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RESEARCH ARTICLE

Semi-active Suspension Control with PSO Tuned LQR Controller Based on MR Damper

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ABSTRACT - As the Linear Quadratic Regulator (LQR) approach is applied extensively in the system control of automobile suspension, the accuracy improvement of the weighting Q and R matrices is getting concern. The Particle Swarm Optimization (PSO) algorithm is being introduced to identify parameters and optimize matrix Q and R in order to fix the insufficiency of these experienced values because of the fast convergence and a more accurate solution. In this article, a quarter car model and a Bouc-Wen-based magnetorheological (MR) damper model are developed to combine the control of PSO identification and PSO-LQR controller in the semi-active suspension system. The MR damper was performed with an experimental test for running identification using experimental data as input into the Bouc-Wen model to obtain six unknown parameters, where the parameters were estimated with the PSO algorithm. Since the numerical model has been done with all parameters clear, the need for damping force from suspension is obtained by means of running the model using an input current. In the employment of PSO for damper model and vehicle control, the dual applications succeeded in verifying the feasibility of parameter identification in the MR damper and successfully tuned the LQR controller in the semi-active suspension, which decreases the vehicle body acceleration and displacement so that the improvement of ride comfort and drive stability achieved.

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Parameter identification

1.0 INTRODUCTION

Suspension acts an important role in the chassis system of a vehicle, which works to hold the body of the vehicle, to keep the connection of wheels and to reduce vibration from road disturbance. In general, the suspension system is divided into three kinds, passive suspension, active suspension, and semi-active suspension, respectively. The passive suspension consists of a strong spring (used to store energy) and a hydraulic damper (shock absorber). The active suspension has a force generator that provides continuously variable damping force, which performs the best dynamic compensation for ride comfort and drive stability. However, the active suspension has great shortcomings, including the manufacturing cost and energy consumption. For semi-active suspension, it makes the balance of active suspension and passive suspension; it provides real-time continuous damping outputs and has both lower energy and lower cost [1]. Semi-active dampers can directly work as passive dampers if the controlled damper loses function. One of the most attractive semi-active devices is the MR damper; it has great advantages, such as large damping force, fast response, low energy cost, low manufacturing cost and small size [2]. The MR damper is being used in the study for both experimental and simulation works. For the simulation work, the damper model is using PSO algorithm to identify parameters, where the parameters highly have a dependency on the experimental data, which are used to calculate for reducing the PSO searching range of these parameters.

Parameter identification is a usual and useful method to deliver experimental to simulated work. Ikhouane [3] used an analytical method to describe entirely in the parameter identification. However, it's not a universally acceptable method due to the complex processing and the grand assumptions applied to this method. Later, Kwok [4] simplified the identification by using the GA algorithm in the parameter identification in which the crossover and mutation were adaptive. Zaman et al. [5] implemented a firefly algorithm to estimate parameters using the Bouc-wen model; they compared the performance with the GA algorithm and had the conclusion that the firefly algorithm works more efficiently than GA. Liu et al. [6] worked with PSO for parameter estimation in synchronous motors. However, the PSO can be generally used in the identification process in many fields if more precise parameters are not needed in the modeling application. To improve the accuracy of parameter identification, PSO is implemented in this article.

ElMadany et al. [7] developed the LQR methodology for 1/4 vehicle model by deriving from optimal control theory and the singular value of inequality matrix. Gigih et al. [8],[9] proposed the PSO algorithm to optimize a neural network strategy in suspension controlling. In 2008, PSO was used to achieve different performance aims for PD robust controllers [10]. Then, Rajeswari et al. [11] applied the PSO algorithm to tune Fuzzy Logic Controller in an active suspension [12]. A few years later, PSO tuned LQR strategy was used in an active mass damper [13]. Assahubulkahfi et al. [14] obtained

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better performance with a full-car model system running PSO tuned LQR controller than passive suspension, and the traditional LQR-controlled suspension, where PSO combined LQR is indicated as the best solution for the stability of car body movement. Due to the successful optimization implemented in the full-car model, the authors would suppose that the same method improves the quarter model. Also, the improvement effect of GA and PSO algorithms is usually addressed to make comparisons. PSO has a great efficiency on computational convergence, meanwhile has an apparent superiority in dealing with complex systems of Q matrix[14], [15]. Generally, the weighting matrix Q is more dependent on experience values in standard LQR strategy, and they are getting pretty complex when employed in vehicle control systems. To solve this problem, the PSO algorithm is introduced to search plenty of particles and select suitable matrix coefficients for LQR. Also, random pavement excitations are used as input for simulation analysis to verify the effectiveness of the optimized control strategy.

In the past, researchers either focused on the damper itself for modeling as well as parameter estimation or concentrated on the aspects of vehicle controlling, rarely applying the same algorithm to both. Also, due to previous researchers' work in comparison of the Genetic Algorithm (GA), PSO algorithm and Firefly Algorithm(FA) in other applications, PSO indicated having great advantages of computing cost and best performance as well as cost functions[16], the authors would like to perform PSO to optimize MR damper estimation and suspension system. However, previously PSO was employed for active suspension or full car models rather design than a semi-active suspension with 1/4 model. In this article, the authors employ dual PSO to identify MR damper and control the semi-active automobile using a quarter car model.

2.0 DAMPER BASED OPTIMIZATION

2.1 Mechanical model of Bouc-wen

Bouc-Wen model was first proposed as a hysteretic model by Bouc in 1967; then Wen promoted it in 1976. It is one of the simplest mathematical models for large hysteretic behavior, as shown in Figure 1.

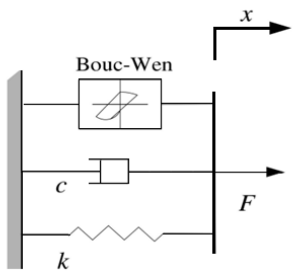


Figure 1. MR Damper of Bouc-Wen Model

The Bouc-Wen model consists of a hysteretic system, a damping component and an elastic element, the damping force equations of which is described as:

$$\begin{cases} F = c\dot{x} + k(x - x_0 + \alpha z) \\ \dot{z} = -\gamma \cdot |\dot{x}| \cdot z \cdot |z|^{n-1} - \beta \cdot \dot{x} \cdot |z|^n + \kappa \cdot \dot{x} \end{cases} \tag{1}$$

where F is the MR damping force, c is the damping coefficient, k is the stiffness of the mechanism, $\dot{x}$ and x are for the velocity and displacement of the piston, x_0 stands for the initial position of the piston, and z is for the evolutionary variable. α, γ, β, κ and n are adjusting parameters for hysteretic shape. γ and β only affect the shape of the hysteresis loop but have no influence on the damping force. Also, the parameter n only affects the smoothness of the curve transitioning from the elastic area to the plastic area [4],[17].

2.2 Test on MR damper

The MR damper was tested at BWI laboratory Moraine, and hysteresis loops were generated under amplitude 25 mm with excitation current 0A, 0.75A, 1.5A, 3A and 5A. The characteristic curves of force-velocity (F-v) and force-displacement (F-x) are shown in Figure 2 and Figure 3, respectively. Look through the F-v curve; it's a typical hysteretic characteristic that damping force is non-zero at the 0 speed of the piston. The loop area is expanding more visible with the excitation current getting larger. That means when the current is a non-changing steady value, the area enclosed by the curve indicates that the magnetorheological fluid deliveries work by outputting the damping force, and the scale of the graphic area shows how much energy the damper can absorb from vibration. So, at a higher speed zone, the energy absorbed is getting saturation. On the other hand, looking through the F-x curve, it shows a significant nonlinear between damping force and piston displacement. Generally, the damping force is going larger while the excitation current gets larger. However, when the excitation current increases from 3A to 5A, the rising trend of damping force becomes slower, and soon achieve to saturation.

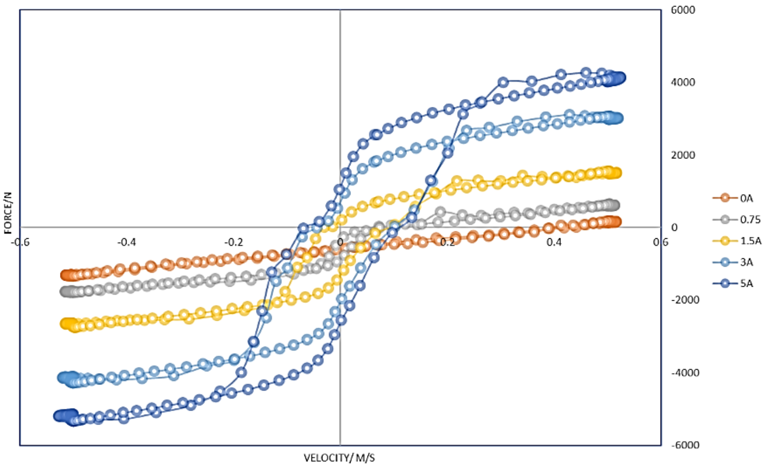


Figure 2. Velocity vs damping force

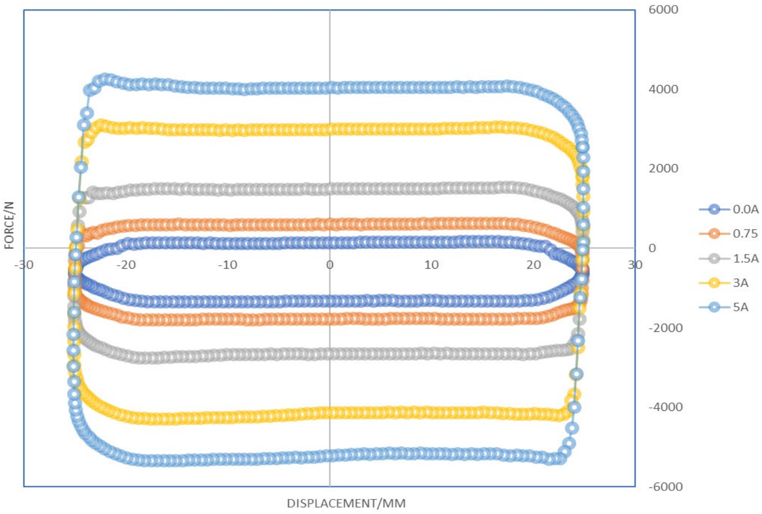


Figure 3. Displacement vs damping force

3.0 IDENTIFY PARAMETERS OF MR DAMPER USING PARTICLE SWARM OPTIMIZATION

The PSO algorithm was proposed by James Kennedy and Russell Eberhart in 1995 [18], where it was improved from Bird Swarm Algorithm and Fish Swarm Algorithm. The PSO algorithm processes each solution as an individual case, which is called a particle, and the dynamic position of which is determined by the fitness value. There are two best solutions for each particle by keeping search; one of them is called individual best (P_{best}) of i particle, and the other is called global best (G_{best}) of the group particles. The purpose of applying the PSO algorithm is to get its advantages of fast convergence, high precision and easy-to-implement [19].

In the Bouc-Wen model, there are eight unknown parameters, which are c, k, x_0, α, γ, β, κ and n, respectively. Here, F can be transformed into the equation as follows.

$$F = c\dot{x} + kx - kx_0 + k\alpha z \tag{2}$$

Further,

$$F \doteq c\dot{x} + kx + k\alpha z \tag{3}$$

x_0 has fewer effects on the damping force and also has absolutely nothing to the evolutionary variable z. Therefore, x_0 is assumed to be 0 for parameter identification in this article. For the parameter n, it is assumed to be two due to a fixed MR fluid used in the MR damper. The coefficient n values are stable by changing the suitable MR fluid of the specified damper. Therefore, there are six parameters needed to get identified; c, k, α, γ, β and κ.

The criteria to see if all the six parameters have achieved good identification is the matching level between testing data and simulation data. Also, it can be called fitness function, which is a population standard deviation as in Eq. (4).

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(F_t(i) - F_s(i))^2} \tag{4}$$

where N is the sample number for testing and simulation data, $F_t(i)$ stands for the damping force of experimental testing, and $F_s(i)$ is the value of simulation.

Then, the important step of PSO is able to keep updating for the individual best solution (P_{best}) and the global best solution (G_{best}). During the searching or updating process, the velocity and position are the two critical aspects as formulated in Eq.(5) and Eq. (6), respectively [20]-[22].

$$v_{t+1} = \omega_t v_t + c_1 r_1 (P_{best} - x_t) + c_2 r_2 (G_{best} - x_t) \tag{5}$$

$$x_{t+1} = v_{t+1} + x_t \tag{6}$$

where v is the velocity, x stands for the position, r_1, r_2 account for random numbers, and t is the current iteration. The ω accounts for a function of inertia weighting as the formula in Eq. (9) with the meaning of searching capability of the PSO algorithm. c_1, c_2 are learning factors that can be described as in Eq. (7) and Eq. (8).

$$c_1 = \frac{t}{it}\left(c_{1\_0} - c_{1\_end}\right) + c_{1\_end} \tag{7}$$

$$c_2 = \frac{t}{it}\left(c_{2\_0} - c_{2\_end}\right) + c_{2\_end} \tag{8}$$

where c_{1_0}, c_{2_0} stand for the initial learning factor, c_{1_end}, c_{2_end} are completed learning factors in the end, and it is the total times of iteration.

$$\omega = \omega_{max} - \frac{t}{it}(\omega_{max} - \omega_{min}) \tag{9}$$

Initial inertia weighting ω_{max} is always taking a larger value to help improve searching capability, meanwhile final inertia weighting ω_{min} is taking smaller value with the help of local searching capability. A smaller weighting factor ω has a higher ability in local searching and a lower function for global searching. On the other hand, a larger weighting factor ω has weaker searching in the local domain and more powerful for global searching.

In the parameter estimation process for MR damper, the setting parameters for the algorithm are shown as particles = 20, iteration= 50, dimensions = 6, c_1 = 1, c_2 = 1 and ω = 0.6. Meanwhile, for the use of PSO in the vehicle control for LQR optimization, these parameters are set as particles = 30, iteration= 80, dimensions = 3, c_1 = 2, c_2 = 2 and ω = 0.9. Basically, the number of particles is the larger, the better. However, the balance of running time and slight global improvement should be taken into account. Usually, the number of particle and iterations should be considered a larger value for a complex system, which is the reason to set the larger valve for vehicle control comparing with damper estimating. In the testing by the authors, c_1 and c_2, the learning factor, the value of which is affecting not too much to the system, but the value for c_1 and c_2 should be the same and none zero so that the system has equal ability for c_1 and c_2 to solve the object system in order to get the best solution.

The group parameters setting above are just the best choices for the systems mentioned in this article. However, the actual determination of the above parameters should be slightly adjusted per the complexity of each object as well as the quantity of the variables.

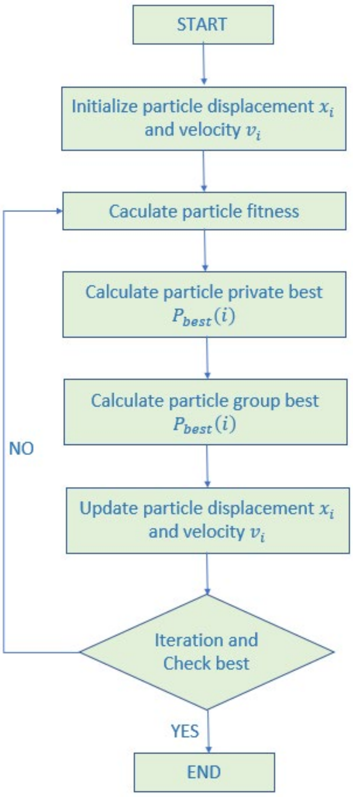


Figure 4. Scheme of PSO applied in the damper

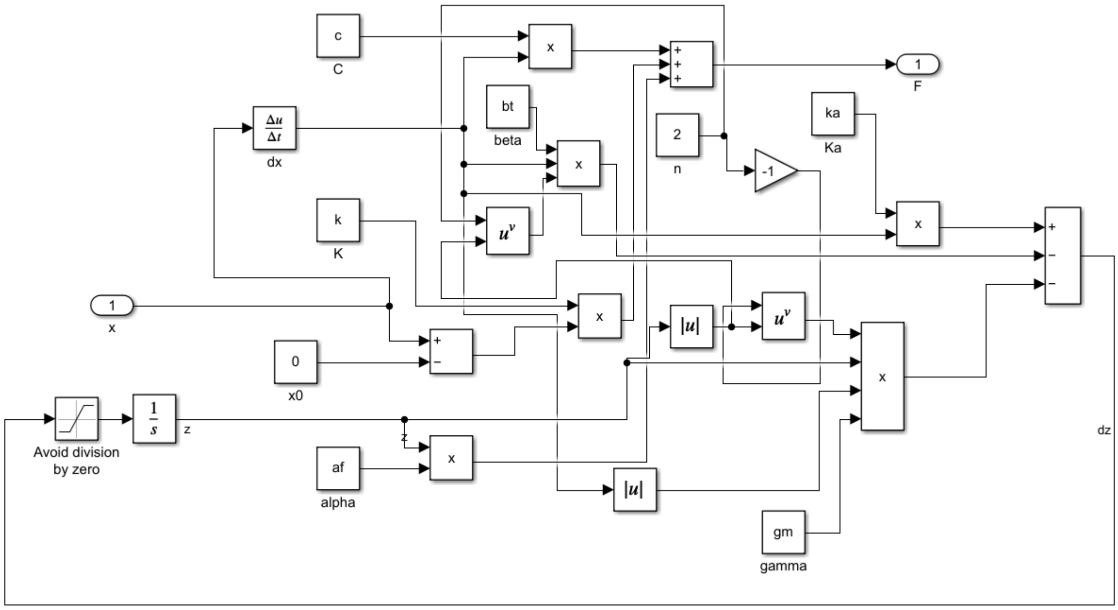


Figure 5. Parameter identification Block of Bouc-wen

After the parameter identification, six unknown parameters can be identified to compare with the simulated damping force. Moreover, the most important curve-fitting work can be done when the six parameters (c, k, α, γ, β and κ) are replaced with excitation current so that we can run suspension performance with force instead than offline; looking for the related current and damping force to implement the active suspension as a semi-active suspension.

$$\begin{cases} c = 0.01I^3 + 0.075I^2 - 0.35I + 0.63 \\ k = -5.6I^3 + 49.17I^2 - 54.28I + 33.98 \\ \alpha = 7.26I^3 - 501I^2 + 373I + 239.1 \\ \gamma = -11.51I^3 - 56.04I^2 - 93.32I + 1022 \\ \beta = -0.94I^3 + 12.99I^2 - 49.3I + 64.09 \\ \kappa = -5.6I^3 + 49.17\,I^2 - 54.28I + 3.98 \end{cases}$$

4.0 SUSPENSION BASED OPTIMIZATION

4.1 Quarter-Car Suspension Model

With the purpose of the analysis for vertical movement of the vehicle body and tire, a 1/4 vehicle model with 2 DoF is developed to address related problems, as shown in Figure 6. The 1/4 car model is not only much simpler than a 1/2 car model or a full car model but also has good accuracy in the simulation of body acceleration, suspension deflection and tire dynamic loads with an excitation from the road.

According to Newton's law of mechanics [23], mathematical equations are obtained from Figure 6 as in the following Eq. (10).

$$\begin{cases} M_s\ddot{X}_s + K_s(X_s - X_u) + C_s(\dot{X}_s - \dot{X}_u) + F_s = 0 \\ M_u\ddot{X}_u + K_s(X_u - X_s) + C_s(\dot{X}_u - \dot{X}_s) + K_t(X_u - X_r) - F_s = 0 \end{cases} \quad (10)$$

where M_s and M_u are the sprung mass and unsprung mass, K_s, K_t stand for spring stiffness of suspension and stiffness of the tire, C_s is the damping coefficient of uncontrollable part, F_s is a variable damping force controlled by LQR in this article. X_s, X_u and X_r are the displacements of sprung mass, unsprung mass, and road displacement of disturbance, respectively.

In order to make it convenient to get solution process, the state space equation is introduced as the following Eq. (11).

$$\begin{cases} \dot{X} = AX + BU + EW \\ Y = CX + DU \end{cases} \quad (11)$$

X is the state variable matrix, where $(X_s - X_u)$ is suspension dynamic displacement, $(X_u - X_r)$ is the tire dynamic displacement, Y is the output variable matrix, U is the controllable damping force.

$$X = \left(X_s - X_u,\ X_u - X_r,\ \dot{X}_s,\ \dot{X}_u,\ X_r\right)^T \quad (12)$$

$$Y = \left(\ddot{X}_s,\ X_s - X_u,\ X_u - X_r\right)^T \quad (13)$$

W is the road excitation displacement input called filtered Gaussian white noise, which can be described as the expression in Eq. (14).

$$\dot{X}_r = -2\pi f_0 X_r + 2\pi\sqrt{G_0 v}\, w \quad (14)$$

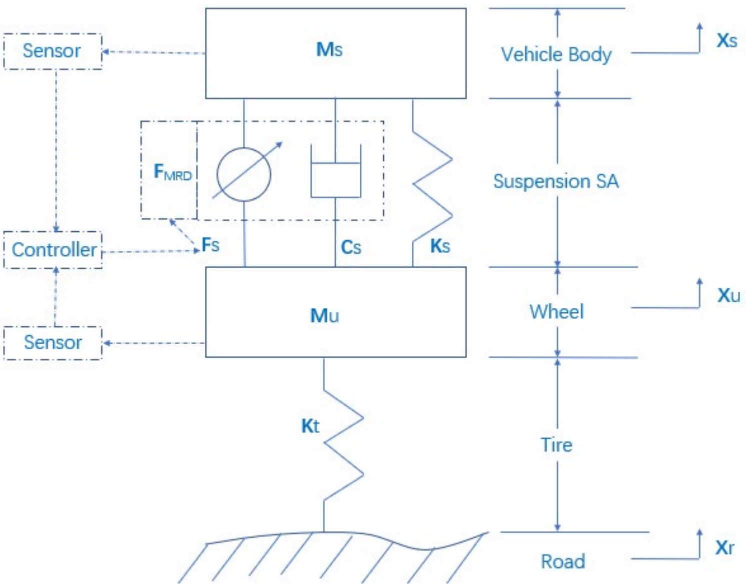


Figure 6. Quarter vehicle model of semi-active suspension

where f_0 is the lower cutoff frequency of road-passing signal, G_0 is the coefficient of road roughness, v is the velocity of the vehicle, and w is the white noise time domain variable.

Following results are obtained by calculation of transforming Eq. (10) to Eq. (14).

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -2\pi f_0 \\ -K_s/M_s & 0 & -C_s/M_s & C_s/M_s & 0 \\ K_s/M_u & -K_t/M_s & C_s/M_s & -C_s/M_s & 0 \\ 0 & 0 & 0 & 0 & 2\pi f_0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -1/M_s \\ 1/M_u \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & -K_s/M_s & K_s/M_s & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1/M_s \\ 0 \\ 0 \end{bmatrix} \qquad E = \begin{bmatrix} 0 \\ -2\pi\sqrt{G_0 v} \\ 0 \\ 0 \\ 2\pi\sqrt{G_0 v} \end{bmatrix}$$

5.0 CONTROLLER DEVELOPMENT WITH PSO IMPROVED LQR

Based on a 1/4 car model, there are three critical indicators to achieve for optimizing suspension performance, body vertical acceleration, suspension deformation and wheel deformation. Such indicators are showing inversely related to suspension performance, which means the smaller the indicators, the better the performance. So, mathematic functions should be introduced to clarify how such three indicators work, they are expressed in Eq. (15).

$$\begin{cases} J_1 = \lim\limits_{T\to\infty} \dfrac{1}{T} E\left\{ \displaystyle\int_0^T \ddot{X}_s^{\,2}\, dt \right\} \\ J_2 = \lim\limits_{T\to\infty} \dfrac{1}{T} E\left\{ \displaystyle\int_0^T (X_s - X_u)^2\, dt \right\} \\ J_3 = \lim\limits_{T\to\infty} \dfrac{1}{T} E\left\{ \displaystyle\int_0^T (X_u - X_r)^2\, dt \right\} \end{cases} \tag{15}$$

where T is a limitation equal to infinity and, $E\{\dots\}$ is expectation operation. Combining performance indicators J_1, J_2, J_3 into a single indicator, J is a better method to identify useful matrix helping to obtain solutions.

$$J = \lim_{T\to\infty} \frac{1}{T}\, E\left\{ \int_0^\infty \left[q_1 \ddot{X}_s^{\,2} + q_2 (X_s - X_u)^2 + q_3 (X_u - X_s)^2 \right] dt \right\} \tag{16}$$

where q_1, q_2, q_3 are weighting factors, equation (16) can be transformed as in Eq. (17).

$$J = \int_0^\infty (Y^T Q_0 Y) dt = \int_0^\infty (Q^T W X + U^T R U + 2X^T N U) dt \tag{17}$$

where

$$Q_0 = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \quad R = D^T Q_0 D \quad W = C^T Q_0 C \quad N = C^T Q_0 D$$

Per optimal control theory considering system feedback [24], the controllable force expression is described as Eq. (18).

$$\begin{cases} U = -KX = F_s \\ K = SR^{-1}B^T \end{cases} \tag{18}$$

where S is the solution obtained from the Riccati equation which is transformed from Eq. (17), K is the optimal control feedback gain matrix, U is generated by LQR controller as an optimal damping force F_s

$$SA + A^T S - SBR^{-1}B^T S + Q = 0 \tag{19}$$

Observing from Eq. (15) to Eq. (19), the controlled performance using the optimal control law is affected by weighting factors, q_1, q_2 and q_3. The Q matrix is developed with multiple attempts based on the designer's experimental work. Thus, the PSO algorithm is used for optimizing the weighting parameter. As the PSO method mentioned above in the implementation of parameter identification Eq. (4) to Eq. (9), it does not have a repetition of introduction in this section.

A fitness function in Eq. (20) is addressed to deal with suspension problem, this function is compared based on the performance of passive suspension.

$$\sigma_{(q)} = \left(\frac{\sigma_{\ddot{X}}}{\sigma_{pas_{\ddot{X}}}} + \frac{\sigma_{X_s - X_u}}{\sigma_{pas_{X_s - X_u}}} + \frac{\sigma_{X_u - X_r}}{\sigma_{pas_{X_u - X_r}}} \right)_{q=q1,q2,q3}^{min} \tag{20}$$

where $\sigma_{\ddot{X}}$, $\sigma_{X_s-X_u}$ and $\sigma_{X_u-X_r}$ are the root mean square (RMS) for body vertical acceleration, suspension deformation and wheel dynamic deformation; meanwhile, they contain weighting factor q_1, q_2 and q_3. These three parameters should be lower than passive indicators σ_{pas} as per Eq. (21).

$$\begin{cases} \sigma_{\ddot{X}} < \sigma_{pas_{\ddot{X}}} \\ \sigma_{X_s-X_u} < \sigma_{pas_{X_s-X_u}} \\ \sigma_{X_u-X_r} < \sigma_{pas_{X_u-X_r}} \end{cases} \tag{21}$$

To have a better convergence and efficiency, q_1, q_2 and q_3 should be set a constraint range between 0.01 to 10^6.

6.0 SIMULATION RESULTS OF SUSPENSION

The simulation is conducted based on a quarter-car model using an LQR controller with PSO-optimized weighting factors, as in Figure 7. The road roughness is using C class road [25], and other input parameters are documented below in Table 1.

Table 1. Input parameters for simulation

Parameters	Value	Unit
Sprung mass M_s	360	kg
Unsprung mass M_u	50	kg
Suspension stiffness K_s	25000	N/m
Tyre stiffness K_t	200000	N/m
Vehicle speed v_0	80	km/h
Lower cut-off frequency f_0	0.1	Hz

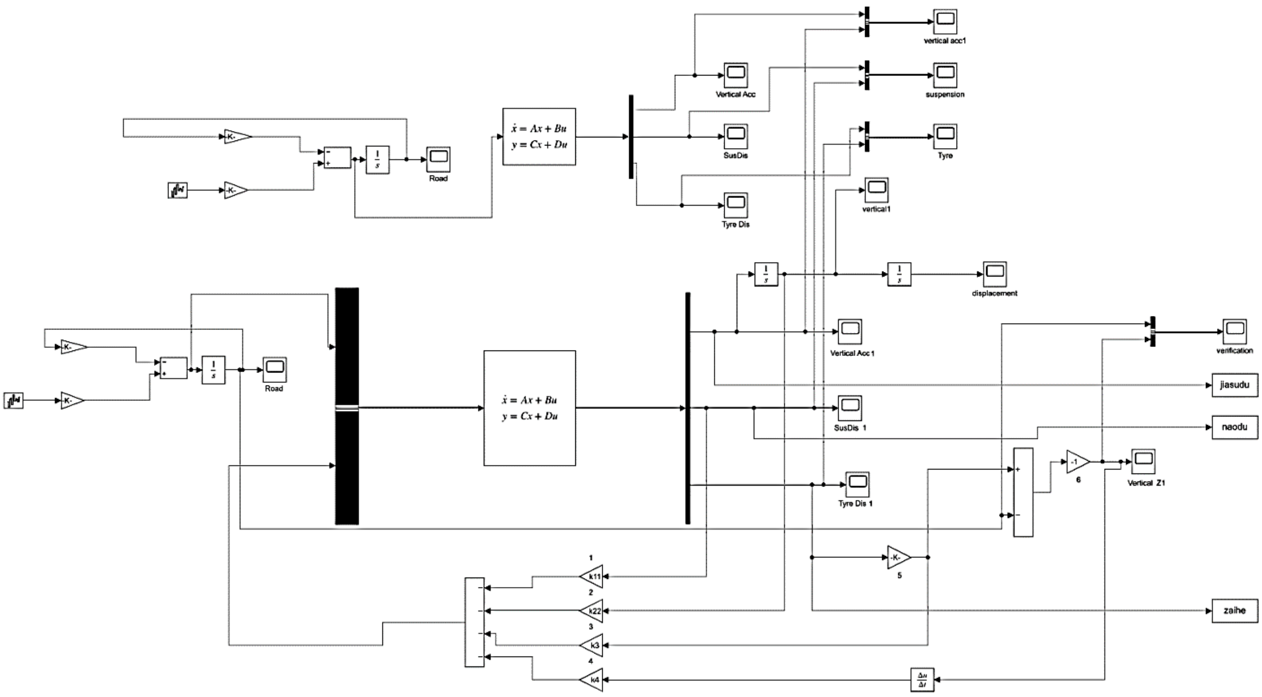
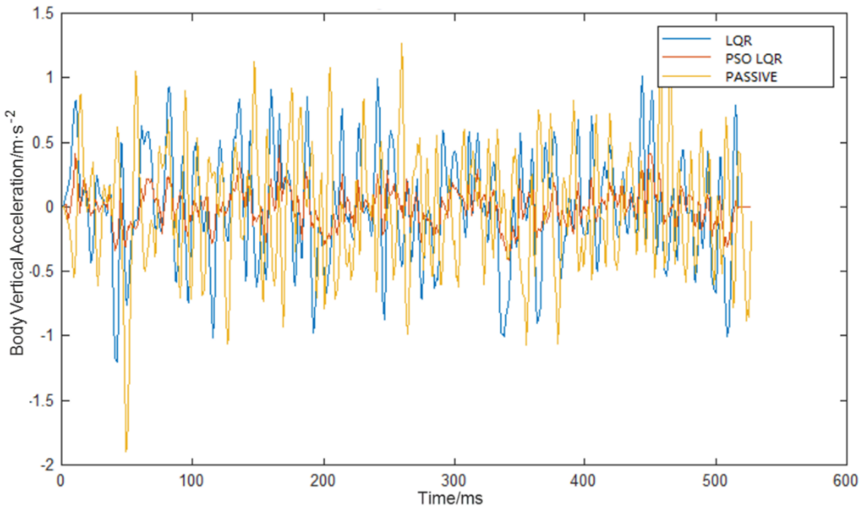
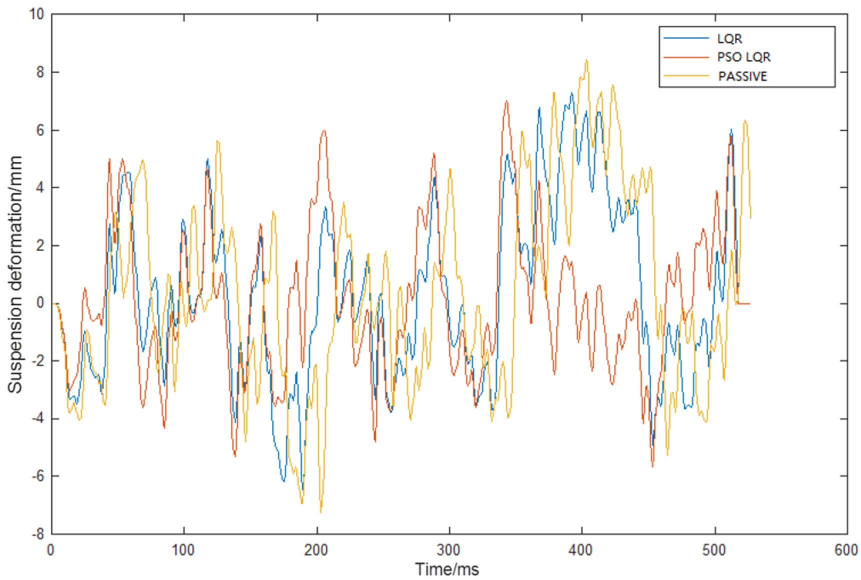


Figure 7. Simulink Scheme of 1/4 model

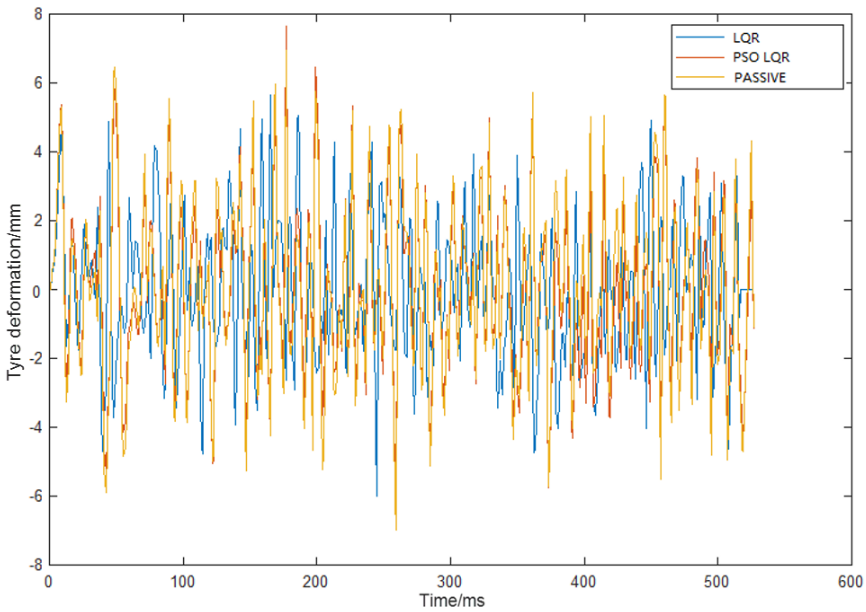
Setting initial particle dimension 3, maximum iteration 80, learning factors $c_1 = 2$, $c_2 = 2$, inertia weighting factor $\omega = 0.9$(The PSO parameters are stated in Section 3.0. Best solutions are obtained after the simulated work is completed, so the best (q_1, q_2, q_3)= (8.554, 2.132e4, 1e-4). Then the program substitutes the best q_1, q_2, q_3 into LQR solver to run final results, we have three critical indicators to judge the performance of a suspension, which are respectively body vertical acceleration, suspension deformation and tyre deformation. To have more gains, we should compare the indicators with passive suspension, traditional LQR controlling approach, and PSO-tuned LQR method, as in Figure 8.



(a) body vertical acceleration



(b) suspension deformation



(c) tyre deformation

Figure 8. Comparison of suspension performances with different strategies

After the comparison works were completed, it was obvious to see that the suspension with a strategy of PSO-tuned LQR significantly reduced the body vertical acceleration. Moreover, it had a good reduction of suspension deformation and tyre dynamic deformation than the performances of the other two strategies. Thus, it indicates from the results that the control method of PSO-tuned LQR with a suspension works effectively, and it enhances the ability of the damper to absorb the energy of vibration. In this part of the work for running suspension analysis, a quarter car model is introduced to simulate the MR damper-based semi-active suspension in which PSO tuned LQR controlling method was successfully implemented to reduce the three critical judging indicators which are the body vertical acceleration, suspension deformation and tyre deformation. They have been improved by 37.8%, 24.2% and 15.8% through the comparison with passive suspension performance; meanwhile, they have been respectively improved by 17%, 16.67% and 7.76% by comparing traditional LQR controlling strategy, as given in Table 2.

Table 2. Comparisons of improvement with RMS

STRATEGY	Body vertical acceleration	Suspension deformation	Tyre deformation
PASSIVE	0.37	0.0033	240
LQR	0.28	0.0030	219
PSO LQR	0.23	0.0025	202
Improvement compared with Passive	37.8%	24.2%	15.8%
Improvement compared with LQR	17%	16.67%	7.76%

7.0 CONCLUSIONS

The purpose of identification parameters for MR damper is to build an ideal model which can produce a damping force that semi-active suspension needed by using an offline single excitation current to replace these six unknown parameters. It demonstrates the feasibility of the PSO algorithm that is able to have efficient parameter identification. The numerical model was significantly simplified, and we were able to run 1/4 model suspension with an ideal damping force using the related excitation current to replace semi-active suspension with executive active suspension. The results significantly indicate that the optimization aims for accuracy improving ride comfort, and handling stability have been successfully well achieved using PSO tuned LQR approach. The dual PSO optimization is feasibly employed for an LQR semi-active car system equipped with an MR damper, resulting in the improvement of body vertical acceleration, suspension deformation and tyre deformation.

8.0 REFERENCES

[1] S. Park, A. Turnip, and K.S. Hong, "Application of LQR control theory to the design of modified skyhook control gains for semi-active suspension systems," In 2009 ICCAS-SICE, IEEE, 2009, pp. 4698-4703.

[2] M. Zapateiro, F. Pozo, H.R. Karimi, and N. Luo, "Semiactive control methodologies for suspension control with magnetorheological dampers*," IEEE/ASME Transactions on Mechatronics*, vol. 17, no. 2, pp. 370-380, 2011.

[3] F. Ikhouane, and J. Rodellar, *Systems with hysteresis: Analysis, identification and control using the Bouc-Wen model*. Chichester, England: John Wiley & Sons, 2007.

[4] NM. Kwok, Q.P. Ha, M.T. Nguyen, J. Li, and B. Samali, "Bouc–Wen model parameter identification for a MR fluid damper using computationally efficient GA," *ISA Transactions*, vol. 46, no. 2, pp. 167-179, 2007.

[5] M.A. Zaman, and U. Sikder, "Bouc–Wen hysteresis model identification using modified firefly algorithm," *Journal of Magnetism and Magnetic Materials*, vol. 395, pp. 229-233, 2015.

[6] L. Liu, W. Liu, and D.A. Cartes, "Particle swarm optimization-based parameter identification applied to permanent magnet synchronous motors," *Engineering Applications of Artificial Intelligence*, vol. 21, pp. 1092–1100, 2008.

[7] ElMadany, M. Mohamed, and S. Zuhair, and Abduljabbar, "Linear quadratic Gaussian control of a quarter-car suspension," *Vehicle System Dynamics*, vol. 32, no. 6, pp. 479-497, 1999.

[8] J. Theunissen, A. Tota, P. Gruber, M. Dhaens, and A. Sorniotti, "Preview-based techniques for vehicle suspension control: A state-of-the-art review," *Annual Reviews in Control*, vol. 51, pp. 206-235, 2021 Jan 1

[9] G. Priyandoko, M. Mailah, and H. Jamaluddin, "Particle swarm optimization neural network based modelling of vehicle suspension system," presented at Regional Postgraduate Conference on Engineering and Science (RPCES 2006), Universiti Teknologi Malaysia, 2006.

[10] H.M. Soliman, M.A. Awadallah, and M.N. Emira, "Robust controller design for active suspensions using particle swarm optimization," *International Journal of Modelling, Identification and Control,* vol. 5, no. 1, pp. 66-76, 2008.

[11] K. Rajeswari, and P. Lakshmi, "PSO optimized fuzzy logic controller for active suspension system," In 2010 International Conference on Advances in Recent Technologies in Communication and Computing, 2010, pp. 278-283.

[12] L. Ovalle, H. Ríos, and H. Ahmed, "Robust control for an active suspension system via continuous sliding-mode controllers," *Engineering Science and Technology, an International Journal,* vol. 28, pp. 101026, 2022.

[13] F. Amini, NK. Hazaveh, and A.A. Rad, "Wavelet PSO-based LQR algorithm for optimal structural control using active tuned mass dampers," *Computer-Aided Civil and Infrastructure Engineering*, vol. 28, no. 7, pp. 542-557, 2013.

[14] M. Assahubulkahfi, Y.M. Sam, A. Maseleno, and M. Huda, "LQR tuning by particle swarm optimization of full car suspension system," *International Journal of Engineering & Technology*, vol. 7, pp. 328-331, 2018.

[15] S. Liu, Q. Sun, L.W. Hou, N. Niu, and LL Sun, "Optimal control of active seat suspension systems using acceleration based particle swarm optimization," In 2017 Chinese Automation Congress (CAC), 2017, pp. 6777-6782.

[16] A. Kumar, D.K. Gupta, S.R. Ghatak, and S.R. Prusty, "A comparison of PSO, GA and FA-based PID controller for load frequency control of two-area hybrid power system," In Smart Technologies for Power and Green Energy: Proceedings of STPGE 2022, 2022, pp. 281-292.

[17] M.H. Harun, "Hybrid fractional modified skyhook controller optimized by particle swarm optimization for railway secondary lateral suspension," Ph.D thesis, Universiti Teknologi Malaysia, Johor Bharu, 2022.

[18] J. Kennedy, "Swarm intelligence," in *Handbook of nature-inspired and innovative computing*, A.Y. Zomaya, Eds. Boston, MA: Springer, 2006, pp. 187-219.

[19] H. Guoliang, L. Hao, and L. Gang, "Parameter identification for Bouc-Wen model of magnetorheological damper based on particle swarm optimization and least square method," *Journal of Magnetic Materials and Devices*, vol. 51, no. 5, pp. 30-35, 2020.

[20] Y. Yang, Y. Ding, and S. Zhu, "Parameter identification of MR damper model based on particle swarm optimization," In Proceedings of the 11th International Conference on Modelling, Identification and Control (ICMIC2019) 2020, pp. 555-563.

[21] A. Elsawaf, H. Metered, T. Vampola, and Z. Šika, "Parameter identification of magnetorheological damper using particle swarm optimization," *International Journal of Earthquake Engineering*, vol. 2, no. 1, pp. 1-6, 2015.

[22] M.A. Razman, G. Priyandoko, and A.R. Yusoff, "Bouc-Wen model parameter identification for a MR fluid damper using particle swarm optimization," *In Advanced Materials Research*, vol. 903, pp. 279-284, 2014.

[23] S. Cui, *Vehicle system dynamics and simulation*. Beijing, China: Peking University Press, 2014.

[24] KD. Rao, and S. Kumar, "Modeling and simulation of quarter car semi active suspension system using LQR controller," In Proceedings of the 3rd International Conference on Frontiers of Intelligent Computing: Theory and Applications (FICTA) 2014 2015, pp. 441-448.

[25] X. Wang, and D. Bao, "Application of improved particle swarm algorithm in LQR semi-active suspension," *Journal of Mechanical Science and Technology for Aerospace Engineering*, vol. 3, pp. 1-7, 2022.