVIFN using the Nearest Weighted Intuitionistic Interval Approximation (NWIIA) provides a flexible environment for fuzzy numbers, which allows for the transformation of a form of fuzzy number into another form of fuzzy number. Notably, this NWIIA function was derived from the α- and β-cuts of TrIFN.

(c) The derived conversion function has illustrated its applicability in developing the fuzzy AHP method using TrIFN. The proposed fuzzy AHP method is, in fact, simpler than other fuzzy AHP methods such as Buckley's fuzzy AHP. The proposed fuzzy AHP method has also converted TrIFN into IVIFN, which is then crispified using the score functions before calculating the priority weights.

In the future, the AHP model based on TrIFN can be integrated with other decision-making methods, for instance, the analytical network process (ANP), the technique for order of preference by similarity to ideal solution (TOPSIS), decision-making trial and evaluation laboratory (DEMATEL), and multi-criteria optimization and compromise solution (VIKOR) for better selection problem results. While AHP is used to calculate the criteria weight, ANP, TOPSIS, DEMATEL, or VIKOR is used for selecting the best alternative. The proposed method can also be used to solve other selection problems in the future.

ACKNOWLEDGEMENTS

Institution

The authors would like to thank Universiti Malaysia Pahang Al-Sultan Abdullah for providing the laboratory facilities.

Fund

The authors would like to thank Universiti Malaysia Pahang Al-Sultan Abdullah for providing financial support under UMPSA Postgraduate Research Grants Scheme (PGRS) No. PGRS220301.

Individual Assistant

NA

AUTHOR CONTRIBUTIONS

Nik Muhammad Farhan Hakim Nik Badrul Alam (Conceptualization; Formal analysis; Writing- original draft), Ku Muhammad Naim Ku Khalif (Validation; Writing- review & editing), Nor Izzati Jaini (Writing- review & editing).

DECLARATION OF ORIGINALITY

The authors declare no conflict of interest to report regarding this study conducted.

REFERENCES

[1] Aliev RA, Guirimov BG, Huseynov OH, Aliyev RR. A consistency-driven approach to construction of Z-number-valued pairwise comparison matrices. Iranian Journal of Fuzzy Systems. 2021;18(4):37–49.

[2] Partovi FY, Burton J, Banerjee A. Application of analytical hierarchy process in operations management. International Journal of Operations & Production Management. 1990;10(3):5–19.

[3] Saaty TL. The Analytic Hierarchy Process. New York, NY: McGraw-Hill; 1980.

[4] Kahraman C, Onar SC, Oztaysi B. Fuzzy multicriteria decision-making: A literature review. International Journal of Computational Intelligence Systems. 2015 May;8(4):637–66.

[5] Li W, Yu S, Pei H, Zhao C, Tian B. A hybrid approach based on fuzzy AHP and 2-tuple fuzzy linguistic method for evaluation in-flight service quality. Journal of Air Transport Management. 2017;60:49–64.

[6] Dožić S. Multi-criteria decision making methods: Application in the aviation industry. Journal of Air Transport Management. 2019;79:101683.

[7] Havle CA, Kılıç B. A hybrid approach based on the fuzzy AHP and HFACS framework for identifying and analyzing gross navigation errors during transatlantic flights. Journal of Air Transport Management. 2019;76:21–30.

[8] van Laarhoven PJM, Pedrycz W. A fuzzy extension of Saaty's priority theory. Fuzzy Sets and Systems. 1983;11(1-3):229–41.

[9] Aliyev R, Temizkan H, Aliyev R. Fuzzy analytic hierarchy process-based multi-criteria decision making for universities ranking. Symmetry. 2020;12(8):1351.

[10] Deng Y. Chaos, Solitons & Fractals. Chaos Solitons & Fractals. 2016;91(C):549–53.

[11] Buckley JJ. Fuzzy hierarchical analysis. Fuzzy Sets and Systems. 1985;17(3):233–47.

[12] Chang DY. Applications of the extent analysis method on fuzzy AHP. European Journal of Operational Research. 1996;95(3):649–55.

[13] Liu Y, Eckert CM, Earl C. A review of fuzzy AHP methods for decision-making with subjective judgements. Expert

Systems with Applications. 2020;161(1):113738.
[14] Atanassov KT. Intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1986;20(1):87–96.
[15] Xu Z, Liao H. Intuitionistic Fuzzy Analytic Hierarchy Process. IEEE Transactions on Fuzzy Systems. 2014;22(4):749–61.
[16] Zhang C, Li W, Yang H. A new drug risk assessment model of comprehensive hospitals. International Conference on Biomedical Engineering and Biotechnology. 2012;1165-8.
[17] Liao H, Xu Z. Consistency of the fused intuitionistic fuzzy preference relation in group intuitionistic fuzzy analytic hierarchy process. Applied Soft Computing. 2015;35:812–26.
[18] M. Deepika, Kannan ASK. Global supplier selection using intuitionistic fuzzy Analytic Hierarchy Process. International Conference on Electrical, Electronics, and Optimization Techniques 2016. 2016;2390–5.
[19] Sadiq R, Tesfamariam S. Environmental decision-making under uncertainty using intuitionistic fuzzy analytic hierarchy process (IF-AHP). Stochastic Environmental Research and Risk Assessment. 2007;23(1):75–91.
[20] Abdullah L, Najib L. Sustainable energy planning decision using the intuitionistic fuzzy analytic hierarchy process: choosing energy technology in Malaysia. International Journal of Sustainable Energy. 2016;35(4):360-77.
[21] Buyukozkan G, Feyzioglu O, Gocer F. Evaluation of hospital web services using intuitionistic fuzzy AHP and intuitionistic fuzzy VIKOR. International Conference on Industrial Engineering and Engineering Management IEEM. 2016;6451:607–11.
[22] Tavana M, Zareinejad M, Di Caprio D, Kaviani MA. An integrated intuitionistic fuzzy AHP and SWOT method for outsourcing reverse logistics. Applied Soft Computing. 2016;40:544–57.
[23] Büyüközkan G, Göçer F, Feyzioğlu O. Cloud computing technology selection based on interval-valued intuitionistic fuzzy MCDM methods. Soft Computing. 2018;22(15):5091–114.
[24] Wu J, Chiclana F. Non-dominance and attitudinal prioritisation methods for intuitionistic and interval-valued intuitionistic fuzzy preference relations. Expert Systems With Applications. 2012;39(18):13409–16.
[25] Atanassov K, Gargov G. Interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1989;31(3):343–9.
[26] Abdullah L, Najib L. A new preference scale mcdm method based on interval-valued intuitionistic fuzzy sets and the analytic hierarchy process. Soft Computing. 2014;20(2):511–23.
[27] Dogan O, Deveci M, Canıtez F, Kahraman C. A corridor selection for locating autonomous vehicles using an interval-valued intuitionistic fuzzy AHP and TOPSIS method. Soft Computing. 2020;24:8937-53.
[28] Büyüközkan G, Havle CA, Feyzioğlu O. A new digital service quality model and its strategic analysis in aviation industry using interval-valued intuitionistic fuzzy AHP. Journal of Air Transport Management. 2020;86:101817.
[29] Verma R, Chandra S. Interval-valued intuitionistic fuzzy-analytic hierarchy process for evaluating the impact of security attributes in fog based internet of things paradigm. Computer Communications. 2021;175:35–46.
[30] Kaur P. Selection of vendor based on intuitionistic fuzzy analytical hierarchy process. Advances in Operations Research. 2014;2014(1):1–10.
[31] Nirmala G, Uthra G. AHP based on triangular intuitionistic fuzzy number and its application to supplier selection problem. Materials Today: Proceedings. 2019;16:987–93.
[32] Tian D, Chen J, Wu X. A two stage risk assessment model based on interval-valued fuzzy numbers and risk attitudes. Engineering Applications of Artificial Intelligence. 2022;114:105086.
[33] Gong R, Ma L. A new score function and accuracy function of interval-valued intuitionistic fuzzy number and its application. Systems Engineering - Theory & Practice. 2019;39(2):463–75.
[34] Mahapatra GS, Mahapatra BS. Intuitionistic fuzzy fault tree analysis using intuitionistic fuzzy numbers. International Mathematical Forum. 2010;5(21):1015–24.
[35] Nagoor Gani A, Mohamed V. A method of ranking generalized trapezoidal intuitionistic fuzzy numbers. International Journal of Applied Engineering Research. 2015;10(10):25465–73.
[36] Chaira T. Fuzzy set and its extension: The intuitionistic fuzzy set. John Wiley & Sons, Inc.; 2019.
[37] Zhang Q, Sun D. Some notes on possibilistic variances of generalized trapezoidal intuitionistic fuzzy numbers. AIMS Mathematics. 2021;6(4):3720–40.
[38] Kumar M. Applying weakest t-norm based approximate intuitionistic fuzzy arithmetic operations on different types of intuitionistic fuzzy numbers to evaluate reliability of PCBA fault. Applied Soft Computing. 2014;23:387–406.
[39] Banerjee S, Roy T. Arithmetic operations on generalized trapezoidal fuzzy number and its applications. Turkish Journal of Fuzzy Systems. 2012;3(1):16–44.
[40] Cheraghi M, Eslami Baladeh A, Khakzad N. A fuzzy multi-attribute HAZOP technique (FMA-HAZOP): Application to gas wellhead facilities. Safety Science. 2019;114:12–22.

DATA ANALYTICS AND APPLIED MATHEMATICS
E-ISSN: 2773-4854
VOLUME 5, ISSUE 2, 2024, 1 – 16
DOI: https://doi.org/10.15282/daam.v5i2.11030

RESEARCH ARTICLE

Appendix

Table 23. Comparison of sensitivity analysis results when the criteria weights are increased by 10%, 20% and 30%

Method	**Nirmala and Uthra [31]**														
Increment	10%					20%					30%				
Alternative	C	Q	S	CT	R	C	Q	S	CT	R	C	Q	S	CT	R
Vendor A	0.3309	0.3311	0.3313	0.3315	0.3314	0.3305	0.3310	0.3314	0.3317	0.3316	0.3301	0.3308	0.3315	0.3320	0.3318
Vendor B	0.3373	0.3371	0.3373	0.3370	0.3372	0.3375	0.3370	0.3374	0.3368	0.3373	0.3376	0.3369	0.3375	0.3366	0.3373
Vendor C	0.3318	0.3318	0.3314	0.3315	0.3314	0.3320	0.3320	0.3312	0.3315	0.3311	0.3323	0.3323	0.3310	0.3314	0.3309
Ranking	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ A \succ C$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ A \succ C$	$B \succ A \succ C$	$B \succ A \succ C$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ A \succ C$	$B \succ A \succ C$	$B \succ A \succ C$
Consistency	80%					40%					40%				
Method	**Proposed**														
Increment	10%					20%					30%				
Alternative	C	Q	S	CT	R	C	Q	S	CT	R	C	Q	S	CT	R
Vendor A	0.3280	0.3274	0.3284	0.3290	0.3287	0.3276	0.3266	0.3285	0.3298	0.3290	0.3273	0.3257	0.3286	0.3305	0.3294
Vendor B	0.3404	0.3403	0.3405	0.3398	0.3403	0.3406	0.3403	0.3407	0.3393	0.3404	0.3408	0.3404	0.3409	0.3388	0.3404
Vendor C	0.3316	0.3323	0.3311	0.3312	0.3310	0.3318	0.3331	0.3308	0.3309	0.3306	0.3320	0.3339	0.3305	0.3307	0.3302
Ranking	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$	$B \succ C \succ A$
Consistency	100%					100%					100%				

*CORRESPONDING AUTHOR | Nik Muhammad Farhan Hakim Nik Badrul Alam| ✉ farhanhakim@uitm.edu.my