INTERNATIONAL JOURNAL OF ENGINEERING TECHNOLOGY AND SCIENCE (IJETS) Vol.8 (1) Dec 2017 DOI: http://dx.doi.org/10.15282/ijets.8.2017.1.7.1082

Opportunistic Maintenance Policy for a Multi-Component System Subject to Random Failures

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Abstract- In this paper, an opportunistic maintenance policy (OMP) for a multi-component system is studied. The objective is to minimize the maintenance cost while guaranteeing a minimum level of reliability for the system and for each of its components. Each component is subject to random failures and at most one spare part of it should be kept in stock or ordering at any time. The lifetime of this system will be divided into several periods. At the beginning of each period, the set of actions (among many others) must be determined in order to achieve the objective mentioned above. The policy OMP is characterized by two parameters; the first one is the scheduled time for spare ordering and the second one is the period of realization of the maintenance action (if any). These parameters will be derived from the joint optimization of maintenance cost and the inventory cost for each component. Finally, a numerical example to explain the proposed maintenance policy and the optimization procedure is provided.

Index Terms- Opportunistic maintenance, preventive maintenance, Multi-component system, Reliability.

I. INTRODUCTION

Nowadays, maintenance is the most important issue in each critical activity and the challenges of maintenance are growing especially with the sophisticated industrial systems, which are more complex and multifunctional. Consequently, in the last decades, researchers and practitioners start developing new tools and approches to deal with problems in the areas of maintenance and reliability. The maintenance activity is dependent on the availability of the necessary resources [18]. Nevertheless, most analytical models assume that these resources are always available. Therefore, most of researches has treated spare parts management and maintenance optimization separately or sequentially [15, 25, 16, 26, 14], and even nowadays, little attention has been paid to the joint optimization of the spare parts provisioning and maintenance actions [6]. Such a way, practitioners and researchers are missing the opportunity to achieve substantial savings. Indeed, recent studies have shown that joint optimization of spare parts inventory and maintenance strategies generate substantial gains in availability and cost [1, 4, 5, 13, 24, 27]. In these cases, the demand for spare parts arises from both preventive and corrective maintenance.

Recent studies show that the effective use of opportunistic maintenance is beneficial to the overall performance of the production system [21, 28, 7, 11, 8]. In addition, the grouping of maintenance actions for a set of components is suitable when the cost of access to these components of the same class are expensive. On the other hand, the cost of downtime can also be reduced by effective utilization of opportunistic maintenance.

Generally, after undertaking a maintenance action, the system will not be as good as new; the imperfect preventive maintenance concept is used here to model this reality. Pham and Wang [20] have reported that more than 40 mathematical imperfect maintenance models have been proposed over the last 30 years for estimating the reliability measures and determining the optimum maintenance policies. They have proposed various methods and optimal policies on the imperfect maintenance.

In this paper, an opportunistic maintenance policy (OMP) for a multi-component system subject to random failures is proposed. This policy consists in determining at the beginning of each period the maintenance actions to be undertaking on the system. The actions selected are the ones that minimize the maintenance cost while guaranteeing a minimum level of reliability. For each component C_i, the scheduled time for spare ordering t_i and the scheduled time for preventive replacement T_i are determined using the joint optimization of maintenance cost and the inventory cost. Without loss of generality assume that $T_1 \leq T_2 \leq \cdots \leq T_n$. According to preventive maintenance scheduling for each component, the system must be maintained at each instant jT_i ($i \in \mathrm{N}, j \in \mathrm{IN}^*$). Unfortunately, this could be practically inconceivable. For this reason, a single period T_s, common to all the elements of the system, will be chosen. The same way, a common time t_s will be chosen to ordering spare parts.

This paper is organized as follow. Section 2 presents some notations and describes the policy for a single-component system. For this system, the expected cost rate and optimization procedure used to determine the optimal ordering time and the optimal replacement time. Then, based on the results obtained for this single-component system, in section 3, the policy OMP for a multi-component system is developed. To demonstrate the effectiveness of this policy, numerical examples are given in section 4. Finally, section 5 concludes the paper.

II. THE JOINT OPTIMIZATION OF SPARE PARTS AND PREVENTIVE MAINTENANCE FOR A SINGLE-COMPONENT SYSTEM

2.1. Model description

In this model, a machine consists of a single component subject to random failures. This extreme case can be considered from that moment the considered component is much more important than others. Furthermore, this model assume that one spare part of that component should be kept in stock or in ordering at any time. Armstrong and Atkins [1] analyze the joint optimization of maintenance and inventory to operate this system at the lowest possible long-run average cost rate. The system (i.e. the component) is replaced at failure or at age T, whichever occurs first. The ordering for a spare is placed at a scheduled time d or at failure time. Then it's delivered after a deterministic lead time L ($d \leq T - L$). The component will take over its operation as soon as it is delivered. The objective is to determine jointly the ordering time (d) and the replacement time (T) to minimize the expected cost rate.

2.2. Cost model

Since each replacement is a regeneration point, the time between successive replacements can be regarded as one cycle. The expected cost per cycle is the sum of the replacement, holding and shortage costs.

There exist the following four mutually exclusive and exhaustive possibilities in every cycle. For each scenario m, the total average cost $N_m(d, T)$ and the average duration of the corresponding cycle $D_m(d, T)$ are evaluated.

Scenario 1: The operating unit fails before the scheduled ordering time d.
The order for a spare is placed immediately and the spare is delivered after a lead time L. Thus it is necessary to assume the corrective replacement cost and the shortage cost.

$$N_1(d,T) = (c_r + c_s.L)\int_0^d f(x)\mathrm{d}x \tag{1}$$
$$D_1(d,T) = \int_0^d (x+L)f(x)\,\mathrm{d}x \tag{2}$$

Scenario 2: The operating unit fails between t and the arrival of the ordered spare $d + L$.
The regular order for a spare is placed at time d and the spare is delivered after a lead time L. The corrective replacement cost and the shortage cost must be assumed.

$$N_2(d,T) = c_r\int_d^{d+L} f(x)\,\mathrm{d}x + c_s\int_d^{d+L}(d+L-x)f(x)\,\mathrm{d}x \tag{3}$$
$$D_2(d,T) = (d+L)\int_d^{d+L} f(x)\,\mathrm{d}x \tag{4}$$

Scenario 3: The operating unit fails between $d + L$ and the scheduled preventive replacement time T.
The spare part is delivered and stored. So, the corrective replacement cost and the holding cost will be considered.

$$N_3(d,T) = c_r\int_{d+L}^{T} f(x)\,\mathrm{d}x + c_h\int_{d+L}^{T}(x-d-L)f(x)\,\mathrm{d}x \tag{5}$$
$$D_3(d,T) = \int_{d+L}^{T} xf(x)\,\mathrm{d}x \tag{6}$$

Scenario 4: The operating unit survive until time T.
In this case, the preventive replacement cost and the holding cost must be assumed.

$$N_4(d,T) = c_p\int_T^{\infty} f(x)\,\mathrm{d}x + c_h\int_T^{\infty}(T-d-L)f(x)\,\mathrm{d}x \tag{7}$$
$$D_4(d,T) = T\int_T^{\infty} f(x)\,\mathrm{d}x \tag{8}$$

Since the cost per cycle is the sum of (1),(3), (5) and (7), then the expected cost per cycle, N(d,T), is given by:

$$N(d,T) = [c_p + c_h(T-d-L)] + (c_r - c_p)F(T) - c_h\int_{d+L}^{T} F(x)\,\mathrm{d}x + c_s\int_d^{d+L} F(x)\,\mathrm{d}x \tag{9}$$

Thus the expected cycle length, D(d,T) , is computed as follows.

$$D(d,T) = T - \int_0^d F(x)\,\mathrm{d}x - \int_{d+L}^{T} F(x)\,\mathrm{d}x \tag{10}$$

From the renewal reward theorem [2], the expected cost rate for an infinite time span is the expected cost per cycle divided by the expected cycle length.
Hence the expected cost rate, denoted C(d,T) , is

$$C(d,T) = N(d,T)/D(d,T) = \frac{[c_p + c_h(T-d-L)] + (c_r - c_p)F(T) - c_h\int_{d+L}^{T} F(x)\,\mathrm{d}x + c_s\int_d^{d+L} F(x)\,\mathrm{d}x}{T - \int_0^d F(x)\,\mathrm{d}x - \int_{d+L}^{T} F(x)\,\mathrm{d}x} \tag{11}$$

The following subsection presents the resolution of this equation to determine the optimal point (d,T).

2.3. Optimization Procedure

Since $C(d,T)$ is unimodal and pseudo-convex in (d,T) as shown by Armstrong and Atkins [1], the optimal values of d and T, can be obtained using a numerical computing software.
This problem is formulated as a constrained nonlinear optimization : Find values of (d,T) that minimize $C(d,T)$ and subject to the constraints $T \geq (d+L)$.

$$\begin{cases} \min_{(d,T)} C(d,T) \\ (d_0; T_0) = (0; 0) \\ T \geq d + L \end{cases} \tag{12}$$

The function fmincon of Matlab software is used to search the minimum of $C(d,T)$. This function finds a constrained minimum of a scalar function of several variables starting at an initial estimate.
This model is effective for a single-component system, however, in reality, production machines are multi-component systems. Consequently, an opportunistic maintenance policy for multi-component system is studied.

III. OPPORTUNISTIC MAINTENANCE POLICY FOR MULTI-COMPONENT SYSTEMS

3.1. Maintenance actions and Reliability

Without loss of generality, three possible actions A1, A2 and A3 are assumed to designate respectively a preventive replacement, an imperfect maintenance and a "no action" event.

3.1.1. Perfect Preventive Maintenance

Perfect Preventive Maintenance (A1) is a maintenance action which restores the equipment to the as same as new condition.
In this condition, the reliability of i^{th}component in the j^{th} period is defined as :

$$R_j^i(t) = R_0 R_i\big((t-(j-1)T)\big); \qquad (j-1)T \leq t \leq jT \tag{13}$$

At the beginning of the j^{th} period (i.e $t=(j-1)T$), the reliability of the component C_i is equal to R_0.

3.1.2. Imperfect Preventive Maintenance

Imperfect Preventive Maintenance (A2) is a maintenance action which significantly improves the equipment condition, even without bringing the equipment to a seemingly new condition (better than old) [11].
In this case, the reliability of i^{th}component in the j^{th} period is defined as :

$$R_j^i(t) = R_{0,j}^i R_i\left(\frac{1}{\delta_i}(t-(j-1)T)\right); \qquad (j-1)T \le t \le jT \tag{14}$$

Where :

- δ_i is the improvement factor of component C_i which is set between 0 and 1. It can be regarded as the ratio of the life of the surviving parts to their original life.
- $R_{0,j}^i$ is the initial reliability of component C_i in the j^{th} period.

According to age reduction model [23], the initial reliability of component C_i in the j^{th} period can be expressed as

$$R_{0,j}^i = R_{f,j-1}^i + \delta_i(R_{0,j-1} - R_{f,j-1}^i) \tag{15}$$

Where $R_{0,j-1}^i$, $R_{f,j-1}^i$ are respectively the initial and final reliabilities of component C_i in the $(j-1)^{th}$ period.

To explain the rationale of equation (15) and illustrate the effects of various actions to reliability, a system whose failure belong to Weibull distribution was used as an example. The system reliability was expressed as

$$R(t) = R_0 e^{-(\frac{t}{\theta})^{\beta}} \tag{16}$$

Where (θ, β) denote the scale and the shape parameters of the Weibull distribution, respectively. For this example, θ= 4000, β = 2.5, T = 2000 and δ= 0.6. Under the given parameters, changes in the reliability function of the machine according to different preventive maintenance (PM) actions is represented in Figure. 1.

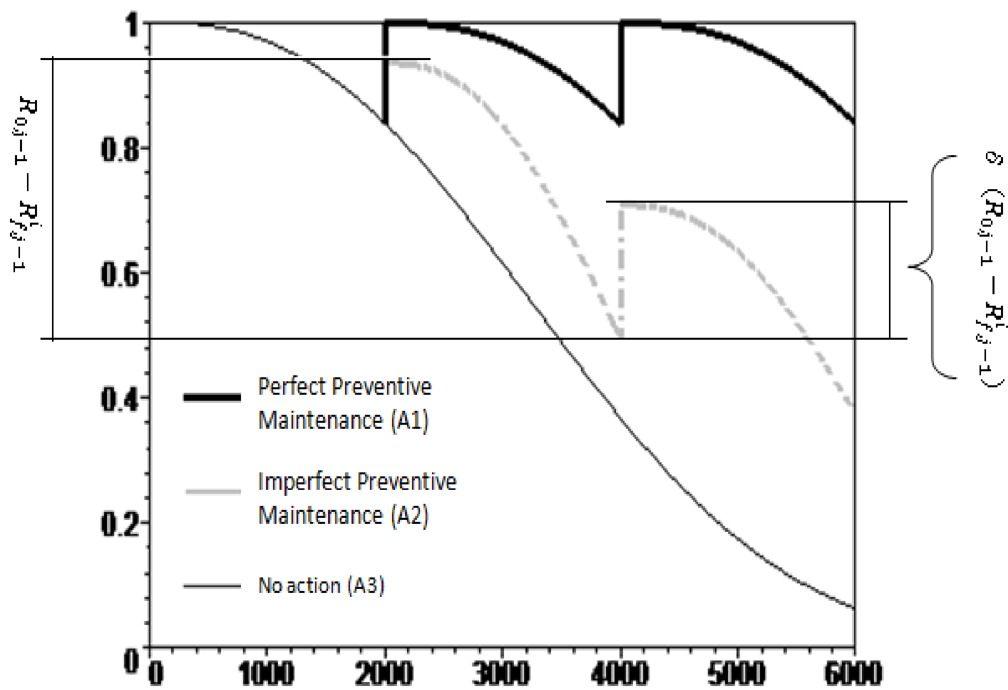


Figure 1: The changing of reliability on different PM actions.

3.1.3. No action event (A3)

In this case, the decision is postponed to the next period.

3.2. Description of the opportunistic maintenance policy (OMP)

In practice, for some critical system (bottleneck machine), the manager must ensure the availability of the machine and the availability of its critical components. For example, a minimum level of reliability may be required for the system and for each component.
The contribution of this work is the simultaneously satisfaction of both criteria: minimize the maintenance cost and maximize availability of the system. Moreover, among the actions that satisfy these criteria and maximizes the maintenance benefit are considered. In the work of [23], the actions are chosen independently from each other and the model is based on an age replacement policy without taking into consideration the inventory management.
In this work, a system is composed of several components, the (d_i, T_i) (optimal ordering and replacement times) of each component C_i can be derived by optimization procedure described in section 2. For a system, if the components are replaced depending on their T_i individually, the system availability would be largely reduced and the maintenance cost would be higher due to system shut-down. To avoid this problem, the concept of opportunistic maintenance will be incorporated. According to this concept, the preventive maintenance of the system is carried out at each interval of time T_s, such that the system preventive maintenance interval T_s is the minimum replacement time among all the T_i, T_s=Min{ T_i} and the scheduled time for spare ordering is d_s=Min{ d_i}. Along the lifetime of the system, the kind of action to adopt for each component C_i should be determined. The decision of the actions is depending on the status of reliability.
For example, if the component's reliability at the next period is less than the set minimum reliability level, i.e. $R_i(2T_s) < R_{min}$; the component need to be maintained on this period. The latter is decided depending on the results of the maintenance-benefit analysis of the system.
The maintenance benefit of any component in the j^{th} period is defined by

$$B_{i,k} = \frac{\int_{j*T_s}^{\infty} R_{j+1}^{i}(t)\,\mathrm{d}t - \int_{j*T_s}^{\infty} R_{j}^{i}(t)\,\mathrm{d}t}{C_{i,k}} \qquad (17)$$

Where the subscript i denotes the i^{th} component, k denotes the type of maintenance action and $C_{i,k}$ is the maintenance cost of action k.
The action that leads to the maximum maintenance benefit, i.e. $B_i^* = Max(B_{i,k})$, would be selected for the component.
Therefore the opportunistic maintenance policy can be described as follows:

Step 1: Determine the optimal ordering and replacement times (d_i, T_i) of all components by the optimization procedure described in section 2.4.

- Determine the period of the system T_s and the scheduled time for spares ordering d_s.
- Put j = 1.

Step 2: Put $T_j = j * T_s$.

a) If $R_s(2T_s) \geq R_{min}^{s}$ and $R_i(2T_s) \geq R_{min}$ for all C_i, then no action is needed. Move to step 4.

b) If $R_s(2T_s) \geq R_{min}^{s}$ and at least there exist one component C_q satisfying $R_q(2T_s) < R_{min}$, then the maintenance action should be taken for this component. The action chosen is the one which maximizes the maintenance-benefit of component C_q.

c) If $R_s(2T_s) < R^s_{min}$, then maintenance actions should be taken for the components from the set $\{A1, A2, A3\}$. The actions chosen are those that maximize the maintenance-benefit for the system.

Step 3: Calculate the reliability and the maintenance cost of the whole system based on actions adopted in step 2.

Step 4: While $T_j < T_L$, put $j = j + 1$ and go to step 2.

The procedure ends when $T_j \geq T_L$

IV. NUMERICAL EXAMPLE

A concrete example will be helpful in illustratingt he interest of this maintenance policy. The following assumptions will be used:

1. The lifetime distribution of each component can be determined.
2. Failures can be removed by imperfect preventive maintenance or perfect preventive maintenance.
3. The improvement factor and the maintenance costs of the components can be identified.

For this example a series system which consists of three components is used to explain the policy OMP. The lifetime of each components is supposed to be a Weibull distribution, then the reliability of component C_i was expressed as

$$R_i(t) = R_0 e^{-(\frac{t}{\theta_i})^{\beta_i}} \tag{18}$$

Where (θ_i, β_i) denote the scale and the shape parameters of the Weibull distribution, respectively. The supposed related parameters (θ_i, β_i) and the maintenance costs in the example are listed in Table 1. The improvement factor is the same for all components and it is set to $\delta = 0.7$. The lifetime of the system is set to $T_L = 8500h$ and the initial reliability of all components is set to $R_0 = 0.999$.

Table 1: The supposed parameters of the components in the example

Components	θ_i	β_i	c_p	c_r	c_h	c_{imp}	c_s	L
1	2400	2.5	1000	2000	40	450	400	20
2	2600	3	1200	2400	50	540	400	20
3	3200	3.2	1400	2800	60	630	400	20

According to the given parameters, the optimal ordering and replacement times (d_i, T_i) of the components can be obtained by the optimization procedure described in section 2. The numerical calculation gives $d = \{834,1035,1370\}$ and $T = \{854,1055,1390\}$

The period of the system would be $T_s = 854$ and the ordering time $d_s = 834$.

The preventive maintenance cost of the component 2 and 3 is intentionally modified by multiplying them by 0.9, this is the reason why: at the time T_s, the technicians and tools are already mobilized to work on the machine. So, it would take less time to improve the reliability of the component 2 or 3.

Next, the maintenance benefit of the system is calculated similarly to Equation (15), using the fact that the reliability of the system at time t is the product of the reliability of each component and the

maintenance cost is the sum of the maintenance cost of each component.
The maintenance benefit of the system and of each component are as shown in Table 1
The actions that lead to the maximum of the maintenance benefit are reported in Table 3.

Table 2: The maintenance benefits of system

Period	Time (h)	PM actions			maintenance benefit of system
		C_1	C_2	C_3	
j=1/j=6	854/5120	$A1$	$A1$	$A3$	0.281
		$A1$	$A1$	$A1$	0.246
		$A1$	$A1$	$A2$	0.258
		$\boldsymbol{A1}$	$\boldsymbol{A2}$	$\boldsymbol{A3}$	***0.285***
		$A1$	$A2$	$A1$	0.197
		$A1$	$A2$	$A2$	0.216
j=2/j=7	1708/5978	$A1$	$A1$	$A1$	0.354
		$\boldsymbol{A1}$	$\boldsymbol{A1}$	$\boldsymbol{A2}$	**0.360**
		$A1$	$A2$	$A1$	0.314
		$A1$	$A2$	$A2$	0.348
j=3/j=8	2562/6832	$A1$	$A1$	$A1$	0.266
		$A1$	$A1$	$A2$	0.250
		$A1$	$A2$	$A1$	0.220
		$\boldsymbol{A1}$	$\boldsymbol{A2}$	$\boldsymbol{A2}$	**0.275**
j=4/j=9	3416/7686	$A1$	$A1$	$A1$	0.348
		$\boldsymbol{A1}$	$\boldsymbol{A1}$	$\boldsymbol{A2}$	**0.350**
		$A1$	$A2$	$A1$	0.306
		$A1$	$A2$	$A2$	0.337
j=5/j=10	4270/8500	$\boldsymbol{A1}$	$\boldsymbol{A1}$	$\boldsymbol{A1}$	**0.300**
		$A1$	$A1$	$A2$	0.290
		$A1$	$A2$	$A1$	0.260
		$A1$	$A2$	$A2$	0.275

Table 3: The maintenance schedules of the machine using the maintenance benefits of system

Period	Time(h)	PM actions			System reliability	Maintenance cost
		C_1	C_2	C_3		
j=1	854	$A1$	$A2$	$A3$	0.88	1540
j=2	1708	$A1$	$A1$	$A2$	0.72	2710
j=3	2562	$A1$	$A2$	$A2$	0.82	2170
j=4	3416	$A1$	$A1$	$A2$	0.75	2710
j=5	4270	$A1$	$A1$	$A1$	0.82	3340
j=6	5124	$A1$	$A2$	$A3$	0.88	1540
j=7	5978	$A1$	$A1$	$A2$	0.72	2710
j=8	6832	$A1$	$A2$	$A2$	0.82	2170
j=9	7686	$A1$	$A1$	$A2$	0.75	2710
j=10	8500	$A1$	$A1$	$A1$	0.82	3340
$R^s_{min} = 0.65;\ R_{\min} = 0.8;\ Total\ cost = \$24940$						

According to these results, the total cost of the preventive maintenance of the policy OMP is $24940. The maintenance cost of the system and the maintenance actions applied to each component using PM of [23] are reported in Table 4. The OMP policy cost is 8% less expensive than the policy suggested by Tsai and al. [23] and 11% less expensive than an individual management of each component based on the model suggested in section 2.
Moreover, the reliability changing of the system and its components in the example are shown respectively in Figure. 2 and Figure. 3. According to Figure. 2, the reliability of the system is always greater than the minimum reliability of system R^{s}_{min}. At each period T_s, the component 1 undergoes Perfect Preventive Maintenance (A1). The component 2 alternately undergoes Imperfect Preventive Maintenance (A2) and Perfect Preventive Maintenance (A1). But the component 3, along the lifetime of the system, undergoes three actions: No action event (A3), Imperfect Preventive Maintenance (A2) and Perfect Preventive Maintenance (A1). For three component, the reliability is always greater than the minimum reliability R_{min}. So, the policy OMP ensures the availability of the system at a lower cost.

Table 4: The maintenance schedules of the machine using the maintenance benefits of component [24]

period	Time(h)	PM actions			System reliability	Maintenance cost
		C_1	C_2	C_3		
j=1	854	$A1$	$A1$	$A3$	0.88	2080
j=2	1708	$A1$	$A1$	$A1$	0.72	3340
j=3	2562	$A1$	$A1$	$A3$	0.88	2080
j=4	3416	$A1$	$A1$	$A1$	0.72	3340
j=5	4270	$A1$	$A1$	A3	0.88	2080
j=6	5124	$A1$	$A1$	$A1$	0.72	3340
j=7	5978	$A1$	$A1$	$A3$	0.88	2080
j=8	6832	$A1$	$A1$	$A1$	0.72	3340
j=9	7686	$A1$	$A1$	$A3$	0.88	2080
j=10	8500	$A1$	$A1$	$A1$	0.72	3340
$R_{\min} = 0.8;\ Total\ cost = \$27100$						

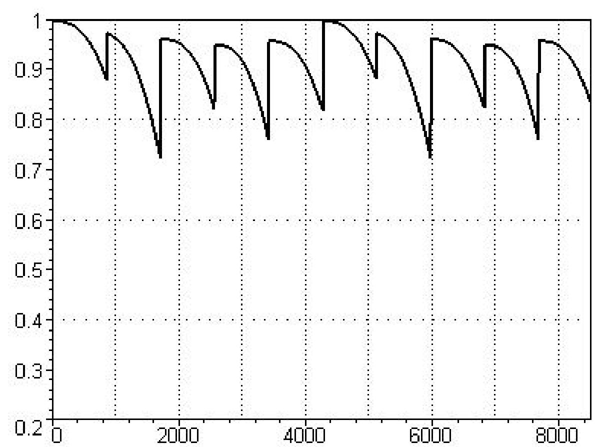
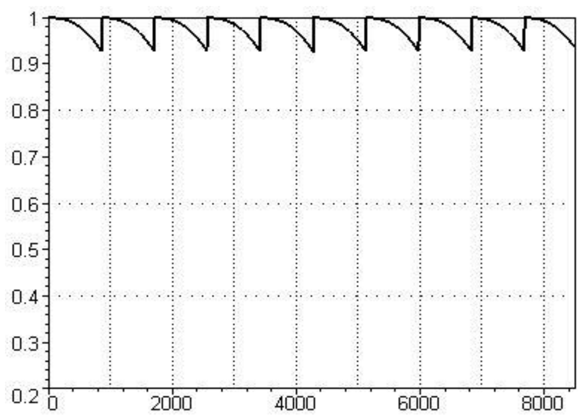
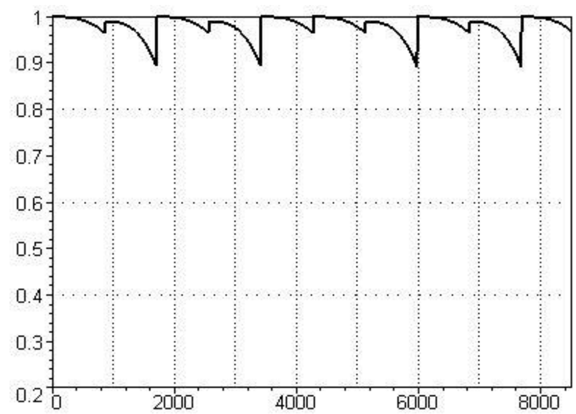


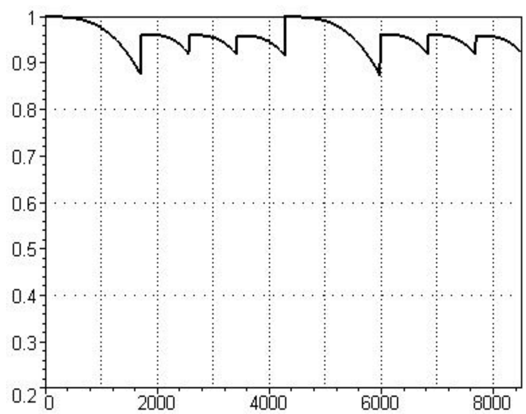
Figure 2: The reliability changing of system under the policy OMP



Component1



Component2



Component3

Figure 3: The reliability changing of components under the policy OMP

V. CONCLUSION

In this paper, an opportunistic maintenance policy (OMP) is proposed for a multi-component system. This policy is based on the joint optimization of maintenance cost and the inventory for the whole components. In this policy, at a periodic interval, the opportunity to undergo a specific maintenance actions to all components of the system has been investigated in order to optimize both the cost and the reliability. Based on a numerical example, this policy is more effective than a separate management of each component. Moreover, from a practical point of view, the single scheduled time for spare ordering is easier to implement than a multiple ordering points.

The main idea is to maintain the reliability of the system above R_{min}^{s} and the reliability of each component above a given level of reliability at a lower cost. In the future research, the gain of the benefit depending on the system architecture will be studied (parallel, bridge, parallel-series). A sensitivity analysis would be carried out to investigate the robustness of the model whenever a variation in the minimum level of reliability system occurs.

Nomenclature

$f(x)$ Probability density function
$F(x)$ Cumulative distribution function
$\overline{F}(x)$ Survivor function of time to failure
d Scheduled time for spare ordering
L Lead time between order and receipt of a spare part
T Scheduled time for preventive replacement $(T \geq d + L)$
T_L The lifetime of system
c_p Preventive replacement cost
c_r Corrective replacement cost $(c_r \geq c_p)$
c_{imp} Imperfect preventive maintenance cost
c_h Holding cost of a spare per unit time
c_s Downtime cost per unit time due to spare shortage
C_i Component i
$R_s(t)$ Reliability of system at time t
$R_i(t)$ Reliability of component C_i at time t
$R_j^i(t)$ Reliability of component C_i in the j^{th} period at time t
R_0 Initial reliability of every new component
R_{min}^s Minimum level of reliability of system
R_{min} Minimum level of reliability of every component
$C(d,T)$ Expected cost rate for an infinite time span

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