

A Review on Various Control Strategies and Algorithms in Vehicle Suspension Systems

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ABSTRACT - Automotive suspension systems provide passenger safety, ride comfort and vehicle handling in passenger and commercial vehicles. Through extensive research in the last couple of decades coupled with the recent advancements in technology, the improvement in vehicle handling and ride comfort have been significant by using various control strategies in semi-active and fully active suspension systems. Despite a significant number of articles available on the enhancement and improvement of vehicle suspension systems, there is certainly a lack of knowledge on various control strategies and algorithmic techniques used in the vehicle suspension system. Thereby, to address the gap, this review consecutively attempts to comprehensively explore the various research work conducted on the various control strategies used in vehicle suspension systems.

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1.0 NOMENCLATURE

M_s	Sprung mass (kg)
M_{us}	Unsprung mass (kg)
X_s	Sprung mass displacement (m)
X_{us}	Unsprung mass displacement (m)
C_s	Suspension damping coefficient (Ns/m)
C_t	Tyre damping coefficient (Ns/m)
K_s	Suspension spring coefficient (N/m)
K_t	Tyre spring coefficient (N/m)
F_a	Force actuator (N)
X_t	Road profile

2.0 INTRODUCTION

Technology advancements in the automotive industry focus predominantly on delivering low energy consumption, better passenger comfort and road-holding capability, especially in commercial road vehicles. The design and control system of the vehicle suspension systems play an important part in providing comfort and stability at a minimum cost [1]. The vibrations caused by the load on the vehicle and road irregularities are the main reasons for the undesirable conditions faced by the vehicle control system [2]. Designing a robust and effective control system has been a long-standing engineering problem in vehicle suspension systems [3]. There are three types of suspensions used in the modern world, namely passive, semi-active and active systems. Passive suspension systems, consisting of a spring and fixed damper, provide little comfort and handling characteristics to the passengers on even and uneven road conditions. Through extensive research, the inadequacies of the passive suspension system are neglected by fitting adjustable dampers.

Self-adjusting dampers in active suspension systems are supplemented with force actuators like hydraulic and electromagnetic actuators. Damping in the system can be varied continuously based on the input of the force actuator. Force actuators can generate and store the energy required for the smooth operation of the active suspension, which is not possible in semi-active and passive suspension systems [4]. The active suspension system gives the best overall performance compared with semi-active and passive suspension systems. Active suspension systems are very complex to manufacture and require high maintenance costs; external energy is required for their operation, which is not feasible in commercial passenger vehicles.

Semi-active suspension is preferred over other suspension systems because of its low price and less energy consumption in the automobile industry [5]. Semi-active suspensions are far superior in achieving damping control on varying road profiles compared with passive suspension systems [6]. Semi-active suspensions possess components of passive systems, but the differentiating factor between the two systems is that the semi-active systems dissipate the energy, making the system intrinsically stable [7]. Hamersma et al. discovered that an anti-lock braking system (ABS) performed better when fitted with semi-active suspension [8]. Even though ABS control the vehicle in heavy braking conditions,

there is a performance change on uneven road surfaces. An SUV model fitted with semi-active suspension reduced the vehicle stopping distance by 9 meters [9].

The ride performance limitation, high cost, and other complexities associated with active suspension systems are overcome using semi-active suspension systems [11]. Control algorithms, semi-active dampers, accelerators, and ride height sensors are the main parts of semi-active suspension systems. The active suspension system consists of the following components - actuators, hydraulic pumps and accumulators. In this paper, the focus is on control systems and algorithms used in semi-active and active suspension systems. Many studies on H_∞ state feedback controllers [12], robust control active suspension control strategy [13,14] and fuzzy control systems were carried out in vehicle suspension systems [16]. The controlled suspension systems are integrated into hybrid and electric passenger car segments through extensive research and development. The merits and limitations of vehicle suspension systems are presented in Table 1.

Table 1. Merits and limitations of vehicle suspension systems

Parameters	Passive suspension	Semi-active suspension	Active suspension
Construction	easy	difficult	very difficult
Cost	low	medium	high
Ride comfort	bad	good	very good
Vehicle handling	bad	good	very good
Life expectancy	low	high	very high
Maintenance	low	medium	high
Weight	light	medium	heavy
Energy requirement	low	medium	very high
Performance	low	medium	very high

Semi-active dampers use a combination of hydro-mechanical devices to dissipate the energy using limited power. Energy-dissipating devices used to obtain the desired damping in the vehicle suspension systems are as follows

- i. Solenoid valves/servo-valves dampers
- ii. Magnetorheological (MR) dampers
- iii. Electromagnetic dampers
- iv. Electrorheological (ER) dampers

A servo valve damper consists of a hydraulic actuator to control the orifice area. Servo valves have high response time, complex design and are more expensive in comparison with solenoid valves. But the overall efficiency of the servo dampers is much better than solenoid dampers [17]. The limitations of the servo dampers are overcome by using magnetorheological or electrorheological dampers [18]. Magnetorheological and electrorheological dampers are used due to their faster time response (40 ms), larger force range and a wider portion of control and cost-effectiveness [19]. Passive system, Semi-active system and Active system are shown in Figure 1. (a), (b), and (c) respectively.

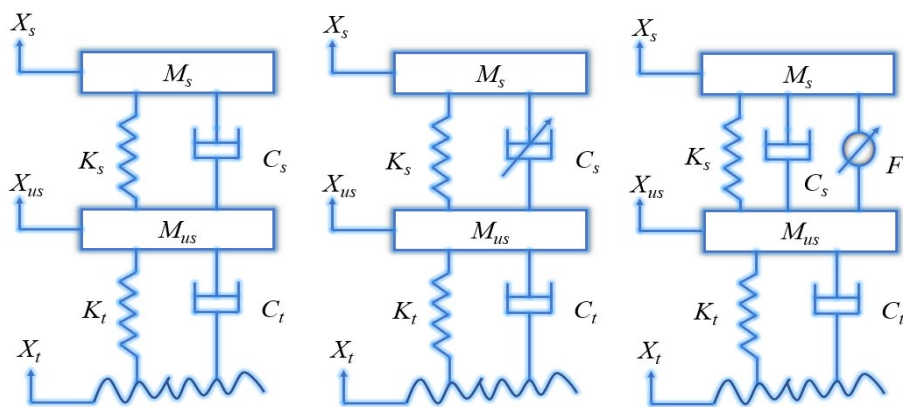


Figure 1. (a) passive system, (b) semi-active system, (c) active system

3.0 CONTROL SYSTEM

A control system refers to a comprehensive assemblage of components (viz., controller, current convertor, accelerometer, and signal amplifier) operated under predefined systematic procedures. These control systems are specifically devised to govern and regulate the behavior of a dynamic system or process of automotive suspensions, with the ultimate objective of attaining predetermined outcomes or optimizing performance [20]. They are also referred to as the set of mechanical and electronic devices that control the system through control loops. These control loops could be open-loop or closed-loop, depending on the system requirements. It directs, regulates and commands the system to get the desired output. It can be tuned and analyzed in ways suitable for the system to function properly, with minimum transient errors. Based on the input signals used, they can be continuous time systems/discrete-time systems. Single Input

Single Output (SISO) systems are used when one input and one output are required. Multiple Input Multiple Output (MIMO) systems are used when more than one input and one output are required. These control systems have a wide range of applications, such as commercial, passenger and autonomous vehicles where passenger safety is paramount. A controller should adhere to the following objectives [21].

- i. **Stability:** The system should always produce an output response when excited with an input signal. This system is called a bounded input bounded output (BIBO) system. This is especially significant for inherently unstable systems. The stability of a system is classified into three types.
 - Absolute stable system
 - Conditionally stable system
 - Marginally stable system
- ii. **Tracking:** The control system should track the desired reference value (setpoint). The system response should have minimal transient and steady-state error.
- iii. **Robustness:** The control system should exhibit stability and tracking despite external disturbances. The system must remain stable at all times, closer to the setpoint assigned, irrespective of internal or external disturbances.

In a vehicle suspension system, where the operating conditions are continuously changing, an open-loop control system is not a possibility. Thus, to get the desired response under all operating conditions, closed-loop control systems are preferred. As these systems adjust automatically to the needs of the system objectives, they are also called automatic control systems. Controllers can be divided into three categories: (i) classical controllers; consider only past and current system behavior. This system is reactive to any disturbance or deviation in the signals within the system, (ii) predictive controllers; can anticipate deviations from the system using a system model to predict the future trajectory from the reference input, (iii) repetitive controllers calculate the future path of the next cycle of the system based on the behavior of the previous cycle which can help in calculating an optimal trajectory for the next cycle [22]. The block diagram of the active suspension control system is shown in Figure 2

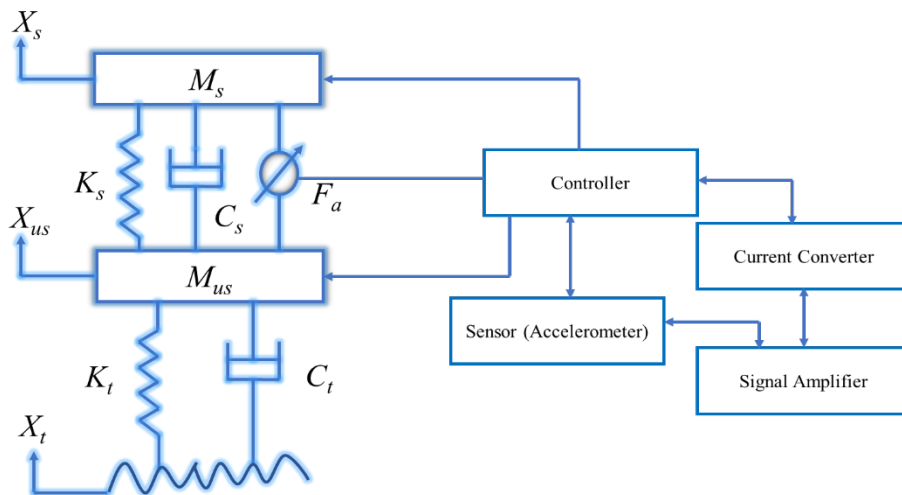


Figure 2. Block diagram of active suspension control system

4.0 SUSPENSION CONTROLLER

Suspension control systems are of paramount importance in augmenting the entire driving experience through the provision of a dynamic and adjustable suspension configuration. Suspension controllers monitor the vehicle’s behavior, road conditions, and driving dynamics in real-time using data from numerous sensors such as accelerometers, wheel position sensors, and occasionally even cameras [23]. The controller uses this data and accordingly enables respective quick modifications to the suspension system in order to attain seamless operation, eventually leading to effective performance. Suspension controllers control the active suspension systems, in which the damping parameters of each wheel can be adjusted independently with respect to the data input. By regulating the damping force, the controller may control how much the suspension system compresses and rebounds in reaction to road irregularities and vehicle movements. Many automobiles equipped with electronic suspension systems have multiple driving modes (for example, Comfort, Sport, and Eco) [24]. The suspension controller modifies the suspension settings in accordance with the driving mode selected. Suspension controllers can also adjust the suspension to account for variations in vehicle weight. When moving huge amounts of freight or passengers, the controller may tighten the suspension to maintain proper ride height and handling. Some modern active suspension controllers may also alter the suspension settings while cornering to counter body roll and improve stability, boosting the vehicle’s handling qualities. Suspension controllers can adjust suspension settings to prevent front-end “dive” during braking and rear-end “squat” during acceleration, enhancing braking performance and traction. In a similar vein, semi-active suspensions control systems, the suspension properties can be altered but only to a certain extent, the limitations herein are apparent.

Some suspension controllers are designed to anticipate impending road conditions or driving scenarios based on data obtained. Instead of reacting to instant inputs, they can now change the suspension proactively. Suspension controllers are a crucial component in modern automobiles that contribute to a more comfortable, safe and dynamic driving experience. They use innovative technology to change the suspension system on the fly, reacting to changing conditions and driver preferences. They do, however, have limitations and can be altered by things such as sensor accuracy, software issues, and extreme weather [25].

5.0 CONTROL STRATEGIES

Generally, passenger vehicles are fitted with semi-active suspension systems to overcome the monotonous working of passive systems and to override the high cost, complications in design and high-power consuming nature of active suspension systems. Control approaches such as skyhook control, ground hook control, hybrid skyhook-ground hook control, acceleration driven damper control (ADDC), linear optimal control (LOC), model predictive control (MPC), adaptive control, robust control, H_2/H_∞ control, fuzzy logic controller (FLC), linear quadratic regulator (LQR), neural network methods like artificial neural networks (ANN), convolutional neural networks (CNN) and preview control systems are used to enhance the vehicle ride experience to the passengers [26]. The architecture of control systems is shown in Figure 3.

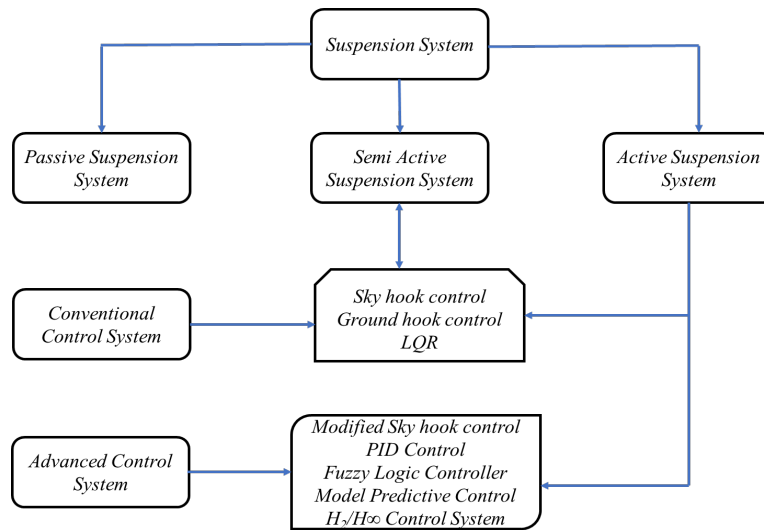


Figure 3. Control systems – architecture

Mahmoud et al. investigated a passenger car on three different control strategies - PID, LQR and FLC and under three different road irregularities fitted with active suspension systems. Fuzzy logic controllers performed better compared with PID and LQR controllers, in which suspension movement was reduced by 61%, thereby increasing the stability of the vehicle. The displacement and acceleration of the sprung masses were reduced by 23.8% and 52% respectively [27]. Bao Tran et al. studied on quasi – Linear Parameter Varying (qLPV) using H_2 and H_∞ conditions [28]. Due to the recent advancement in relatively inexpensive signal processors, the practical applications of vehicle suspension systems have been possible. Various control strategies are implemented into the suspension system to achieve vibration control and to initiate practical and easy implementation of the system.

5.1 Skyhook Control System

Disturbances arising on the vehicle body due to the unevenness of road profiles can be reduced using the skyhook control strategy. Karnopp in 1974, developed and patented the skyhook control strategy. The compression and rebound nature of the damper gets affected, depending upon the vehicle’s body movement. The skyhook control strategy was introduced as an alternative to the passive suspension system. The skyhook model only focuses on the sprung mass. This control strategy is ideal only if the ride performance and the comfort of the occupants are concerned. The skyhook damper is an imaginary damper that simulates the disturbances caused in the vehicle. An off/on switch can be enabled to change the damper setting to low or high damping, depending upon the driver’s requirements [29]. Sky Hook control system is shown in Figure 4(a).

$$M_s \ddot{X}_s + C_{sky} \dot{X}_s + K_s (X_s - X_{us}) = 0 \tag{1}$$

$$C_{sky} \dot{X}_s = C_s (\dot{X}_s - \dot{X}_{us}) \tag{2}$$

$$C_s = C_{sky} \dot{X}_s / (\dot{X}_s + \dot{X}_{us}) \tag{3}$$

$$M_{us} \ddot{X}_{us} + K_t (X_{us} - X_t) = K_a (X_s - X_{us}) \tag{4}$$

where C_{sky} is the damping coefficient of the skyhook damper

$$C_s = \begin{cases} C_{min} & \text{if } \dot{X}_s (\dot{X}_s - \dot{X}_{us}) \leq 0 \\ C_{sky} & \text{if } \dot{X}_s (\dot{X}_s - \dot{X}_{us}) > 0 \end{cases} \quad (5)$$

where C_{min} is the minimum damping coefficients of the semi-active damper and C_{sky} = maximum damping coefficients of the semi-active damper.

Lu et al. studied the skyhook-LQR adaptive approach with a road adaptive algorithm on the semi-active suspension vehicle. On varying road conditions, the control strategy achieved better ride comfort with minimum computational costs [30]. Liu et al. studied a novel concept of skyhook control to improve the vehicle suspension system performance. It had high robustness and performed better compared with other suspension systems [31]. Díaz-Choque et al. formulated an optimal controller in a semiactive suspension system. The results show that the controller worked better by increasing the tire displacement, but the sprung mass acceleration was reduced [32]. Núñez and Muñoz analyzed the skyhook controller with the compact and grip levels in the vehicle suspension system [33]. Moaaz and Ghazaly et al. found a performance increase in ride comfort and vehicle handling with skyhook suspension [34]. Gupta et al. evaluated the performance using the modified skyhook control strategy. The simulation was done for a gain of 1000, 3000 and 5000 and by changing the control variable values from 0 to 1 with increments of 0.25 [35].

5.2 Ground Hook Control System

The ground hook control strategy deals with the deflection of the unsprung mass; the system is connected to the ground, but this cannot be achieved and must be approximated. A passive damper is changed into a semi-active suspension by the on-off-ground hook strategy control by hooking between the tyre and the ground. The Ground hook control system deals with the unsprung mass of the vehicle. An off/on switch can be enabled as follows [29]. Ground hook control system is shown in Figure 4(b).

$$C_{in} = \begin{cases} C_{min} & \text{if } -\dot{X}_s (\dot{X}_s - \dot{X}_{us}) \leq 0 \\ C_{ground} & \text{if } -\dot{X}_s (\dot{X}_s - \dot{X}_{us}) > 0 \end{cases} \quad (6)$$

where C_{min} is the minimum damping coefficient of the semi-active damper and C_{ground} is the maximum damping coefficient of the semi-active damper

Moaaz and Ghazaly observed the ground hook control system offers better ride comfort and vehicle handling performance [34]. Díaz-Choque studied that the optimal ground hook controller increases the mobility of the vehicle, but passenger comfort is reduced due to the increase in sprung mass acceleration [32]. M. Aladdin and J. Singh evaluated the modified skyhook control strategy. Simulation was done for a gain of 1000, 3000 and 5000 and by changing the control variable values from 0 to 1 with increments of 0.25. The findings show passenger comfort is reduced, but the handling of the vehicle is increased. As the ground hook controller deals with unsprung mass acceleration and velocity, the payload of the vehicle can be increased [36].

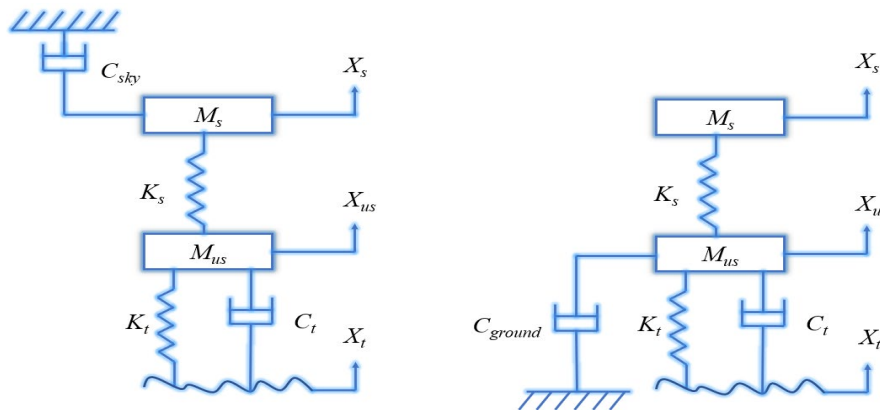


Figure 4. (a) SkyHook control system, (b) ground hook control system

5.3 Hybrid SkyHook – Ground Hook Control System

An alternative to the skyhook and ground hook control system, the hybrid skyhook-ground hook control system has been shown to take advantage of both skyhook and ground hook controls. By having hybrid control, the system can be programmed in such a way it changes between skyhook and ground hook seamlessly. The hybrid control system can switch between low and high damping levels [29]. Hybrid skyhook - ground hook control system is shown in Figure 5.

$$C_{hybrid} = [\mu C_{sky} + (1 - \mu) C_{ground}] \quad (7)$$

$$C_{in} = \begin{cases} C_{min} & \text{if } \dot{X}_s (\dot{X}_s - \dot{X}_{us}) \leq 0 \\ C_{sky} & \text{if } \dot{X}_s (\dot{X}_s - \dot{X}_{us}) > 0 \\ C_{min} & \text{if } -\dot{X}_s (\dot{X}_s - \dot{X}_{us}) \leq 0 \\ C_{ground} & \text{if } -\dot{X}_s (\dot{X}_s - \dot{X}_{us}) > 0 \end{cases} \quad (8)$$

where, μ is relative relation between skyhook and ground hook control. If $\mu = 1$, hybrid control is switched to pure skyhook control and if $\mu = 0$, hybrid control is switched to pure ground hook control.

Mulla et al. analyzed and compared skyhook, ground hook and hybrid control strategies with the passive and semiactive suspension system. When $\alpha = 1$, the skyhook strategy is enabled, giving better ride comfort, but comprising on vehicle handling. When $\alpha = 0$, the ground hook is enabled, giving better performance on vehicle handling with ride comfort limitations. Using the hybrid system provides the best attributions of skyhook and ground hook control [37]. Gupta et al. studied hybrid control strategies enabled with a semi-active suspension system consisting of on-off controls. The proposed hybrid controller provides better isolation at higher frequencies [35].

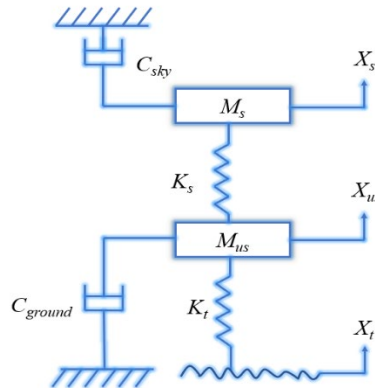


Figure 5. Hybrid skyhook - ground hook control system

5.4 Accelerator-Driven Damper Control System

Accelerator driven damper control system is an upgrade to the existing skyhook and ground hook control systems. Savaresi et al. proposed a control system called acceleration-driven damper (ADD) control, which outperforms the SH-based algorithms. It minimizes the vertical body acceleration, as the AAD design is well equipped to improve comfort, but has some limitations in road holding [38,39].

$$C_{in} = \begin{cases} C_{min} & \text{if } \ddot{X}_s (\dot{X}_s - \dot{X}_{us}) \leq 0 \\ C_{max} & \text{if } \ddot{X}_s (\dot{X}_s - \dot{X}_{us}) > 0 \end{cases} \quad (9)$$

Many papers have been published combining the skyhook system with ADD controller. Birhan Abebaw Negash et al. compared the skyhook (SH) and mixed skyhook-acceleration driven damper (SH-ADD) controlling algorithms using a semi-active magnetorheological damper in a quarter-car model. The skyhook-controlled MR damper improved sprung mass and vertical displacement by 54.49% and by 58.22% when the MR damper was combined with a mixed SH-ADD controller [40,41].

5.5 Fuzzy Logic Control System

Fuzzy logic control systems were first introduced by Lotfi Zadeh in 1965 [42]. Fuzzy logic algorithms are used due to their simple and flexible design [43]. Fuzzy logic provides very valuable flexibility for reasoning as it needs very little data as input. The fuzzy controller rules are generally formulated in qualitative or linguistic terms [44]. Fuzzy logic controllers usually outperform other controllers when complexity, nonlinear, or undefined systems are involved. The output functions are very smooth despite a wide range of variations of input. This system can have multiple inputs and outputs depending on the profile of the problem statement. The control system provides a very efficient solution to complex problems. This system has gained prominence as one of the best control systems due to its easy adaptability and high performance. One of the major limitations of the fuzzy logic system is its difficulty in setting up accurate fuzzy guidelines, which can be an arduous task. The fuzzy rationale can be error-prone, and the fuzzy framework needs to be tested with equipment to produce the best results. The Fuzzy logic controller has the following stages of operation – rule base, fuzzification, inference engine and defuzzification. The layout of the fuzzy logic control system is shown in Figure 6.

Yatak et al. analyzed the hybrid fuzzy controller in the active suspension system to improve ride comfort - road holding parameters of the vehicle. Simulation results show that improvement is possible with the proposed controller [45]. Taskin et al. studied an active suspension system with an MISO fuzzy logic controller on a quarter-car test rig. Results show that it performs better with classical FLC on time and frequency responses [46]. Qazi et al. analyzed the fuzzy logic controller

on a semi-active suspension system. The ride comfort of the suspension was studied using three different damping coefficient limits of 3000,4000 and 5000 Ns/m. The simulation results were satisfactory when 4000 Ns/m damping limit suspension was used [47]. Pang et al. investigated the fuzzy-smith compensation control method in semi-active suspension systems using time delay [48]. Joshua Robert et al. analyzed the performance of a model-based conventional controller with a model-free intelligent Fuzzy Controller on body displacement and body acceleration. The results of the fuzzy control perform better on vehicle body displacement and acceleration.

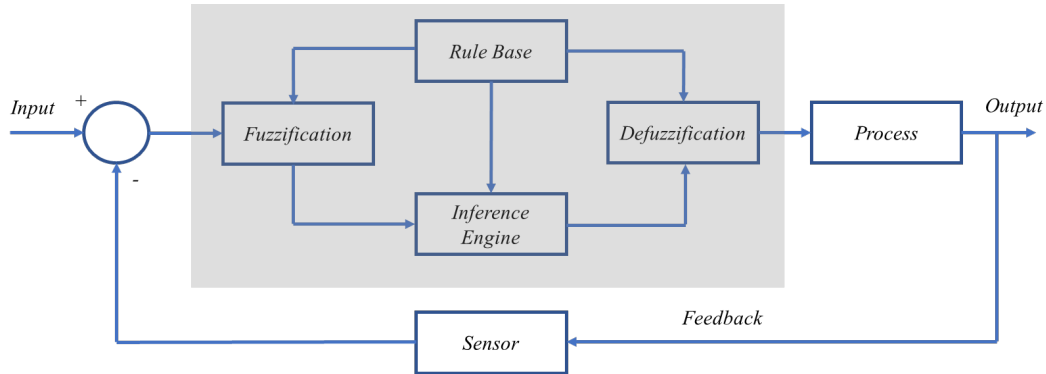


Figure 6. The layout of the fuzzy logic control system

Numerous types of research are carried out in suspension systems using fuzzy logic controllers [49]. Pekgökgöz et al. studied a genetic algorithm using FLC for the active suspension of cars. Vehicle body deflection was better with a fuzzy-logic enabled controller than with a PID controller [50]. Mustafa et al. proposed a time delay estimation technique (TDE) with the fuzzy logic controller to enhance the ride comfort on the active suspension system [51]. Palanisamy et al. investigated the fuzzy-logic technique on the active suspension vehicle [52]. Sakman et al. designed and investigated the FLC active suspensions. The active suspension had better overall performance than the passive system. This approach solves the degeneration problem in the suspension system [53]. Rezaee and Pajohesh studied fuzzy logic control in an active suspension system using MATLAB/Simulink. Results show that the ride comfort of the vehicle improved due to the reduction in sprung mass acceleration [54]. Hung et al. proposed the fuzzy logic controller for a quarter car of spring-mass damper system using MATLAB/Simulink, which provided good vehicle stability and ride comfort [55].

6.0 PID CONTROL SYSTEM

Elmer Sperry, in 1911, developed the first Proportional-Integral-Derivative (PID) controller. It is a feedback control system that delivers the desired output. PID controller is a linear control system and one of the actively used controllers in passenger car vehicles. PID controller has three subdivisions - proportional, integral and differential. Depending upon the actions required from the PID control system, any subdivision of the PID controller can be used. PID controller becomes proportional-derivative (PD), and proportion-integral (PI). In some cases, either controller proportional (P) or controller derivative (D) is needed for the feedback output. In 1942, Ziegler and Nichols introduced tuning rules to enhance the performance of PID controllers further. A small amount of noise in the control system can cause signal variations in the output value. The PID control algorithm is a robust and simple algorithm that offers higher accuracy and better performance in suspension systems. When the control gain is set to small, the P controller has steady-state errors. As the control gain increases, some errors creep up with the stability of the feedback loop. In a vehicle suspension system, high proportional gain happens when the rise time is reduced. Low proportional gain reduces overshoot and oscillations, which is difficult to execute in all systems. The derivative of the PD or PID control system should be set as zero to overcome the noise disturbance when the frequency is increased. To counteract and minimize the problem, an electronic signal filter is added to the system. These filters remove the unwanted noise disturbances arising within the system and, at the same time, enhance wanted signals in the system [56]. The PID control system is shown in Figure 7.

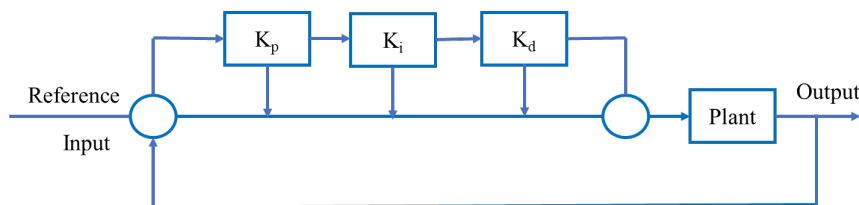


Figure 7. PID control system

Zhao et al. studied ride comfort, vertical acceleration, suspension dynamic travel and tire dynamic load using particle swarm optimization with a PID control system in MATLAB [57]. Sharkawy et al. compared the passive and active suspension of a passenger car using the PID controller design approach. Active suspensions, ride comfort and vehicle handling performance improved [58]. Nassar and Matrood investigated a modified PID controller on a quarter-car model system. Dynamic responses such as acceleration, car body displacement, velocity and suspension stroke were tested. The

simplicity of the modified PID controller gains makes it easier to control output signals when compared with fuzzy and fuzzy-PID controllers [59]. Shafiei simulated a ¼ car active suspension system in Simulink/MATLAB, to reduce the vehicle body displacement and acceleration and to improve road holding and ride comfort of the vehicle using Ziegler–Nichol’s method. The vehicle body experienced less force comparatively than passive suspension [60].

Nagarkar studied an ¼ GA-based FLC active suspension system with a PID passive suspension system. Results show a reduction in RMS acceleration and an increase in ride comfort [61]. Tian & Nguyen evaluated the performance of the PID controller with an 8-DOF car model at speeds ranging from 40 to 60 km/h. Different excitations of the road surfaces were conducted. The acceleration responses of the driver’s seat body angles were greatly reduced. Their RMS values with the PID controller are reduced by 19.3, 18.3, and 24.7 % respectively [62].

6.1 Linear Quadratic Regulator Control System

The linear quadratic regulator (LQR) is a feedback controller that provides optimal control feedback for linear systems. Linear quadratic regulators (LQR) and linear quadratic Gaussian (LQG) controllers are also called LQ controllers [63]. The feedback gains are achieved by Riccati equations. For better control of the LQR controller, the inputs and outputs of the system have to be predefined [64] [65]. It minimizes the cost function and balances the tracking performance. This system performs efficiently only if the system has accurate information on the system parameters. Systems alternate between linear and nonlinear systems, so the accuracy of the system is flawed [66]. Adaptive control techniques, like the model reference adaptive control (MRAC) or self-tuning regulator (STR) are used to rectify the flaws in control system operation [67]. LQR assumes the system to be free of disturbances and measurement noise, which are impossible in real systems. Disturbance rejection techniques like feedforward control or Kalman filter can be used to reduce external disturbances and noise. Stochastic LQR (SLQR) can help optimize the trade-off between the expected performance and the controller effort [68]. The controller has to solve the matrix Riccati equation, which can be time-consuming and numerically unstable. The controller needs to be updated and redesigned frequently to adapt to the changes in the system dynamics, system uncertainty and constraints. This increases the computational burden, which can be nullified by using approximation and simplification techniques to reduce the complexity and computation elements in the LQR controller [69]. The linear quadratic regulator control system is shown in Figure 8.

$$\dot{x} = Ax + Bu \tag{10}$$

$$\dot{y} = Cx + Du \tag{11}$$

$$J = \frac{1}{2} T \int_0^{\infty} x^T Qx + u^T Ru + x^T Nu \, dt \tag{12}$$

where Q, R and N are symmetric, time-invariant matrices containing system parameters and weighted matrixes. Q is semi-positive definite; R & N positive definite [64].

The feedback control law is given by:

$$u = - Kx \tag{13}$$

The feedback matrix K can be expressed as:

$$K = R^{-1} (B^T P + N) \tag{14}$$

where the P matrix satisfies the Riccati equation.

$$A^T P + P A - (PB + N) R^{-1} (B^T P + N) + Q = 0 \tag{15}$$

Nagarkar et al. studied the modeling, control and optimization of PID and LQR control ¼ car suspension system in MATLAB/Simulink using GA-based optimization technique. Vehicles traveling at 80 kmph produced better ride comfort [70]. Tran et al. analyzed the closed multi-loop algorithm in the active suspension system using the LQR controller. For cyclic oscillations, the values of displacement and acceleration are about 25.25% and 32.47%, respectively. For random oscillators, values of displacement and acceleration are about 43.37% and 73.23% respectively [71]. Kumar and Vijayarangan developed an active suspension for a passenger car in a quarter-car model using the conventional method (CM) and acceleration-dependent method (ADM). Root Mean Square (RMS) passenger acceleration was reduced by 54.23% with the active CM system, and with the active ADM system, it was reduced by 93.88% and suspension stroke was reduced by 37.5% [72]. Nguyen studied the LQR control algorithm for the active suspension system. Values of displacement and acceleration are about 2.68% and 43.00% respectively. Ride comfort and vehicle stability improved in alternating driving conditions if the LQR control algorithm was used [73].

Nagarkar and Vikhe studied the ¼ car Macpherson strut suspension system to improve ride control using the genetic algorithm (GA) approach. RMS controller force was reduced by 20.42%, with a slight increase of 3.65 % in sprung mass acceleration [74]. Rao studied linear quadratic regulator (LQR) and PID controllers in a semi-active ¼ car suspension system. Sprung mass acceleration, sprung mass displacement, unsprung displacement, and suspension deflection were the parameters examined. Semi-active suspension systems with LQR and PID control systems performed better on settling time and peak overshoot for wheel deflection, wheel position, suspension deflection and body position [75].

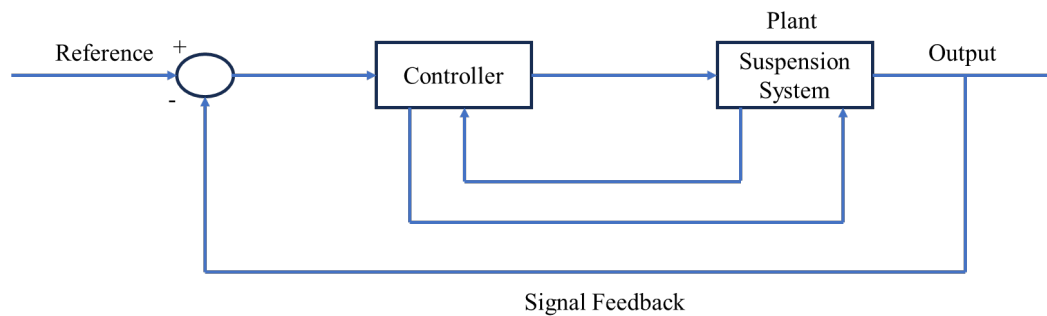


Figure 8. Linear quadratic regulator control system

Nagarkar and Vikhe studied vibration dose value (VDV), RMS sprung mass acceleration, maximum sprung mass displacement and maximum suspension space deflection weighting matrices LQR controller using GA. Sprung mass acceleration was reduced by 24.18%. VDV was reduced by 16.54%, 40.79% and 67.34% respectively [76]. Ye and Zheng analyzed LQR optimal control and mixed H_2/H_∞ control strategies of semi-active suspension control. Suspensions of LQR control and mixed H_2/H_∞ control improved comfort and control stability on uneven road conditions [77].

6.2 H_2/H_∞ Control System

H_2/H_∞ optimization is a method for a closed-loop transfer function with proper feedback to minimize disturbances and parameter variations [78]. H_∞ control systems are preferred over classical control techniques in their adaptability in multivariate systems [79]. A mathematical understanding is needed to apply the control system successfully for the model of the system to be controlled. The resulting controller is an optimal controller which fits with a cost function, system constraints and disturbances. The control system produces errors when settling time; energy expenditure has to be evaluated concerning the input signals of the system [80]. Savaresi et al. studied control algorithms and optimal/robust methods on vehicle suspension systems [29]. Ghazaly et al. developed a model using the H_∞ technique on an active suspension quarter car model. Using step, sinusoidal and random road profile as parameters, car body acceleration, suspension deflection, and tyre deflection performance improved using a semi-active suspension system [81]. Strohm and Christ compared H_∞ controllers on vehicle suspension systems using preview information on road irregularities [64]. Tudon-Martinez et al. studied semi-active force using two linear parameter varying H_∞ (LPV- H_∞) filters [82]. Shao et al. proposed an H_∞ control with actuator faults and time delay in active suspensions. Output feedback controllers improved vehicle suspension and motor performance for actuator thrust losses. Output feedback controllers II had better performance comparatively when compared with output feedback controller I and the passive suspension when actuator thrust losses and time delays occurred [83].

Vela proposed an H_∞ observer for unknown road disturbance using SA damper force estimation on an ER damper (QoV). The main objective is to reduce the cost and reduction in the usage of sensors in the design process [84]. Jibril studied the H_2 and H_∞ optimal controller in a quarter-car model using MATLAB/Simulink and examined suspension deflection body acceleration on varying road surfaces. Sprung mass displacement decreased using H_∞ optimal controller [85].

6.3 Robust Control System

Robust control system emphasis is on system reliability and robustness [86]. These control systems are used in the requirement of system stability and to perform the intended functionality of the system in a defined environment without failure [87]. The sensor in the feedback control system plays a vital role in the proper functioning of the system, as a sensor failure can disrupt the working process [88]. The requirement of system parameter dynamics is not essential for the operation of the robust control system, due to which the robustness may be limited. The feedback system robustness can be greatly improved if the system can be provided with parameter dynamics, time interaction and uncertainty of the system [89]. The safety of autonomous vehicles when tasked with uncertain disturbances is improved [90]. Mahmoodabadi and Nejadkourki studied a quarter-car model using a fuzzy adaptive robust proportional-integral-derivative (PID) controller in an active suspension system. Body acceleration and relative displacement are the parameters considered for the analysis [91]. Bai and Wang studied the optimal control strategy using linear quadratic regulator (LQR) and sliding-mode control on the CarSim and MATLAB/Simulink [92]. Zhou et al. studied the mixed H_2/H_∞ robust control methodology on the four-wheel steering (4WS) system. The mixed H_2/H_∞ controller was designed for the betterment of system performance, stability, and robustness of the 4WS vehicle stability control system. H_2/H_∞ controller improved the handling stability of the 4WS vehicle [93]. The robust control system is shown in Figure 9.

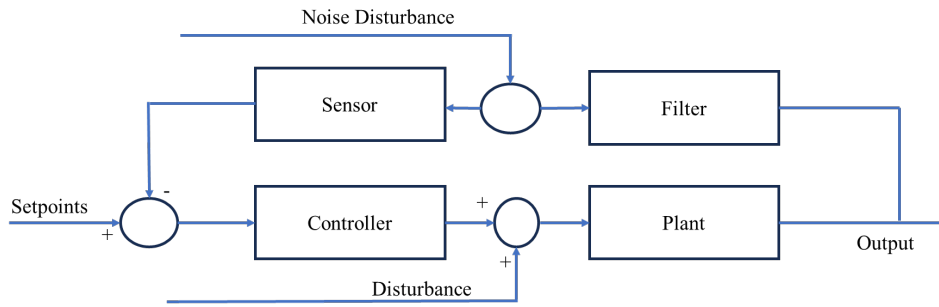


Figure 9. Robust control system

6.4 Model Predictive Control System

Model predictive control systems are advanced system which ensures the satisfaction of given controller sample time, prediction and control horizons, system constraints, and weights, thereby minimizing cost functional. Model-based predictive control (MPC) makes use of a process model to forecast system behaviour [22]. Model predictive control systems can be divided into generalized predictive control, robust model predictive control and nonlinear model predictive control. MPC can perform operations effectively in SISO and MIMO systems. It determines the optimum actions necessary to perform large (MIMO) systems simultaneously. MPC can be tuned easily, can perform computational tasks quickly, and can handle multiple constraints. The limitation of using the MPC control system is that a huge number of model coefficients are needed to get a response. Some models of MPC can perform only in stable and open-loop systems. The predictive horizon of the system should be formulated properly; if not, the control performance of the signals will be poor even if the selected model is correct. Due to continuous interaction between variables, which may be difficult for a single MPC system to manage, they can be split into many smaller systems. It is easier to evaluate issues in any of the multiple segmented MPCs from a larger MPC system [94]. The model predictive control system is shown in Figure 10.

Madhavan Rathai studied the NMPC scheme for a ¼ car semi-active suspension system. The controlled variables, manipulated variables and feedforward variables are calculated in the MPC system. MPC systems are used in industries that require high-speed memory, processors and fast optimization algorithms [95]. Meanwhile, Theunissen et al. proposed a preview control strategy for an active suspension system using an explicit model predictive controller (e-MPC). Without preview control, on the 4Hz frequency range, the vertical acceleration root-mean-square (RMS) value had a reduction of ~10% compared with a skyhook controller. With preview control, performance improved from 8 to 21% [96]. Das et al. examined the performance of model predictive control in active suspension [97].

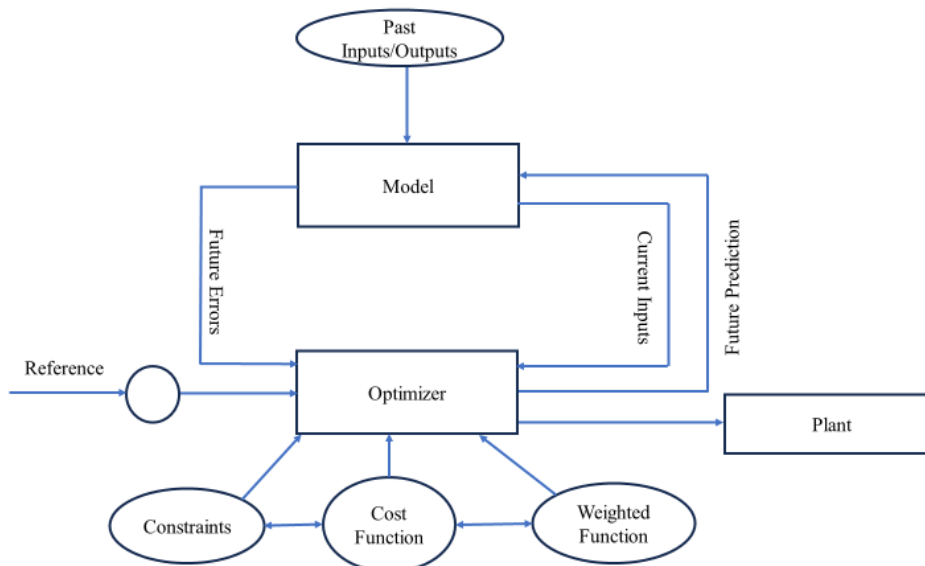


Figure 10. Model predictive control system

An explicit model predictive control (EMPC) was designed to navigate the fast dynamics, actuator constraints and couplings in semi-active suspension, as it requires low computation time [98]. Shao et al. studied a half-car vehicle in an active suspension system using distributed model predictive control (DMPC). The vehicle’s vertical and pitch motion were examined. Comfort in the vehicle is improved by the distributed model predictive (DMPC) controllers [99]. Madhavan Rathai investigated a quarter car system using model predictive control (MPC) on the INOVE test platform. Performance in the vehicle was much better with an MPC controller than with a modified Skyhook controller [100].

6.5 Preview Control System

The preview control system utilizes a feedforward control system in which the road disturbances of adjacent vehicles are measured using the Wiener filter theory [101]. Built-in mechanisms of the vehicle suspension control system are reactive to measured or estimated vehicle responses to road disturbances through various onboard sensors—the road profile disturbance changes to a measured external input. The performances of the control system are affected due to the fast dynamics of vehicle suspension motion, low bandwidth of signal actuators and the time delays between input and output control buses. These can be greatly enhanced when road preview control is used. Preview control systems gather information such as road elevation at various points, typically at the vehicle corner. As the vehicle passes by each corner on the road, the pump pressurizes the respective actuator to feed the required energy into the system to improve its overall performance. The identification of transient road inputs by mapping the road conditions using laser scanners and mono/stereo cameras is the most important parameter in the preview control system. Line feedback control and Wiener-filter-based controllers evaluate the performance of a vehicle suspension system with road irregularities with road preview [102]. Bender in 1968, proposed preview information in active vehicle suspensions. Several research works on the preview control problem have been reported from various viewpoints. H_∞ preview control and robust preview control have been extensively studied over recent years. Kaldas MM et al. examined road preview control strategies on varying road profiles in vehicle suspension systems [103]. Li and Liu investigated the hydro-pneumatic active suspension system in a $\frac{1}{2}$ vehicle model for the improvement in riding comfort and road handling. From the results, the wheel preview control method showed better overall performance than the partial preview control method [104].

Youn et al. studied the fully tracked vehicle (FTV) with the preview control system in active and semi-active suspension. In the continuous and frequency domains, the skyhook suspension system performed better [105]. Canale et al. studied using preview information on model predictive control (MPC) techniques for a semi-active suspension [63]. Ahmed and Svaricek proposed a preview control system with lidar sensors, using Fast Fourier Transform (FFT) and model predictive control (MPC) on a half-car model. Simulation and experimental results showed better performance on ride comfort when adaptive robust controller, skyhook approach, passive suspension, and optimal preview controller are compared [106]. Desikan et al. analyzed image processing techniques using the fuzzy logic model with preview control on a semi-active car suspension. Ride comfort and vehicle stability were better when ER dampers were used [107]. Bei et al. studied the neural networks algorithm using a PID controller on a $\frac{1}{2}$ vehicle semi-active suspension with wheelbase preview where, at increasing speeds, the PID-controlled system with preview reduced vehicle vibration on a semi-active suspension system. At lower speeds, pitch angular acceleration and rear vertical acceleration decreased significantly, but the ride comfort of the vehicle improved [108]. De Bruyne et al. analyzed a preview control strategy based on a hybrid MPC on a quarter-car vehicle model. Simulation results showed that the hybrid MPC approach recovered 8% of the comfort performance [109]. Simulation on an active suspension system with preview control and a feedforward controller with skyhook logic (to measure the ride comfort of the vehicle over a bump) showed the enhancement of ride comfort is higher using an active suspension control system coupled with skyhook feedback control logic [110].

6.6 Limitations of Suspension Controller

Suspension controllers are responsible for controlling the behavior and characteristics of the suspension system, which contributes to a more comfortable, safe, and dynamic driving experience. They use cutting-edge technology to dynamically adjust the suspension system in response to changing conditions and driver preferences. Despite their appealing features, there exist certain limitations as follows, which open up further avenues for in-depth research:

- i. The complex electronic and mechanical components are susceptible to faults and failures, thereby jeopardizing the functioning of the suspension system.
- ii. Most current suspension controllers rely on numerous sensors to collect data about the vehicle's motion, road conditions and other factors. The controller's ability to optimize the suspension system may be compromised if these sensors fail or provide erroneous data. The need for sensor calibration is touted to be an additional limitation in this context.
- iii. While suspension controllers can adjust suspension settings in real time, their capacity to adapt to unexpected changes in road conditions or driving dynamics is often limited. Rapid changes may not provide enough time for the device's controller to make optimal adjustments, potentially resulting in compromised handling or discomfort.
- iv. Extreme situations, such as extremely rough off-road terrain, extremely high speeds, or big loads, may exceed the suspension system's designed boundaries, diminishing the controller's efficiency. The need for an efficient suspension system, regardless of any bad road conditions, could be found only in high-cost vehicles, which put forth an economic constraint.
- v. When suspension controllers fail, fixing or replacing them can be expensive and demand the use of specialized equipment; when compared to ordinary non-electronic suspension systems, this can result in higher maintenance costs. It was also reported that the process is time-consuming and laborious as well. Likewise, these limitations arise on installing any aftermarket parts/components as well.
- vi. Some modern suspension controllers can learn and adapt to the preferences and habits of the driver. The learning process, however, may take some time, and the controller may not always effectively predict the driver's intentions or adapt entirely to their preferences. Accounting for this, being user-friendly is a big challenge here.

- vii. Suspension controllers require a solid power source as well as functional electrical connections. Electrical problems, such as voltage drops or shorts, might impair the controller's functioning and cause it to fail.
- viii. Suspension controllers frequently employ software or firmware that may require updates to increase performance, adapt to new situations, or address faults. Failure to update the controller's software may result in missed possibilities or potential vulnerabilities.
- ix. The inclusion of modern suspension controls and accompanying technological advances might add to the vehicle's overall cost. This cost may be higher than typical suspension systems, compromising the vehicle's affordability.

Contempt of these restrictions, suspension controllers continue to adapt and improve, providing greater comfort, safety, and performance in a variety of driving situations. To make informed judgments regarding their automobiles, vehicle owners must grasp both the perks and downsides of these systems beyond any existing or forthcoming advances in research on suspension control systems.

7.0 CONCLUSIONS

Various control strategies and algorithms used in vehicle suspension systems were discussed, right from the basic skyhook, hybrid systems, and PID controllers to the more advanced controllers like MPC, H_2/H_∞ , and preview control systems on theoretical, experimental and simulation viewpoints. The following are the potential conclusions drawn:

Controllers with varying sets of gains are required to regulate the system effectively throughout the operational phase, as nonlinear systems exhibit varying dynamics under varying operating conditions. Control systems enhance by employing multiple sets of gains across the entire range of operating conditions. In the case of PID and Fuzzy logic controllers, integrating the error over a prolonged duration may lead to very large values, resulting in a loss of stability. To counteract this problem, the threshold error value could be increased to an upper limit. Thereby, the upper and lower thresholds can be maintained within the operating range through various techniques. Another method is to limit the size of the buffer of error history and integrate it using a queue data structure to discard the error value that is too old through the first in – first out (FIFO) principle.

Due to high dynamic responses, some controllers like fuzzy logic controllers and H_2/H_∞ tend to produce abrupt responses, which is not desirable. Such actions of the controller cause a loss of stability, and the output values become error-prone. The actual value of the system can get tampered with. Controller outputs must be aligned before the logical operations, signals and values are fed into the plant, which should enable the control system to track the setpoint under all operating conditions. Controllers try hard for the outputs to converge, which is necessary for highly precise systems, but it is not possible to achieve this in all control algorithm settings.

Finally, when it comes to control strategy, the problems arising from longitudinal control prefer fuzzy logic and PID controllers, and lateral control problems prefer MPC and robust controllers to get the desired output. However, the exact choice of choosing a particular control strategy depends upon the specifics of the problem statement. Factors such as damper type, cost analysis, mode of operation, and suspension configuration play a major role in choosing the best optimum control strategy. In moving forward, learning-based control strategies can play a major role in reshaping the vehicle dynamics control system operation of vehicle suspension systems. It would be safe to say these advanced control systems are at the initial stage of research and development with a bright future ahead.

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