# Taguchi Optimization of Geometrical Factors of a Polymer Composite Patch in Crack Repair 

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#### Abstract

For the repair of a crack in thin aluminium sheets, polymer composite patching is one of the better repair techniques. During service, when the load acts, the patch separates from the substrate due to the stresses developed at the interface. This separation of the patch from the substrate largely depends on the geometrical factors of the patch, i.e. length, width and thickness. The optimum geometrical factors need to be incorporated for the effective and economical repair of the cracks. In this work, the optimum combination of the geometrical factors, i.e. length, width and thickness of the polymer composite patch, is obtained using the Taguchi technique with the help of the results generated in the numerical analyses.


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### 1.0 INTRODUCTION

Thin aluminium alloy sheets may have a through-the-thickness crack which grows in the stressed regions under the application of an external load. These cracks propagate with the increase in the applied load, eventually leading to catastrophic failure of the component. The cracks are repaired by bonding a patch made of polymer composites on the cracked region. The bonded patch shares the applied load and prevents crack propagation in the aluminium sheet. But as the applied load increases, the bonded patch starts separating from the aluminium skin due to the interfacial peel and shear stresses. The patch separation is largely influenced by geometrical factors such as patch length, patch width and the number of plies. An optimum combination of these geometrical factors can be obtained by simulating the patch separation process using the finite element method and then analysing the results with a suitable technique such as the Taguchi method.

The repair of metallic structures with composite materials has been investigated by several researchers and was first introduced in Australia in the early 1970s and later in the USA in the early 1980s for the repair of military and civil aircraft [1]. Ratwani [2] studied the structural life enhancement options for airframe structures with a focus on composite patch repair. Okafor et al. [3] performed the stress analysis of a single-sided multi-plies boron/epoxy octagonal composite patch. They observed that the maximum stress in the parent material decreased significantly after the application of the patch. Barroso et al. [4] performed the experiments and analysed the failure of adhesively bonded lap joints between aluminium plates and CFRP laminates by using the local stress state model and studied the influence of plate thickness, CFRP thickness, stacking sequence and overlap length on the failure of the joint. The prediction by local stress state model was compared with that by the numerical method and was found to be satisfactory.

Alfano et al. [5] employed the cohesive zone model to study the Mode-I fracture in a precracked bonded double cantilever beam specimen with the finite element code ABAQUS and examined the sensitivity of cohesive zone parameters in predicting the overall mechanical response. Papathanassiou et al. [6] presented a model for the prediction and optimisation of the composite patch repair applied for the restoration of damaged structures and optimised the bonding process in terms of total procedure duration and energy density consumption by using the genetic algorithm technique. Grabovac and Whittaker [7] applied CFRP patches to the structures used in the ship deck and studied the effectiveness of the bonded repair for a service life of 15 years. Pandey and Kumar [8] observed that the failure criteria selected for predicting the position of the failure in the adhesive for the bonded repair as well as the patch geometry were the important parameters. Ricci et al. [9] performed finite element analysis of the bonded patch repair in case of the fatigue loading and observed that the fatigue life of the metallic plate increases due to the bonding of the patch. M. Ramji et al. [10] concluded that the octagonal patch exhibits better performance in reducing the stress intensity factor among the patches with circular, rectangular, square, elliptical and octagonal shapes. Katnam et al. [11] performed a review on the bonded repair of aircraft structures and identified several scientific challenges and opportunities such as advanced nondestructive testing for damage assessment, advanced surface treatments for interface bonding, controlled cure conditions for patch fabrication, accurate analysis and design for optimised repairs and automation for reliable repairs.

Toudeshky et al. [12] analysed the composite repair of precracked aluminium panels by using cohesive elements and concluded that the separation propagation could be decreased by appropriate dimensional and material properties of the composite patch. Gift et al. [13] studied the role of cohesive zone model in analysing crack propagation in different
materials. Wang et al. [14] carried out one cycle accelerated ageing test on the aluminium structure bonded with composite patch and found that there was no failure in the composite patch as well as the interface between the structure and the patch. Rao et al. [15] presented a detailed overview of the Taguchi method in terms of its evolution, concept, steps involved and its interdisciplinary applications and stated the effectiveness of this method in optimising the products and processes. Sheikh [16] studied the effect of variation of the patch width by performing experiments on a cracked aluminium specimen bonded with a patch made of thin unidirectional GFRP and CFRP plies. Aradhi [17] investigated the patch separation process by using cohesive zone model in the finite element code ANSYS for different lengths of the patch made of unidirectional GFRP and CFRP plies bonded to the cracked aluminium specimen. Shinde et al. [18] concluded from the experiments that until the patch remained bonded to the specimen, the J-integral at the crack tip was small. The separation of the patch was initiated at the crack edge, followed by slow crack growth with the increase in the applied load. The patch failed suddenly after the applied stress exceeded the yield stress of the specimen [19]. Rasane et al. [20] arrived at an optimum combination of patch length and width by generating a response surface from the failure stresses obtained in the numerical analyses performed by using ANSYS 15.0. The current study presents the evaluation of the interface separation between a thin pre-cracked aluminium alloy sheet and a bonded patch made of unidirectional GFRP and CFRP plies. The cohesive zone model was employed in the finite element evaluation and the Taguchi technique was used for optimising the geometrical parameters. An optimum combination of the patch length, patch width and patch thickness were obtained.

### 2.0 METHODOLOGY

A thin plate of aluminium alloy 6061-T6 with a through-the-thickness centre crack was repaired with a single-sided FRP patch consisting of a thin unidirectional GFRP ply and multiple CFRP plies. Unidirectional tension tests were carried out on a universal testing machine, and the failure loads obtained were compared with the numerical results obtained through finite element analyses. The commercial finite element code ANSYS 15.0 was used for the numerical simulation of the patched specimen, and only a quarter model was analysed due to the symmetric nature of the patched specimen, as shown in Figure 1.


Figure 1. Quarter model of patched specimen
The fracture analysis of the bonded interface between the cracked sheet and composite patch was studied using the cohesive zone material (CZM) model. The CZM model directly introduces fracture mechanism at the interface by adopting softening relationships between tractions and the separations. The interface surfaces of the bonded materials were represented by a special set of interface elements and thus, the CZM model was used to characterise the constitutive behavior of the interface. The cohesive material law with bilinear nature was used to simulate the interface separation process. A mixed mode failure criterion in terms of the normal and tangential separation was specified to describe the failure of the interface. Rasane et al. [20] have shown that the difference between the experimentally and numerically obtained failure stresses was less than $3 \%$ confirming the credibility of the numerical method and the cohesive zone model for the analysis of the patch separation process. Sheikh [16] and Shinde [18, 19] have shown through experiments that the patch separation occurred at the interface of the skin and the GFRP ply. Normal and shear stresses, developed at the interface between the aluminium skin surface and the GFRP ply of the patch, initiate the separation of the patch. The separation area increased gradually with the increase in the applied load. This growth was slow and contained until the
applied stress reached the yield stress of the aluminium skin. Once the yield stress of the skin was exceeded, there was rapid growth of the separation area, as shown in Figure 2.


Figure 2. Separation area vs applied stress
A number of numerical simulations were performed for various combinations of the geometrical factors. The width of the patch was varied as $30 \mathrm{~mm}, 36 \mathrm{~mm}, 42 \mathrm{~mm}$ and 48 mm , while the length of the CFRP ply was varied as 30 mm , $40 \mathrm{~mm}, 50 \mathrm{~mm}$ and 60 mm . The length of the GFRP ply was kept 20 mm longer than that of the longest CFRP ply for all cases. The thickness of the ply varied from one to three CFRP plies. A total of 48 numerical experiments were performed. Thus, there are three parameters for optimisation; length, width \& thickness of the patch. The increase in length, width and thickness of the patch resulted in an increase in strength of the patch. But increasing the parameters beyond a certain value may not be possible due to practical constraints, e.g. if enough space is not available for increasing the patch length in the practical application, then it is convenient to increase the thickness so as to obtain a similar strength. Thus, it is appropriate to determine the optimum combination of the dimensional parameters.

### 3.0 TAGUCHI ANALYSIS

Taguchi method with orthogonal arrays was used for the optimisation of the geometrical factors in the patch separation process. Taguchi method is used in industries for the robust design of products and processes wherein the product/process designed works well under various operating conditions. The robust design is based on three functions, i.e. orthogonal array, $\mathrm{S} / \mathrm{N}$ ratio and loss function. The Taguchi method involves performing matrix experiments using special matrices called orthogonal arrays to determine the effects of several parameters efficiently. Depending on the number of factors and the factor levels, a suitable orthogonal array is selected. The signal-to-noise ratio of the output responses for all the experiments is determined, and the $\mathrm{S} / \mathrm{N}$ ratio is maximised to obtain the optimum condition [21, 22, 23]. L48 Taguchi orthogonal array [24] was used in this study with different combinations of the geometrical factors, i.e. patch length, width and thickness, as shown in Table 1. The following parameters were used;

| W | $=$ width of patch in mm |
| :--- | :--- |
| LGFRP | $=$ Length of GFRP ply in mm |
| LGFRP 1 | $=$ length of first CFRP ply in mm |
| LGFRP 2 | $=$ length of second CFRP ply in mm |
| LGFRP 3 | $=$ length of third CFRP ply in mm |
| n | $=$ Number of CFRP plies |

The 48 experiments were grouped into 12 sets by combining similar types of experiments together, as shown in Table 2. Factor ' $A$ ', representing the patch length, was assigned to column 1 of the orthogonal array. Twelve levels were formed for the factor 'A' consisting of the patch lengths $30,40,50$, and 60 mm with single ply; $30,40,50$, and 60 mm with two plies and $30,40,50$, and 60 mm with three plies. Thus, there were four levels of single-ply patches, four levels of two plies patches and four levels of three plies patch system. The thickness effect was included in the patch length factor while combining the length groups. The factor ' B ' representing the patch width was assigned to the second column of the orthogonal array. Four levels of factor 'B' were 30, 36, 42 and 48 mm . The other factors C, D, in the orthogonal array were not used in this analysis.

Table 1. L48 orthogonal array [24]

| Sr no | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expt No | CC1P_1 | CC1P _ 2 | CC1P _3 | CC1P _4 | CC1P _5 | CC 1P _6 | CC1P _ 7 | CC1P _ 8 | CC1P _9 | CC1P _10 |
| W | 30 | 30 | 30 | 30 | 36 | 36 | 36 | 36 | 42 | 42 |
| $\mathrm{L}_{\text {GFRP }}$ | 50 | 60 | 70 | 80 | 50 | 60 | 70 | 80 | 50 | 60 |
| $\mathrm{L}_{\text {CFRP1 }}$ | 30 | 40 | 50 | 60 | 30 | 40 | 50 | 60 | 30 | 40 |
| $\mathrm{L}_{\text {CFRP } 2}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\mathrm{L}_{\text {CFRP3 }}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Sr no | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Expt no | CC1P _11 | CC1P _12 | CC1P _13 | CC1P _14 | CC1P _15 | CC1P _16 | CC2P_1 | CC2P_2 | CC2P_3 | CC2P_4 |
| W | 42 | 42 | 48 | 48 | 48 | 48 | 30 | 30 | 30 | 30 |
| $\mathrm{L}_{\text {GFRP }}$ | 70 | 80 | 50 | 60 | 70 | 80 | 50 | 60 | 70 | 80 |
| $\mathrm{L}_{\text {CFRPI }}$ | 50 | 60 | 30 | 40 | 50 | 60 | 30 | 40 | 50 | 60 |
| $\mathrm{L}_{\text {CFRP } 2}$ | 0 | 0 | 0 | 0 | 0 | 0 | 20 | 30 | 40 | 50 |
| $\mathrm{L}_{\text {CFRP3 }}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| Sr no | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Expt no | CC2P_5 | CC2P_6 | CC2P_7 | CC2P_8 | CC2P_9 | CC2P_10 | CC2P_11 | CC2P_12 | CC2P_13 | CC2P_14 |
| W | 36 | 36 | 36 | 36 | 42 | 42 | 42 | 42 | 48 | 48 |
| $\mathrm{L}_{\text {GFRP }}$ | 50 | 60 | 70 | 80 | 50 | 60 | 70 | 80 | 50 | 60 |
| $\mathrm{L}_{\text {CFRP } 1}$ | 30 | 40 | 50 | 60 | 30 | 40 | 50 | 60 | 30 | 40 |
| $\mathrm{L}_{\text {CFRP2 }}$ | 20 | 30 | 40 | 50 | 20 | 30 | 40 | 50 | 20 | 30 |
| $\mathrm{L}_{\text {CFRP3 }}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| n | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Sr no | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| Expt no | CC2P_15 | CC2P_16 | CC3P_1 | CC3P_2 | CC3P_3 | CC3P_4 | CC3P_5 | CC3P_6 | CC3P_7 | CC3P_8 |
| W | 48 | 48 | 30 | 30 | 30 | 30 | 36 | 36 | 36 | 36 |
| $\mathrm{L}_{\text {GFRP }}$ | 70 | 80 | 50 | 60 | 70 | 80 | 50 | 60 | 70 | 80 |
| $\mathrm{L}_{\text {CFRP1 }}$ | 50 | 60 | 30 | 40 | 50 | 60 | 30 | 40 | 50 | 60 |
| $\mathrm{L}_{\text {CFRP } 2}$ | 40 | 50 | 20 | 30 | 40 | 50 | 20 | 30 | 40 | 50 |
| $\mathrm{L}_{\text {CFRP3 }}$ | 0 | 0 | 10 | 20 | 30 | 40 | 10 | 20 | 30 | 40 |
| n | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| Sr no | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 |  |  |
| Expt no | CC3P_9 | CC3P_10 | CC3P_11 | CC3P_12 | CC3P_13 | CC3P_14 | CC3P_15 | CC3P_16 |  |  |
| W | 42 | 42 | 42 | 42 | 48 | 48 | 48 | 48 |  |  |
| $\mathrm{L}_{\text {GFRP }}$ | 50 | 60 | 70 | 80 | 50 | 60 | 70 | 80 |  |  |
| $\mathrm{L}_{\text {CFRPI }}$ | 30 | 40 | 50 | 60 | 30 | 40 | 50 | 60 |  |  |
| $\mathrm{L}_{\text {CFRP2 }}$ | 20 | 30 | 40 | 50 | 20 | 30 | 40 | 50 |  |  |
| $\mathrm{L}_{\text {CFRP3 }}$ | 10 | 20 | 30 | 40 | 10 | 20 | 30 | 40 |  |  |
| n | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |  |  |

Table 2. Modified L48 orthogonal array

| Level | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Expt no | CC1P_1 | CC1P_5 | CC1P_9 | CC1P_13 | CC1P_2 | CC1P_6 | CC1P_10 | CC1P_14 |
| Level | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 |
| Expt no | CC1P_3 | CC1P_7 | CC1P_11 | CC1P_15 | CC1P_4 | CC1P_8 | CC1P_12 | CC1P_16 |
| Level | 5 | 5 | 5 | 5 | 6 | 6 | 6 | 6 |
| Expt no | CC2P_1 | CC2P_5 | CC2P_9 | CC2P_13 | CC2P_2 | CC2P_6 | CC2P_10 | CC2P_14 |
| Level | 7 | 7 | 7 | 7 | 8 | 8 | 8 | 8 |
| Expt no | CC2P_3 | CC2P_7 | CC2P_11 | CC2P_15 | CC2P_4 | CC2P_8 | CC2P_12 | CC2P_16 |
| Level | 9 | 9 | 9 | 9 | 10 | 10 | 10 | 10 |
| Expt no | CC3P_1 | CC3P_5 | CC3P_9 | CC3P_13 | CC3P_2 | CC3P_6 | CC3P_10 | CC3P_14 |
| Level | 11 | 11 | 11 | 11 | 12 | 12 | 12 | 12 |
| Expt no | CC3P_3 | CC3P_7 | CC3P_11 | CC3P_15 | CC3P_4 | CC3P_8 | CC3P_12 | CC3P_16 |

The standard L48 Taguchi orthogonal array was used, as shown in the Table T1 of Appendix. From the results of the numerical analysis, the output response was obtained as the normal stresses $(\mathrm{Y})$ in the skin at the origin and at the point where the inner ply was touching the skin. The normal stress (Y) is shown in Table T2 of Appendix. The interfacial normal and shear stresses ( $\sigma$ in and ois) as well as the separation distance, were also obtained. The next step in the Taguchi method is to determine the signal-to-noise ratio known as the $\mathrm{S} / \mathrm{N}$ ratio. The $\mathrm{S} / \mathrm{N}$ ratio is a measurement scale that represents the ratio of sensitivity to variability. There are two types of the $\mathrm{S} / \mathrm{N}$ ratio: static and dynamic. The patch separation process was analysed by using static as well as dynamic $\mathrm{S} / \mathrm{N}$ ratios based on three criteria; linearity, critical load and the volume of the patch. The static $\mathrm{S} / \mathrm{N}$ ratio was used for the critical load and the volume of the patch while the dynamic $\mathrm{S} / \mathrm{N}$ ratio was used for the linearity check. The P diagram for the dynamic case is shown in Figure 3 [22].


Figure 3. P-diagram for dynamic problems [22]
The signal factors ( M ) are the parameters set by the user of the process to express the intended value for the output/response $(\mathrm{Y})$ of the product. The noise factors $(\mathrm{X})$ are the factors that are difficult to control in the field, and they cause the output $(\mathrm{Y})$ to deviate from the target specified by the signal factor. The control factors $(\mathrm{Z})$ are the parameters that can be specified freely by the designer. In a measurement system, the input-to-output relationship is studied. In a good measuring system, the response of measurement must be proportional to the true value. In a dynamic system, there is always a need to adjust the output to the target by varying a certain input signal. In such a case, adjustability becomes critical for the design. Here it is important that the input/output relationship be proportional or linear. In other words, linearity becomes critical for adjusting systems. The better the linearity and the steeper the input/output relationship, the better the adjustability. A good measurement system must be sensitive to different inputs; thus, the slope showing the input/output relationship must be steep. Therefore, the slope is used as being equivalent to sensitivity. The variability also must be small. For the patch separation process, the $\mathrm{S} / \mathrm{N}$ ratio of the type continuous-continuous, i.e. both the input and the output as continuous variables, was used. When a dynamic $\mathrm{S} / \mathrm{N}$ ratio is used to evaluate a measuring system, the three elements, sensitivity, slope and variability, are combined into a single index, $\mathrm{S} / \mathrm{N}$ ratio allowing the user to evaluate and improve the system. The linearity between the signal factor and the output response was verified by using the dynamic $\mathrm{S} / \mathrm{N}$ ratio in terms of the slope ( $\beta$ ) given by [22,23];

$$
\begin{equation*}
\eta=10 * \log _{10}\left(\beta^{2}\right) \tag{1}
\end{equation*}
$$

To verify the linearity between the input (applied stress, M ) and the output (normal stress developed in the patch, Y ), the relation of normal stress in the patch against the applied stress in the axial direction was plotted for all 48 cases. The input/output relation for the case of a single patch with a length of 30 mm and width of 30 mm with four distinct points is shown in Figure 4.


Figure 4. Input-output linear characteristics
From the figure, it was clear that there was a linear relationship between the applied stress and the normal stress in the patch for four distinct points. The same linear relationship was observed for all the remaining cases. This implied that the skin-patch assembly followed a linear relation under the action of applied load similar to a solid material. Thus the patch separation process satisfied the linearity criterion as per the Taguchi technique. It was also observed from the graph that the variability around the linear curve of the input/output was very small for all the cases, satisfying the criterion of the variability of the Taguchi technique.

Though the sensitivity was not important in this case, it was checked. To check the sensitivity between the input and output, the slope of the straight line in the above graph of normal stress against the applied stress was determined, and the $\mathrm{S} / \mathrm{N}$ ratio for the slope was calculated for all the cases by using Eq. (1) as shown in Table T3 of Appendix. By using the S/N ratios obtained and by performing analysis of variance (ANOVA), the factor effects were calculated for each factor and the factor effect plot for the slope was generated in the MS Excel software, as shown in Figure 5.


Control factors and their levels
Figure 5. Factor effect plot for slope characteristics
From the factor effect plot of the slope characteristics, it was observed that the two maximum $\mathrm{S} / \mathrm{N}$ ratio values for the factor ' A ' were -2.609 and -2.735 , corresponding to the levels A7 and A8 respectively, in the two plies system. The maximum $\mathrm{S} / \mathrm{N}$ ratio for the factor ' B ' was obtained as -2.995 , corresponding to level B 1 . With these maximum $\mathrm{S} / \mathrm{N}$ ratios, the high sensitivity was verified. The main objective with the slope characteristics was to verify the linearity between applied stress as input and normal stress as output. In four-level factors, the width remains constant, but the length varies for the selected level. Since at the given level the thickness remains constant, there is no variation in the $\mathrm{S} / \mathrm{N}$ ratio.

The other objective of the optimisation was to minimise the amount of material used in the crack repair, which was expressed in terms of the volume of the patch. For obtaining the minimum volume condition, a static $\mathrm{S} / \mathrm{N}$ ratio with a smaller the better criterion was used. The P diagram for the static case is shown in Figure 6.


## CONTROL FACTORS

Figure 6. P-diagram for static problems [22]
The static $\mathrm{S} / \mathrm{N}$ ratios are used to study a single target, and it includes two aspects; sensitivity and variability. In the design of a product for a fixed target, there is no need to adjust the target from time to time, and the mean is considered as being equivalent to sensitivity. The value of the $\mathrm{S} / \mathrm{N}$ ratio is expressed in terms of a decibel value. To express $\mathrm{S} / \mathrm{N}$ ratio in decibel form, the logarithm of the mean sum of squares is multiplied by -10 ; that is:

$$
\begin{equation*}
\eta=-10 * \log _{10} \text { (mean of sum of squares) } \tag{2}
\end{equation*}
$$

The $\mathrm{S} / \mathrm{N}$ ratio for volume was calculated by using Eq. (2) as shown in Table T4 of Appendix. By using the $\mathrm{S} / \mathrm{N}$ ratios obtained and by performing the analysis of variance (ANOVA), the factor effects were calculated for each factor and the factor effect plot for volume was generated in the MS Excel software as shown in Figure 7.


Figure 7. Factor effect plot for volume
For volume characteristics, the criterion of smaller the better was applied. From the factor effect plot for volume, it was observed that the $\mathrm{S} / \mathrm{N}$ ratio was at the maximum at -62.268 for level A8 in the two plies system while it was the maximum at -64.278 for level A12 in the three plies system for the factor ' $A$ '. The $\mathrm{S} / \mathrm{N}$ ratio was at the maximum at the level 'B4', and it was -60.957 for the factor 'B'. In 4-level factors, the width remains constant, but the length varies for the selected level. Since, at the given level, the thickness remains constant, there is no variation in the $\mathrm{S} / \mathrm{N}$ ratio.

Then the criterion of the critical load of separation of the patch was applied. Since the critical load associated with the complete failure of the patch needed to be maximised since it delays the failure of the patch, the larger-the-better criterion was used, and the corresponding $\mathrm{S} / \mathrm{N}$ ratio was obtained. The $\mathrm{S} / \mathrm{N}$ ratio for the larger-the-better condition is expressed in terms of a decibel value. To express the $\mathrm{S} / \mathrm{N}$ ratio in decibel form, the logarithm of the mean of the sum of reciprocals squares was multiplied by -10 ; that is:

$$
\begin{equation*}
\eta=-10 * \log _{10} \text { (mean of sum of reciprocals squares) } \tag{3}
\end{equation*}
$$

To maximise the critical load, the $\mathrm{S} / \mathrm{N}$ ratio values for the critical load (Pc) were determined by using the Eq. (3) and are shown in Table T5 of Appendix. By using the $\mathrm{S} / \mathrm{N}$ ratio and by performing the analysis of variance (ANOVA), the factor effects were calculated for each factor and the factor effect plot for the critical load was generated in the MS Excel software, as shown in Figure 8.


Figure 8. Factor effect plot for the critical load
For the critical load characteristics, the larger the better criterion was used. From the factor effect plot, it was observed that the $\mathrm{S} / \mathrm{N}$ ratio was at the maximum at level A 8 of the two plies system. It was also the maximum at level A12 of the three-plies system. The S/N ratio for A8 and A12 were 50.565 and 50.563 , respectively. Even if three CFRP plies are
used for arresting the crack propagation, there is hardly any improvement in the critical loads, i.e. the load at which there is complete failure of the patch. The two plies and the three plies patches have the same strength from the separation point of view, and increasing the number of plies above the two plies system only increases the volume of the patch. For economical repair, the two plies patch proves to have sufficient strength. Thus, with the critical load criterion, the maximum $\mathrm{S} / \mathrm{N}$ ratio was considered at level A8 of the two plies patch. For factor B , the maximum $\mathrm{S} / \mathrm{N}$ ratio was at level B4. It was observed to be 50.353 .

For optimising the patch geometry, all the above criteria were considered and the optimum conditions were obtained, which can be summarised as follows. As per the slope characteristics criterion, level A8 of the factor 'A' satisfied the condition for the slope characteristics. As per the patch volume criterion, level A8 in two plies system and A12 in three plies system satisfied the volume criteria. Since the two plies condition provided sufficient strength as per the critical load condition, the level A12 of three plies patch was not considered, and it was discarded. As per the critical load criterion, level A8 of the two plies system gave the maximum $\mathrm{S} / \mathrm{N}$ ratio corresponding to the critical load. Thus, from the above discussion, it was clear that level A8 satisfied all the criteria required for the economical and efficient repair of the crack. So, level A8 was chosen as the optimum level of factor 'A' for the patch separation process. Level A8 had four combinations of the geometrical parameters labelled as CC2P_4, CC2P_8, CC2P_12 and CC2P_16. One level out of these four was to be selected as the optimum level. CC2P_4, CC2P_8, CC2P_12 and CC2P_16 had the S/N ratio of $50.543,50.565,50.572$ and 50.580 , respectively. The failure stress corresponding to the numerical non-convergence in the finite element analysis was nearly the same for these cases, with a negligible difference, as seen from Table T5 of Appendix A. Any combination of these four can be considered as the optimum one with the critical load criterion. The combination CC2P_4 satisfied the slope condition, and it had the minimum patch volume at 1014 mm 3 . So CC2P_4 was considered as the optimum combination for the patch separation process based on the factor effect of factor ' A '. The patch length corresponding to this level was 60 mm , with two CFRP plies having a patch width of 30 mm .

For the 4-level factor B, level B1 satisfied the requirement of the slope characteristics. As per the volume criterion and the critical load criterion, factor level B4 satisfied the necessary requirements. Since the slope was used only to verify the linearity between the input and the output, the value of the slope did not matter in this case. The other two criteria, the smaller volume and the larger critical load were more important for the patch separation process. So factor level B4 was considered to be the optimum level for factor B based on the volume and the critical load criteria. Factor level B4 had twelve combinations with a patch width of 48 mm contributing to its effect on the patch separation process. The combination CC1P_14 satisfied both the conditions, i.e. maximum critical load at 332.8 MPa and the minimum volume $768 \mathrm{~mm}^{3}$ among all the combinations of factor B4. The dimensions for this combination were: patch width $=48 \mathrm{~mm}$, patch length $=40 \mathrm{~mm}$ and number of plies $=1$.

From the analysis of variance (ANOVA) based on the critical load criterion, the contribution of factor A (71\%) was greater than the contribution of factor B ( $25 \%$ ). So, based on the critical load criterion, the optimum combination was predicted as CC2P_4 with the patch length 60 mm , patch width 30 mm and two numbers of CFRP plies. Based on the volume criterion, the combination CC1P_14 having the minimum volume was considered as the optimum combination with the patch length of 40 mm , patch width of 48 mm and a single CFRP ply. The optimum combination based on both the criterion, i.e. critical load and the volume, was obtained by combining the two experiments, CC2P_4 and CC1P_14. Thus, from the factor effect study of the factors ' A ' and ' B ', the optimum geometrical factors for the patch separation process were determined, as shown in Table 3. With this optimum combination, the verification experiment was performed, and it was observed that the critical load obtained was 338.05 MPa . The range of the critical load predicted by the analysis of variance was 334.5 MPa to 362.2 MPa . It was clear that the critical load of the verification experiment was in the predicted range, and it was acceptable. The volume of the patch for the optimum combination was $1622.4 \mathrm{~mm}^{3}$. The input-output relation was observed to be linear.

Table 3. Optimum geometrical factors

| Optimum patch length <br> $(\mathrm{mm})$ | Optimum patch width <br> $(\mathrm{mm})$ | Number of <br> plies | Critical load <br> $(\mathrm{MPa})$ | Volume <br> $\left(\mathrm{mm}^{3}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 60 mm | 48 mm | 2 | 338.05 | 1622.4 |

Thus, the Taguchi technique was successfully applied to optimise the geometrical factors of the patch separation process.

### 4.0 CONCLUSIONS

This study was carried out to determine the optimum patch size for a through-the-thickness centre crack repaired with a polymer composite patch in a thin aluminium alloy sheet. Numerical analyses were carried out using the cohesive zone model by varying the patch width from 30 mm to 48 mm , patch length from 30 mm to 60 mm and the number of plies from one to three. The variation of the stresses in the patch separation process was obtained. Taguchi analysis was used to plot the factor effect plots. The linearity, sensitivity and variability between the applied stress and the normal stress developed in the patch were checked. The condition for the maximum critical load was ensured and finally, the patch
with the minimum volume was selected as the optimum patch. The optimum geometrical factors were obtained as; patch width of 48 mm , patch length of 60 mm with two plies.

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### 7.0 APPENDIX

The appendix contains the standard L48 orthogonal array and the output, i.e. stresses obtained from numerical analysis and $\mathrm{S} / \mathrm{N}$ ratio obtained by using Taguchi technique.

Table T1. Standard L48 Taguchi orthogonal array [24]

| A | B | C | D | E | F | G | H | I | J | K | L | M |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A1 | B1 | C1 | D1 | E1 | F1 | G1 | H1 | I1 | J1 | K1 | L1 | M1 |
| A1 | B2 | C2 | D2 | E2 | F2 | G2 | H2 | I2 | J2 | K2 | L2 | M2 |
| A1 | B3 | C3 | D3 | E3 | F3 | G3 | H3 | I3 | J3 | K3 | L3 | M3 |
| A1 | B4 | C4 | D4 | E4 | F4 | G4 | H4 | I4 | J4 | K4 | L4 | M4 |
| A2 | B1 | C1 | D1 | E3 | F3 | G3 | H4 | I4 | J4 | K2 | L2 | M2 |


| A2 | B2 | C2 | D2 | E4 | F4 | G4 | H3 | I3 | J3 | K1 | L1 | M1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A2 | B3 | C3 | D3 | E1 | F1 | G1 | H2 | I2 | J2 | K4 | L4 | M4 |
| A2 | B4 | C4 | D4 | E2 | F2 | G2 | H1 | I1 | J1 | K3 | L3 | M3 |
| A3 | B1 | C1 | D1 | E4 | F4 | G4 | H2 | I2 | J2 | K3 | L3 | M3 |
| A3 | B2 | C2 | D2 | E3 | F3 | G3 | H1 | I1 | J1 | K4 | L4 | M4 |
| A3 | B3 | C3 | D3 | E2 | F2 | G2 | H4 | I4 | J4 | K1 | L1 | M1 |
| A3 | B4 | C4 | D4 | E1 | F1 | G1 | H3 | I3 | J3 | K2 | L2 | M2 |
| A4 | B1 | C2 | D4 | E1 | F3 | G2 | H2 | I3 | J4 | K4 | L3 | M1 |
| A4 | B2 | C1 | D3 | E2 | F4 | G1 | H1 | I4 | J3 | K3 | L4 | M2 |
| A4 | B3 | C4 | D2 | E3 | F1 | G4 | H4 | I1 | J2 | K2 | L1 | M3 |
| A4 | B4 | C3 | D1 | E4 | F2 | G3 | H3 | I2 | J1 | K1 | L2 | M4 |
| A5 | B1 | C2 | D4 | E2 | F1 | G3 | H3 | I4 | J2 | K1 | L4 | M3 |
| A5 | B2 | C1 | D3 | E1 | F2 | G4 | H4 | I3 | J1 | K2 | L3 | M4 |
| A5 | B3 | C4 | D2 | E4 | F3 | G1 | H1 | I2 | J4 | K3 | L2 | M1 |
| A5 | B4 | C3 | D1 | E3 | F4 | G2 | H2 | I1 | J3 | K4 | L1 | M2 |
| A6 | B1 | C2 | D4 | E3 | F2 | G1 | H4 | I2 | J3 | K3 | L1 | M4 |
| A6 | B2 | C1 | D3 | E4 | F1 | G2 | H3 | I1 | J4 | K4 | L2 | M3 |
| A6 | B3 | C4 | D2 | E1 | F4 | G3 | H2 | I4 | J1 | K1 | L3 | M2 |
| A6 | B4 | C3 | D1 | E2 | F3 | G4 | H1 | I3 | J2 | K2 | L4 | M1 |
| A7 | B1 | C3 | D2 | E1 | F2 | G4 | H1 | I4 | J3 | K4 | L2 | M3 |
| A7 | B2 | C4 | D1 | E2 | F1 | G3 | H2 | I3 | J4 | K3 | L1 | M4 |
| A7 | B3 | C1 | D4 | E3 | F4 | G2 | H3 | I2 | J1 | K2 | L4 | M1 |
| A7 | B4 | C2 | D3 | E4 | F3 | G1 | H4 | I1 | J2 | K1 | L3 | M2 |
| A8 | B1 | C3 | D2 | E2 | F4 | G1 | H3 | I1 | J4 | K2 | L3 | M4 |
| A8 | B2 | C4 | D1 | E1 | F3 | G2 | H4 | I2 | J3 | K1 | L4 | M3 |
| A8 | B3 | C1 | D4 | E4 | F2 | G3 | H1 | I3 | J2 | K4 | L1 | M2 |
| A8 | B4 | C2 | D3 | E3 | F1 | G4 | H2 | I4 | J1 | K3 | L2 | M1 |
| A9 | B1 | C3 | D2 | E4 | F1 | G2 | H4 | I3 | J1 | K3 | L4 | M2 |
| A9 | B2 | C4 | D1 | E3 | F2 | G1 | H3 | I4 | J2 | K4 | L3 | M1 |
| A9 | B3 | C1 | D4 | E2 | F3 | G4 | H2 | I1 | J3 | K1 | L2 | M4 |
| A9 | B4 | C2 | D3 | E1 | F4 | G3 | H1 | I2 | J4 | K2 | L1 | M3 |
| A10 | B1 | C4 | D3 | E2 | F3 | G4 | H3 | I2 | J1 | K4 | L1 | M2 |
| A10 | B2 | C3 | D4 | E1 | F4 | G3 | H4 | I1 | J2 | K3 | L2 | M1 |
| A10 | B3 | C2 | D1 | E4 | F1 | G2 | H1 | I4 | J3 | K2 | L3 | M4 |
| A10 | B4 | C1 | D2 | E3 | F2 | G1 | H2 | I3 | J4 | K1 | L4 | M3 |
| A11 | B1 | C4 | D3 | E3 | F4 | G2 | H1 | I3 | J2 | K1 | L2 | M4 |
| A11 | B2 | C3 | D4 | E4 | F3 | G1 | H2 | I4 | J1 | K2 | L1 | M3 |
| A11 | B3 | C2 | D1 | E1 | F2 | G4 | H3 | I1 | J4 | K3 | L4 | M2 |
| A11 | B4 | C1 | D2 | E2 | F1 | G3 | H4 | I2 | J3 | K4 | L3 | M1 |
| A12 | B1 | C4 | D3 | E4 | F2 | G3 | H2 | I1 | J3 | K2 | L4 | M1 |
| A12 | B2 | C3 | D4 | E3 | F1 | G4 | H1 | I2 | J4 | K1 | L3 | M2 |
| A12 | B3 | C2 | D1 | E2 | F4 | G1 | H4 | I3 | J1 | K4 | L2 | M3 |
| A12 | B4 | C1 | D2 | E1 | F3 | G2 | H3 | I4 | J2 | K3 | L1 | M4 |

Table T2. Normal stress in the patch at different applied stresses

| Level | Expt no | Normal stress (Y) in patch (MPa) for the specified applied stress (M), MPa |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0 | 4 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 |
| 1 | CC1P_1 | 0 | 3.18 | 21.96 | 43.46 | 64.86 | 89.60 | 143.62 | 155.97 | 163.55 | 156.33 | 150.23 | ---- |
| 1 | CC1P_5 | 0 | 3.15 | 21.66 | 42.81 | 63.83 | 88.07 | 143.13 | 155.93 | 162.77 | 153.06 | 155.30 | 135.31 |
| 1 | CC1P_9 | 0 | 3.13 | 21.45 | 42.34 | 63.08 | 86.95 | 141.16 | 155.21 | 163.31 | 165.49 | 157.29 | 130.20 |
| 1 | CC1P_13 | 0 | 3.11 | 21.29 | 41.98 | 62.49 | 86.08 | 139.66 | 154.88 | 161.25 | 157.79 | 157.60 | 129.20 |
| 2 | CC1P_2 | 0 | 3.27 | 22.55 | 44.64 | 66.61 | 91.97 | 146.82 | 166.39 | 177.17 | 169.61 | 180.16 | 179.39 |
| 2 | CC1P_6 | 0 | 3.22 | 22.13 | 43.74 | 65.20 | 89.91 | 143.79 | 165.40 | 173.72 | 177.78 | 171.32 | 168.35 |
| 2 | CC1P_10 | 0 | 3.18 | 21.82 | 43.06 | 64.12 | 88.34 | 141.19 | 162.02 | 175.65 | 175.86 | 170.49 | 171.38 |
| 2 | CC1P_14 | 0 | 3.16 | 21.58 | 42.53 | 63.27 | 87.10 | 139.07 | 161.01 | 172.38 | 171.92 | 173.79 | 170.57 |
| 3 | CC1P_3 | 0 | 3.34 | 23.07 | 45.68 | 68.15 | 94.08 | 146.91 | 169.78 | 183.41 | 181.14 | 184.02 | 201.10 |
| 3 | CC1P_7 | 0 | 3.28 | 22.55 | 44.59 | 66.46 | 91.63 | 136.46 | 166.62 | 181.68 | 185.71 | 177.05 | 177.05 |
| 3 | CC1P_11 | 0 | 3.23 | 22.15 | 43.73 | 65.11 | 89.69 | 133.44 | 164.28 | 179.07 | 181.74 | 177.32 | 180.92 |
| 3 | CC1P_15 | 0 | 3.20 | 21.83 | 43.03 | 64.01 | 88.10 | 108.01 | 162.57 | 177.24 | 183.51 | 173.79 | 175.02 |
| 4 | CC1P_4 | 0 | 3.41 | 23.55 | 46.64 | 69.59 | 96.05 | 142.77 | 171.35 | 185.88 | 193.32 | 190.08 | 205.94 |
| 4 | CC1P_8 | 0 | 3.34 | 22.97 | 45.41 | 67.69 | 93.33 | 128.42 | 167.94 | 181.54 | 181.45 | 187.24 | 189.88 |
| 4 | CC1P_12 | 0 | 3.29 | 22.49 | 44.41 | 66.13 | 91.10 | 113.21 | 165.09 | 178.65 | 179.96 | 182.90 | 184.63 |
| 4 | CC1P_16 | 0 | 3.24 | 22.10 | 43.57 | 64.82 | 89.21 | 110.80 | 162.97 | 176.43 | 180.88 | 179.69 | 180.03 |
| 5 | CC2P_1 | 0 | 3.61 | 23.75 | 45.63 | 66.59 | 111.24 | 114.19 | 114.32 | 113.10 | 126.79 | 124.40 | ---- |
| 5 | CC2P_5 | 0 | 3.60 | 23.55 | 45.16 | 65.79 | 109.65 | 112.13 | 111.94 | 113.38 | 133.08 | 113.40 | 50.24 |


| 5 | CC2P_9 | 0 | 3.59 | 23.42 | 44.82 | 65.20 | 108.26 | 113.76 | 113.37 | 110.97 | 126.21 | 110.64 | 95.19 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | CC2P_13 | 0 | 3.58 | 23.33 | 44.55 | 64.71 | 107.45 | 112.55 | 112.05 | 110.13 | 128.12 | 113.54 | 91.47 |
| 6 | CC2P_2 | 0 | 3.63 | 23.58 | 45.07 | 65.54 | 88.36 | 121.59 | 124.32 | 123.04 | 143.45 | 129.65 | 103.29 |
| 6 | CC2P_6 | 0 | 3.73 | 24.05 | 45.82 | 66.49 | 89.45 | 117.39 | 118.34 | 127.55 | 125.66 | 148.54 | 129.52 |
| 6 | CC2P_10 | 0 | 3.67 | 23.55 | 44.73 | 64.77 | 86.96 | 116.96 | 117.32 | 128.29 | 119.51 | 136.07 | 118.98 |
| 6 | CC2P_14 | 0 | 3.69 | 23.58 | 44.66 | 64.55 | 86.53 | 117.32 | 114.21 | 133.10 | 121.00 | 138.91 | 113.81 |
| 7 | CC2P_3 | 0 | 3.94 | 25.30 | 48.21 | 70.06 | 94.43 | 115.49 | 130.20 | 131.34 | 150.83 | 143.06 | 142.03 |
| 7 | CC2P_7 | 0 | 3.90 | 24.83 | 47.11 | 68.27 | 91.78 | 112.03 | 130.74 | 128.16 | 148.40 | 134.58 | 161.18 |
| 7 | CC2P_11 | 0 | 3.87 | 24.49 | 46.28 | 66.88 | 89.71 | 109.33 | 131.70 | 124.88 | 143.57 | 148.07 | 150.17 |
| 7 | CC2P_15 | 0 | 3.85 | 24.23 | 45.62 | 65.77 | 88.04 | 107.16 | 132.92 | 121.90 | 139.35 | 158.79 | 141.41 |
| 8 | CC2P_4 | 0 | 3.94 | 25.17 | 47.93 | 69.68 | 93.98 | 115.01 | 139.09 | 133.25 | 155.10 | 142.26 | 136.31 |
| 8 | CC2P_8 | 0 | 3.89 | 24.58 | 46.58 | 67.51 | 90.81 | 110.93 | 137.57 | 133.95 | 152.29 | 150.47 | 136.74 |
| 8 | CC2P_12 | 0 | 3.85 | 24.13 | 45.51 | 65.78 | 88.28 | 107.67 | 138.05 | 131.08 | 150.96 | 140.53 | 146.36 |
| 8 | CC2P_16 | 0 | 3.83 | 23.78 | 44.66 | 64.37 | 86.21 | 104.99 | 126.10 | 131.74 | 150.87 | 144.31 | 151.29 |
| 9 | CC3P_1 | 0 | 3.24 | 21.34 | 41.04 | 59.88 | 98.15 | 99.35 | 94.70 | 107.73 | 118.99 | 106.97 | 99.76 |
| 9 | CC3P_5 | 0 | 3.22 | 21.17 | 40.63 | 59.19 | 99.16 | 100.81 | 95.82 | 104.79 | 123.02 | 113.66 | 51.40 |
| 9 | CC3P_9 | 0 | 3.21 | 21.05 | 40.33 | 58.66 | 98.19 | 102.14 | 98.58 | 100.35 | 116.21 | 112.39 | 89.36 |
| 9 | CC3P_13 | 0 | 3.17 | 20.70 | 39.59 | 57.49 | 99.30 | 97.05 | 101.69 | 102.80 | 101.44 | 115.76 | 89.69 |
| 10 | CC3P_2 | 0 | 3.36 | 21.77 | 41.44 | 60.02 | 80.53 | 108.10 | 104.09 | 105.55 | 123.09 | 110.89 | 102.95 |
| 10 | CC3P_6 | 0 | 3.34 | 21.50 | 40.81 | 58.98 | 78.94 | 105.51 | 101.16 | 105.48 | 122.28 | 106.56 | 128.00 |
| 10 | CC3P_10 | 0 | 3.32 | 21.31 | 40.35 | 58.18 | 77.73 | 108.45 | 104.97 | 100.49 | 114.92 | 101.26 | 115.07 |
| 10 | CC3P_14 | 0 | 3.24 | 20.75 | 39.17 | 56.36 | 75.14 | 103.95 | 106.75 | 108.17 | 101.36 | 114.09 | 97.05 |
| 11 | CC3P_3 | 0 | 3.31 | 21.08 | 39.81 | 57.39 | 76.71 | 114.39 | 114.24 | 118.30 | 115.38 | 112.83 | 104.24 |
| 11 | CC3P_7 | 0 | 3.28 | 20.72 | 38.97 | 56.02 | 74.65 | 90.50 | 112.14 | 115.61 | 114.83 | 114.39 | 120.97 |
| 11 | CC3P_11 | 0 | 3.25 | 20.47 | 38.35 | 54.96 | 73.06 | 88.41 | 110.84 | 114.30 | 115.59 | 110.78 | 111.80 |
| 11 | CC3P_15 | 0 | 3.31 | 20.76 | 38.77 | 55.44 | 73.54 | 88.87 | 112.64 | 112.38 | 107.91 | 120.01 | 112.25 |
| 12 | CC3P_4 | 0 | 3.38 | 21.22 | 39.91 | 57.46 | 76.76 | 93.27 | 122.56 | 125.09 | 124.53 | 132.14 | 128.41 |
| 12 | CC3P_8 | 0 | 3.34 | 20.76 | 38.83 | 55.70 | 74.16 | 89.90 | 122.45 | 121.58 | 120.30 | 136.01 | 131.52 |
| 12 | CC3P_12 | 0 | 3.31 | 20.42 | 38.01 | 54.34 | 72.13 | 87.25 | 123.49 | 121.59 | 117.43 | 125.69 | 135.30 |
| 12 | CC3P_16 | 0 | 3.37 | 20.72 | 38.38 | 54.70 | 72.42 | 87.46 | 102.20 | 120.94 | 119.95 | 123.84 | 135.37 |

Table T3. S/N ratio for Slope characteristics

| Level | Expt no | Slope $(\beta)$ | $\beta^{2}$ | S/N ratio $(\eta)$ | Level | Expt no | Slope $(\beta)$ | $\beta^{2}$ | S/N ratio $(\eta)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CC1P_1 | 0.717 | 0.514 | -2.89 | 7 | CC2P_3 | 0.767 | 0.588 | -2.30 |
| 1 | CC1P_5 | 0.705 | 0.497 | -3.04 | 7 | CC2P_7 | 0.747 | 0.558 | -2.53 |
| 1 | CC1P_9 | 0.695 | 0.483 | -3.16 | 7 | CC2P_11 | 0.731 | 0.534 | -2.72 |
| 1 | CC1P_13 | 0.690 | 0.476 | -3.22 | 7 | CC2P_15 | 0.718 | 0.516 | -2.88 |
| 2 | CC1P_2 | 0.736 | 0.542 | -2.66 | 8 | CC2P_4 | 0.763 | 0.582 | -2.35 |
| 2 | CC1P_6 | 0.720 | 0.518 | -2.85 | 8 | CC2P_8 | 0.738 | 0.545 | -2.64 |
| 2 | CC1P_10 | 0.708 | 0.501 | -3.00 | 8 | CC2P_12 | 0.718 | 0.516 | -2.88 |
| 2 | CC1P_14 | 0.698 | 0.487 | -3.12 | 8 | CC2P_16 | 0.702 | 0.493 | -3.07 |
| 3 | CC1P_3 | 0.753 | 0.567 | -2.46 | 9 | CC3P_1 | 0.657 | 0.432 | -3.65 |
| 3 | CC1P_7 | 0.734 | 0.539 | -2.69 | 9 | CC3P_5 | 0.650 | 0.423 | -3.74 |
| 3 | CC1P_11 | 0.719 | 0.517 | -2.87 | 9 | CC3P_9 | 0.644 | 0.415 | -3.82 |
| 3 | CC1P_15 | 0.706 | 0.498 | -3.02 | 9 | CC3P_13 | 0.630 | 0.397 | -4.01 |
| 4 | CC1P_4 | 0.769 | 0.591 | -2.28 | 10 | CC3P_2 | 0.657 | 0.432 | -3.65 |
| 4 | CC1P_8 | 0.748 | 0.560 | -2.52 | 10 | CC3P_6 | 0.646 | 0.417 | -3.80 |
| 4 | CC1P_12 | 0.730 | 0.533 | -2.73 | 10 | CC3P_10 | 0.636 | 0.404 | -3.93 |
| 4 | CC1P_16 | 0.715 | 0.511 | -2.91 | 10 | CC3P_14 | 0.614 | 0.377 | -4.24 |
| 5 | CC2P_1 | 0.731 | 0.534 | -2.72 | 11 | CC3P_3 | 0.627 | 0.393 | -4.05 |
| 5 | CC2P_5 | 0.722 | 0.521 | -2.83 | 11 | CC3P_7 | 0.612 | 0.375 | -4.26 |
| 5 | CC2P_9 | 0.715 | 0.511 | -2.91 | 11 | CC3P_11 | 0.600 | 0.360 | -4.44 |
| 5 | CC2P_13 | 0.709 | 0.503 | -2.99 | 11 | CC3P_15 | 0.599 | 0.359 | -4.45 |
| 6 | CC2P_2 | 0.719 | 0.517 | -2.87 | 12 | CC3P_4 | 0.627 | 0.393 | -4.05 |
| 6 | CC2P_6 | 0.728 | 0.530 | -2.76 | 12 | CC3P_8 | 0.607 | 0.368 | -4.34 |
| 6 | CC2P_10 | 0.709 | 0.503 | -2.99 | 12 | CC3P_12 | 0.591 | 0.349 | -4.57 |
| 6 | CC2P_14 | 0.706 | 0.498 | -3.02 | 12 | CC3P_16 | 0.588 | 0.346 | -4.61 |
|  |  |  |  |  |  |  |  |  |  |

Table T4. S/N ratio for volume

| Level | Expt no | Volume (V) | $\mathrm{V}^{2}$ | S/N ratio $(\eta)$ | Level | Expt no | Volume (V) | $V^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| 1 | CC1P_1 | 378.0 | 142884 | -51.550 | 7 | CC2P_3 | 846.0 | 715716 | -58.547 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | CC1P_5 | 453.6 | 205753 | -53.133 | 7 | CC2P_7 | 1015.2 | 1030631 | -60.131 |
| 1 | CC1P_9 | 529.2 | 280052.6 | -54.472 | 7 | CC2P_11 | 1184.4 | 1402803 | -61.470 |
| 1 | CC1P_13 | 604.8 | 365783 | -55.632 | 7 | CC2P_15 | 1353.6 | 1832233 | -62.630 |
| 2 | CC1P_2 | 480.0 | 230400 | -53.625 | 8 | CC2P_4 | 1014.0 | 1028196 | -60.121 |
| 2 | CC1P_6 | 576.0 | 331776 | -55.208 | 8 | CC2P_8 | 1216.8 | 1480602 | -61.704 |
| 2 | CC1P_10 | 672.0 | 451584 | -56.547 | 8 | CC2P_12 | 1419.6 | 2015264 | -63.043 |
| 2 | CC1P_14 | 768.0 | 589824 | -57.707 | 8 | CC2P_16 | 1622.4 | 2632182 | -64.203 |
| 3 | CC1P_3 | 582.0 | 338724 | -55.298 | 9 | CC3P_1 | 576.0 | 331776 | -55.208 |
| 3 | CC1P_7 | 698.4 | 487762.6 | -56.882 | 9 | CC3P_5 | 691.2 | 477757.4 | -56.792 |
| 3 | CC1P_11 | 814.8 | 663899 | -58.221 | 9 | CC3P_9 | 806.4 | 650281 | -58.131 |
| 3 | CC1P_15 | 931.2 | 867133.4 | -59.381 | 9 | CC3P_13 | 921.6 | 849346.6 | -59.291 |
| 4 | CC1P_4 | 684.0 | 467856 | -56.701 | 10 | CC3P_2 | 810.0 | 656100 | -58.170 |
| 4 | CC1P_8 | 820.8 | 673712.6 | -58.285 | 10 | CC3P_6 | 972.0 | 944784 | -59.753 |
| 4 | CC1P_12 | 957.6 | 916997.8 | -59.624 | 10 | CC3P_10 | 1134.0 | 1285956 | -61.092 |
| 4 | CC1P_16 | 1094.4 | 1197711 | -60.784 | 10 | CC3P_14 | 1296.0 | 1679616 | -62.252 |
| 5 | CC2P_1 | 510.0 | 260100 | -54.151 | 11 | CC3P_3 | 1044.0 | 1089936 | -60.374 |
| 5 | CC2P_5 | 612.0 | 374544 | -55.735 | 11 | CC3P_7 | 1252.8 | 1569508 | -61.958 |
| 5 | CC2P_9 | 714.0 | 509796 | -57.074 | 11 | CC3P_11 | 1461.6 | 2136275 | -63.297 |
| 5 | CC2P_13 | 816.0 | 665856 | -58.234 | 11 | CC3P_15 | 1670.4 | 2790236 | -64.456 |
| 6 | CC2P_2 | 678.0 | 459684 | -56.625 | 12 | CC3P_4 | 1278.0 | 1633284 | -62.131 |
| 6 | CC2P_6 | 813.6 | 661945 | -58.208 | 12 | CC3P_8 | 1533.6 | 2351929 | -63.714 |
| 6 | CC2P_10 | 949.2 | 900980.6 | -59.547 | 12 | CC3P_12 | 1789.2 | 3201237 | -65.053 |
| 6 | CC2P_14 | 1084.8 | 1176791 | -60.707 | 12 | CC3P_16 | 2044.8 | 4181207 | -66.213 |

Table T5. S/N ratio for the critical load

| Level | Expt no | $\mathrm{P}_{\mathrm{c}}(\mathrm{MPa})$ | $1 / \mathrm{P}_{\mathrm{c}}{ }^{2}$ | $\mathrm{~S} / \mathrm{N}$ ratio $(\eta)$ | Level | Expt no | $\mathrm{P}_{\mathrm{c}}(\mathrm{MPa})$ | $1 / \mathrm{P}_{\mathrm{c}}{ }^{2}$ | S/N ratio $(\eta)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | CC1P_1 | 268.88 | 72296.454 | $1.3832 \mathrm{E}-05$ | 7 | CC2P_3 | 326.56 | 106641.43 | $9.3772 \mathrm{E}-06$ |
| 1 | CC1P_5 | 279.6 | 78176.16 | $1.2792 \mathrm{E}-05$ | 7 | CC2P_7 | 337.34 | 113798.28 | $8.7875 \mathrm{E}-06$ |
| 1 | CC1P_9 | 286.84 | 82277.186 | $1.2154 \mathrm{E}-05$ | 7 | CC2P_11 | 337.81 | 114115.6 | $8.763 \mathrm{E}-06$ |
| 1 | CC1P_13 | 292.08 | 85310.726 | $1.1722 \mathrm{E}-05$ | 7 | CC2P_15 | 337.99 | 114237.24 | $8.7537 \mathrm{E}-06$ |
| 2 | CC1P_2 | 285.2 | 81339.04 | $1.2294 \mathrm{E}-05$ | 8 | CC2P_4 | 336.63 | 113319.76 | $8.8246 \mathrm{E}-06$ |
| 2 | CC1P_6 | 304.91 | 92970.108 | $1.0756 \mathrm{E}-05$ | 8 | CC2P_8 | 337.5 | 113906.25 | $8.7791 \mathrm{E}-06$ |
| 2 | CC1P_10 | 320.54 | 102745.89 | $9.7327 \mathrm{E}-06$ | 8 | CC2P_12 | 337.77 | 114088.57 | $8.7651 \mathrm{E}-06$ |
| 2 | CC1P_14 | 332.8 | 110755.84 | $9.0289 \mathrm{E}-06$ | 8 | CC2P_16 | 338.05 | 114277.8 | $8.7506 \mathrm{E}-06$ |
| 3 | CC1P_3 | 294.28 | 86600.718 | $1.1547 \mathrm{E}-05$ | 9 | CC3P_1 | 271 | 73441 | $1.3616 \mathrm{E}-05$ |
| 3 | CC1P_7 | 327.66 | 107361.08 | $9.3144 \mathrm{E}-06$ | 9 | CC3P_5 | 298.95 | 89371.103 | $1.1189 \mathrm{E}-05$ |
| 3 | CC1P_11 | 338.61 | 114656.73 | $8.7217 \mathrm{E}-06$ | 9 | CC3P_9 | 308.85 | 95388.323 | $1.0483 \mathrm{E}-05$ |
| 3 | CC1P_15 | 338.7 | 114717.69 | $8.7171 \mathrm{E}-06$ | 9 | CC3P_13 | 315.98 | 99843.36 | $1.0016 \mathrm{E}-05$ |
| 4 | CC1P_4 | 301.38 | 90829.904 | $1.101 \mathrm{E}-05$ | 10 | CC3P_2 | 308.25 | 95018.063 | $1.0524 \mathrm{E}-05$ |
| 4 | CC1P_8 8 | 338.46 | 114555.17 | $8.7294 \mathrm{E}-06$ | 10 | CC3P_6 | 336 | 112896 | $8.8577 \mathrm{E}-06$ |
| 4 | CC1P_12 | 338.64 | 114677.05 | $8.7201 \mathrm{E}-06$ | 10 | CC3P_10 | 337.77 | 114088.57 | $8.7651 \mathrm{E}-06$ |
| 4 | CC1P_16 | 338.92 | 114866.77 | $8.7057 \mathrm{E}-06$ | 10 | CC3P_14 | 337.36 | 113811.77 | $8.7864 \mathrm{E}-06$ |
| 5 | CC2P_11 | 276.8 | 76618.24 | $1.3052 \mathrm{E}-05$ | 11 | CC3P_3 | 325.23 | 105774.55 | $9.4541 \mathrm{E}-06$ |
| 5 | CC2P_5 | 292.8 | 85731.84 | $1.1664 \mathrm{E}-05$ | 11 | CC3P_7 | 337.35 | 113805.02 | $8.787 \mathrm{E}-06$ |
| 5 | CC2P_9 | 304.91 | 92970.108 | $1.0756 \mathrm{E}-05$ | 11 | CC3P_11 | 337.77 | 114088.57 | $8.7651 \mathrm{E}-06$ |
| 5 | CC2P_13 | 310.83 | 96615.289 | $1.035 \mathrm{E}-05$ | 11 | CC3P_15 | 337.8 | 114108.84 | $8.7636 \mathrm{E}-06$ |
| 6 | CC2P_2 | 297.68 | 88613.382 | $1.1285 \mathrm{E}-05$ | 12 | CC3P_4 | 336.45 | 113198.6 | $8.834 \mathrm{E}-06$ |
| 6 | CC2P_6 | 320.4 | 102656.16 | $9.7413 \mathrm{E}-06$ | 12 | CC3P_8 | 337.35 | 113805.02 | $8.787 \mathrm{E}-06$ |
| 6 | CC2P_10 | 337.53 | 113926.5 | $8.7776 \mathrm{E}-06$ | 12 | CC3P_12 | 337.81 | 114115.6 | $8.763 \mathrm{E}-06$ |
| 6 | CC2P_14 | 337.89 | 114169.65 | $8.7589 \mathrm{E}-06$ | 12 | CC3P_16 | 337.97 | 114223.72 | $8.7547 \mathrm{E}-06$ |

