

ORIGINAL ARTICLE

Uncertain Parameters Estimation using Multi-Dimensional Analysis and Stochastic Model Updating

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ABSTRACT – Stochastic model updating based on perturbation theory has been widely applied to quantify uncertain parameters in structural systems due to its simplicity and straightforward approach. Nevertheless, the significant requirements for establishing a good correlation in the initial prediction of structural responses and small perturbations in uncertain parameters have become influential in stochastic model updating. The initial assumptions of structural parameters are often unavailable to quantify the input properties due to insufficient information about the structural system. These problems contribute to large errors in initial prediction, causing ill-posedness in sensitivity matrices and convergence difficulties caused by the local minima function in the stochastic model updating approach. In these circumstances, this study attempts to propose a novel scheme to overcome the ill-posed and converging problems in the stochastic model updating by quantifying structural parameters of the assembled structure encompassing high uncertainties such as the stiffness term of the contact joint interface by using a combination of the lattice-based exploration approach and the perturbation-based stochastic model updating method. The lattice-based exploration approach is adopted for generating samples of predicted responses from the assumed initial distribution of random parameters in the interest of improving the initial correlation of the predicted responses for producing well-condition sensitivity. Responses from each sample are evaluated in light of their experimental counterparts to estimate the optimum initial distribution of the random parameters. Then, the initial statistical properties of the parameters can be estimated by rerunning the sampling approach using the optimum distribution. As a result, stochastic model updating using the perturbation approach can be applied efficiently with the new initial distribution. The proposed scheme has been demonstrated on an assembled bolted joint structure, focusing on the contact interfaces. It is found that the proposed scheme managed to produce satisfactory predictions on the distribution of natural frequencies with only 12.5 % of total errors are recorded in comparison with the experimental data.

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INTRODUCTION

Researchers have explored inverse solutions such as model updating methods to solve structural problems relating to parameters identification and quantification [1], [2]. In the scope of engineering design, a large volume of research focuses on the model updating approach in a deterministic manner. The presence of random uncertainties in structural parameters is not being considered for the sake of modelling simplifications [3]–[9]. However, for real applications with vibration-induced conditions such as in automotive, train, aerospace or electronic industries, these simplifications are irrelevant to establish a robust solution for parameters identification and quantification where designs perspective are limited to restrict significance. Therefore, the combination of random uncertainty analysis in structural parameters identification and quantification approach makes stochastic based model updating the preferable method.

The stochastic model updating approach has been used to analyse variability in vibration responses of structures caused by different types of uncertainties. Then, the vibration responses distribution was used to identify and quantify the uncertain random parameters. Various sources of random uncertainties such as variability in identical manufacturing structures, environmental defects, and variation in mechanical joints have been extensively studied in recent [10]–[16]. Most of the studies highlight uncertainties in production tolerance, material variation, and uncertainties in disassembly and reassembly of mechanical joints, showing a significant variability in vibration responses, especially for natural frequencies.

The use of statistical properties in model updating, especially for randomised updating parameters, has consequently caused an increment in computation efforts. There is growing interest regarding the development of efficient solutions for the stochastic model updating approach. Initially, the minimum variance approach implemented by Collin et al. [17] and Friswell [18] were used to handle experimental variability caused by random measurement noise. Then, Beck [19] introduced a bayesian methodology by utilising statistical responses data to predict structural parameters. In the scope of

minimising cost function in stochastic model updating, Steenackers [20] shows an interesting technique where an inverse weighting factor is introduced. In addition, implementation of perturbation theory in stochastic model updating demonstrated by Khodaparast et al. [21] is more practical to use compared to Hua et al. [22] where the need for 2nd order sensitivity matrices was needed eliminated. However, the use of perturbation theory in stochastic model updating has a number of limitations, such as the need for small uncertainties, restriction in the initial starting estimate model, and requirement of sensitive data for updating parameters.

Problems relating to the estimation of contact stiffness on bolted joints have been explored recently and can be found in [22]–[24]. However, due to the multiple sources of uncertainties that influenced the contact joint's stiffness, the estimation approach has been cumbersome. As explained in [25], the pre-torque tightening method may introduce variation in expected tension by 30 % and may release the tension to 40% when exposed to vibration. The author also claimed that the bolt tension may vary inversely regardless of the condition of excitation force. Plenty of studies can be found in [26]–[29] regarding the modelling of bolt joint and contact interfaces. Nevertheless, none of the introduced schemes can be guaranteed as the most efficient schemes to estimate the stiffness of the contact interface accurately. Therefore, attention to a scheme to estimate stiffness terms in the contact interface is required. In this article, a new scheme of stochastic model updating is proposed by utilising statistical-based multi-dimensional analysis with stochastic model updating for the identification and quantification of random parameters. The proposed scheme is implemented to a structure consisting of high uncertainty problems such as the joint contact interfaces with an aim to accurately predict the distribution of stochastic dynamic behaviour of the assembled bolted joint structure and to accurately estimate the distribution of contact stiffness of the joints. Methods involved in the proposed scheme are explained next.

METHODOLOGY OF THE PROPOSED SCHEME

In this study, an elliptical shape of assembled bolted-joint structure was used to demonstrate the proposed scheme. The study involves the use of 3 primary methods: experimental method, numerical modelling method, and optimisation method. Besides that, the proposed scheme for identification and quantification of random parameters using stochastic model updating was distinguished into 4 significant steps as described below:

- i. Measurement test data using modal testing.
- ii. Finite element modelling for contact stiffness of the bolted joint.
- iii. Multi-dimensional analysis using lattice sampling.
- iv. Stochastic model updating.

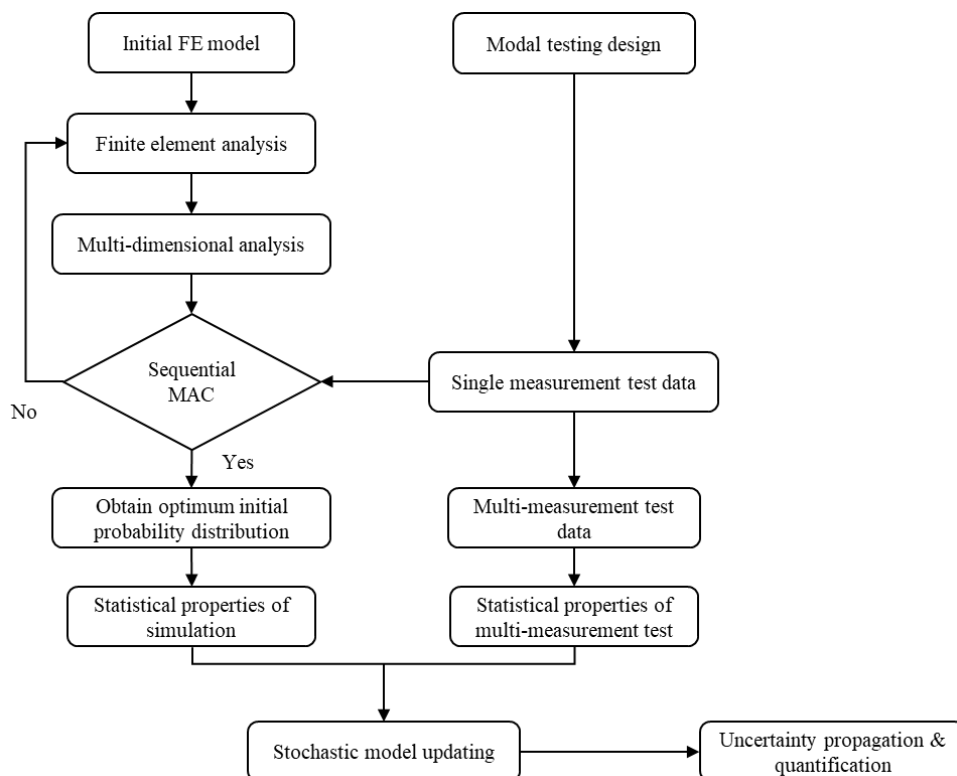


Figure 1. Flowchart of the proposed scheme.

Measurement Test Data using Modal Testing

Modal testing is a method used by structural dynamists to analyse the vibration responses of a test structure. Vibration responses can be analysed either in time, modal or frequency domains. The modal domain is the most preferable in an

aspect to identified dynamic behaviour such as natural frequency, mode shape and modal damping of a structural system [30].

In this study, the assembled bolted-joint structure was tested in the laboratory under free-free boundary conditions. The free-free boundary conditions were used to minimise experimental uncertainties exhibited due to the boundary conditions. Meanwhile, impact hammer and roving accelerometers were used to initiate the relationship between the input and output spectrums. The test setup of the assembled bolted structure is shown in Figure 2. Meanwhile, the frequency response function (FRF) of the single test is illustrated in Figure 3 by implemented input and output relationship as follow

$$\{X(\omega)\} = [H(\omega)]\{F(\omega)\} \tag{1}$$

where $F(\omega)$ is input force exhibited from impact hammer, $X(\omega)$ is output response extracted using accelerometers and $H(\omega)$ is a response model in terms of input-output ratio. The $H(\omega)$ can be expanded as modal model terms and rearranged as:

$$H(\omega) = (-\omega^2 M + i\omega C + K)^{-1} \tag{2}$$

where M , C and K terms is the mass, damping and stiffness matrices of the structural system. From the FRF obtained, damping in the assembled structure can be neglected. For the interest in stochastic responses, the assembled bolted joint structure was disassembled and reassembled to identified variability in the dynamic behaviour. The assembled structure was reassembled 100 times with a frequency of interest was set between 0 Hz to 2000 Hz, which led to 10 mode shapes in the single data set.

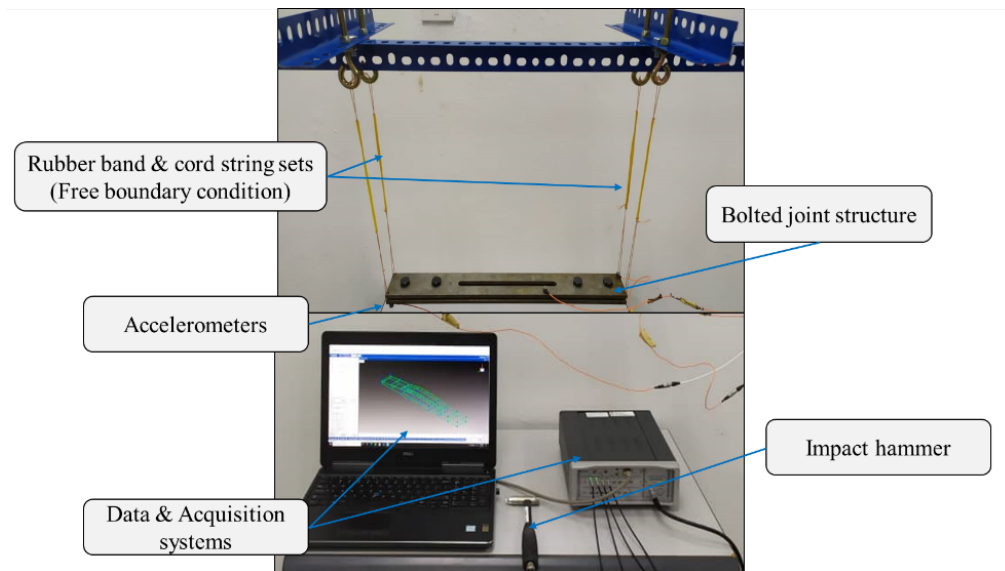


Figure 2. Test setup for single measurement of a bolted joint structure.

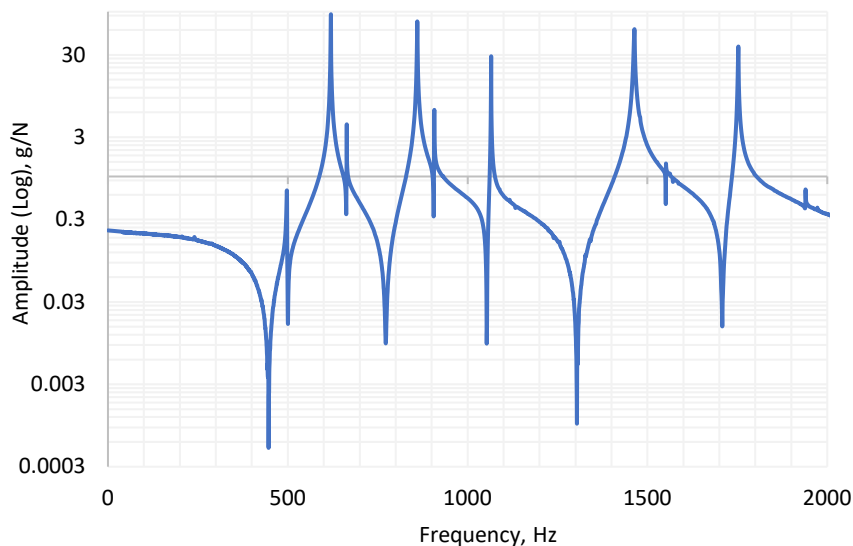


Figure 3. Frequency response function of bolted joint structure.

The bolted joints were tightened under a controlled environment using a torque wrench where a preload was defined for each bolted joint during the reassembly process. Besides, throughout the multi-test series, the test setup, including the excitation and measurement points remained in the same configuration. Controlling the preload on the bolted joint structure and test configurations is important. The source of variability in the structural behaviour can be pointed out to the properties in contact joint interfaces. The variability in the dynamic behaviour of the test structure is shown in Figure 4.

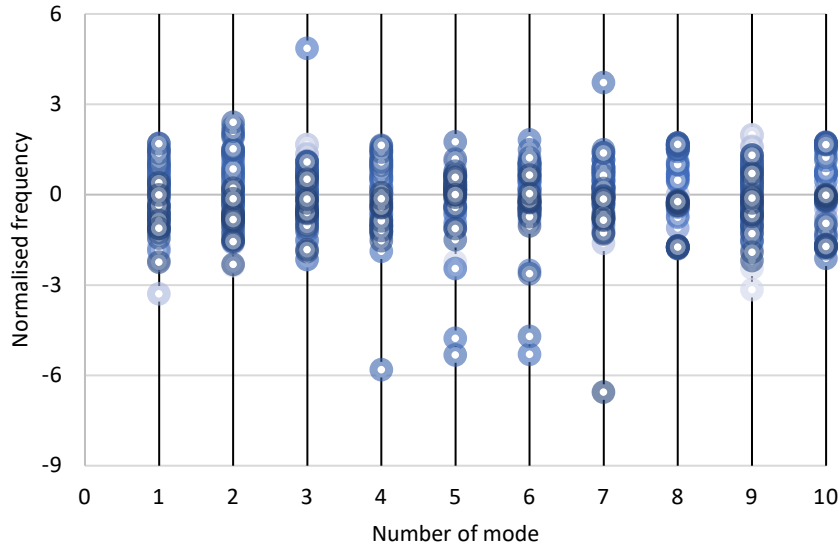


Figure 4. Variability in the dynamic behaviour of bolted joint structure.

Finite Element Modelling for Contact Stiffness of Bolted Joint

The robustness of model updating application is depended on the quality of the initial FE model of structural problems. An appropriate modelling scheme with relevant assumptions is required to make sure certain optimisation processes using the model updating method produce acceptable results with satisfactory accuracy. In this work, the FE model of the bolted joint structure was developed in three steps, which are modelling of (1) structure components, (2) bolt shanks and (3) contact interfaces to make sure all initial conditions were included. Initially, the structural components and bolt shanks were developed as recommended in [31] while their material properties as tabulated in Table 1 were implemented based on the outcome from the study. The study found that initial model updating on the components is required prior to the development of the FE model of the assembled structure to reduce modelling uncertainties. The study also highlights that the CBEAM element is suitable to represent bolt shanks. More studies regarding the modelling of bolt shank using CBEAM element can be found in [32]–[36].

Table 1. Material information for structure components [31].

Parameters	Component A	Component B	Bolt shank
Young’s modulus	215 GPa	214 GPa	200 GPa
Shear modulus	83.72 GPa	81.92 GPa	80 GPa
Density	7937.35 kg/m ³	7904.95 kg/m ³	7900 kg/m ³

In modelling the assembled structure, thin layer elements were proposed to represent as contact joint interfaces. The quality of the behaviour exerted from the contact joint interfaces was depended on a few factors such as structural components properties, material properties of bolt joint, the thickness of the thin layer elements and material properties of the contact interfaces. In this work, the thickness of the thin layer elements was set as 0.18 mm. Meanwhile, due to insufficient information regarding the contact joint interfaces’ parameters, the main problem in this research, initial input parameters were established and tabulated in Table 2. In addition, the thin layer elements were modelled by implemented anisotropic material. The anisotropic material was selected to simulate all the possible stiffness directions in the contact interfaces. The FE model of the structure is presented in Figure 5.

Table 2. Material information of thin layer elements [22].

Parameters	Value
Normal stiffness-z	10 MPa
Tangential stiffness-yz	5 MPa
Tangential stiffness-zx	5 MPa

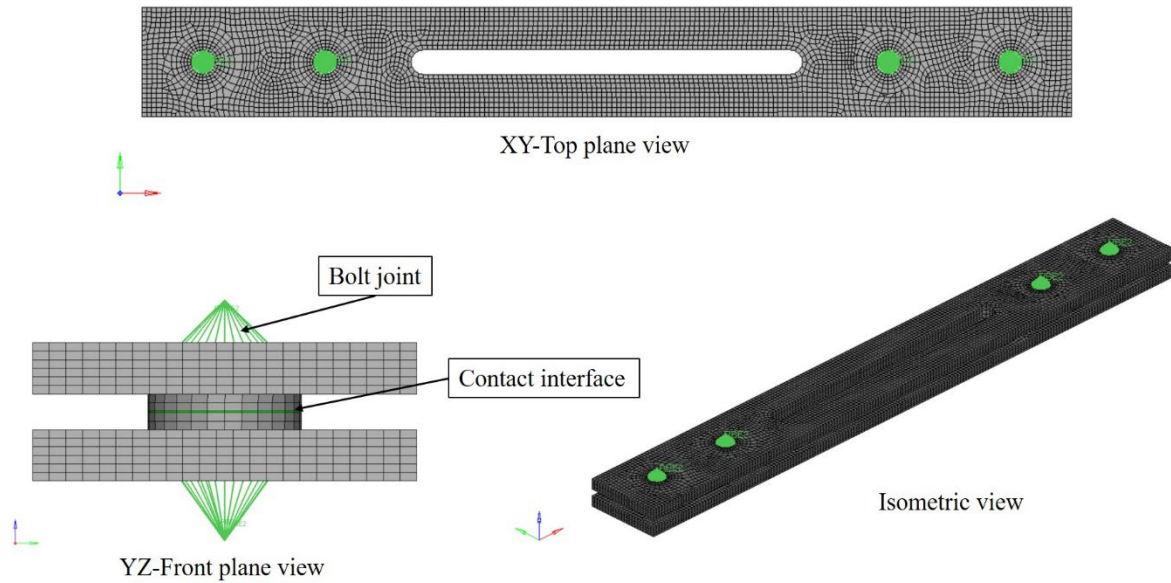


Figure 5. Finite element model of the assembled structure with contact interfaces.

The predicted dynamic behaviour of the assembled structure was obtained by using normal modes analysis. The normal mode analysis can be formulated using the equation of motion of undamped free vibration as:

$$[\mathbf{M}]\ddot{\mathbf{u}}(x, t) + [\mathbf{K}]\mathbf{u}(x, t) = 0 \tag{3}$$

where \mathbf{M} is the mass matrix of FE model and \mathbf{K} is the stiffness matrix of FE model while $\ddot{\mathbf{u}}(x, t)$ and $\mathbf{u}(x, t)$ symbolised acceleration and displacement depending on time. The equation can be rearranged using eigensolution term as:

$$(-(2\pi f_i)^2\mathbf{M} + \mathbf{K})\Psi_i(x) = 0 \tag{4}$$

where f_i and Ψ_i are natural frequencies and mode shapes of the predicted structural system.

Multi-Dimensional Analysis using Lattice Sampling

In order to solve optimisation problems such as ill-posed in sensitivity matrix and convergence difficulty due to local minima function in conventional stochastic model updating method, multi-dimensional analysis using lattice sampling was applied in this study. The multi-dimensional analysis is a statistical approach to analyse predicted data in large dimensions with measurement data counterparts. This approach is essential in identifying the optimum initial distribution of uncertain random parameters using a systematic sampling approach so that the initial starting estimate can be close enough to the true value. The lattice-based quasi-random sampling is a systematic sampling method that provides reliable estimates of output statistics using fewer samples than any other random samplings. The approach is used to spread out sample points equally in dimension space by minimising clumps and empty spaces [37]. The rank-1 lattice points were first introduced by Korobov [38] and used for Monte Carlo simulation. The lattice sampling generates a number of minimum samples with n input variable values for x_n design as:

$$x_n = \frac{11}{10} \left(\frac{(N + 1)(N + 2)}{2} \right) \tag{5}$$

Prior to that, variance analysis was utilised in this study to screen highly significant random parameters where parameters with an F-value ≥ 0.8 were selected as the most significant random parameters. The analysis of variance was implemented using Plackett-Burman (PB) method by evaluating all input parameters in the finite element model. Parameter screening is a prerequisite in uncertainty analysis where the target responses must be sensitive to parameters variations. Usually, conventional experiential judgement and partial derivative-based sensitivity analysis are used in model updating problems [9]. However, variance-based sensitivity analysis was performed in this work due to the robustness in great random problems in which small perturbation requirements can be ignored. The F-value evaluation can be calculated as:

$$F_\theta = F(d_\theta, d_e) = \frac{SS_\theta/d_\theta}{SS_e/d_e} \tag{6}$$

where d_θ and d_e represent the degree of freedom for the parameter θ and the random error e , SS_θ and SS_e Indicated the sums of the square of the parameter θ and the random error e , respectively. After completing with analysis of variance, the lattice samplings were generated using the Gaussian distribution function by omitting low significant random parameters (F-value ≤ 0.79). All the predicted samplings modes were paired with single measurement data using modal assurance criterion (MAC) with threshold value ≥ 0.7 and no cross modes condition. Then, an optimum initial distribution for high significant random parameters was obtained by ranking the predicted modes sampling with measurement data using the MAC correlation coefficient. The MAC formulation can be written as:

$$MAC(e, p) = \frac{|\{\Psi_e\}_n^T \{\Psi_p\}_n|^2}{(\{\Psi_e\}_n^T \{\Psi_e\}_n) (\{\Psi_p\}_n^T \{\Psi_p\}_n)} \tag{7}$$

where φ_e and φ_p denote to the modal vector of experimental and predicted respectively and n represent a number of modes.

Stochastic Model Updating

Subsequently obtaining the optimum initial distribution of highly significant random parameters using multi-dimensional analysis, the lattice algorithm was implemented again to the FE model containing optimum initial distribution for generated optimum statistical properties of the predicted model. After that, the stochastic model updating algorithm can be utilised to estimate the accurate distribution of predicted random parameters by inversely calculating the random parameters using statistical properties of predicted and measured natural frequencies. The stochastic model updating using perturbation approach is formulated as follow

$$\bar{\theta}_{i+1} + \Delta \theta_{i+1} = \bar{\theta}_i + \Delta \theta_i + (\bar{T}_i + \Delta T_i)(\bar{Z}_e + \Delta Z_e - \bar{Z}_i + \Delta Z_i) \tag{8}$$

where θ , T , and Z denotes the parameters vectors, transformation matrix and output responses. On the other hand, $\bar{\theta}$ and Δ represent mean and random terms while subscripts e and i represent measured and predicted data. The transformation matrix used in the stochastic model updating algorithm can be expressed as:

$$\bar{T}_i = (\bar{S}_i^T W_e \bar{S}_i + W_\theta)^{-1} \bar{S}_i^T W_e \tag{9}$$

where S term represents as sensitivity matrix obtained by evaluating mean parameters with measured responses and W is a weighting matrix. The parameter weighting matrix was not required regularisation in this work since the ill-posed problem has been minimised using multi-dimensional analysis. The determination of the mean and covariance matrix of random parameters using the perturbation approach can be characterised as Eq. (10) and Eq. (11) by separating the 0th and 1st order terms of Eq. (8). Finally, the random parameter covariance matrix can be obtained using Eq. (12)

$$\Delta^0: \bar{\theta}_{i+1} = \bar{\theta}_i + \bar{T}_i(\bar{Z}_e - \bar{Z}_i) \tag{10}$$

$$\Delta^1: \Delta \bar{\theta}_{i+1} = \Delta \theta_i + \bar{T}_i(\Delta Z_e - \Delta Z_i) + \Delta T_i(\bar{Z}_e - \bar{Z}_i) \tag{11}$$

$$C_{\theta\theta_{i+1}} = C_{\theta\theta_i} - C_{\theta Z_i} \bar{T}_i^T + \bar{T}_i C_{ee} \bar{T}_i^T - \bar{T}_i C_{Z\theta} + \bar{T}_i C_{ii} \bar{T}_i^T \tag{12}$$

In this work, Latin Hypercube Sampling (LHS) is utilised to obtain statistical properties such as mean and standard deviation of the predicted dynamic behaviour of the assembled structure. There are 1000 samples were generated using LHS. The appropriate number of samples required for predicting the stochastic dynamic behaviour of the assembled structure was obtained from the sampling convergence study. Figure 6 shows the graph of the convergence study based on the standard deviation of natural frequencies. From the figure, it has been found that the predicted natural frequencies of the assembled structure are stable after achieving 1000 samples and above. Therefore, for reducing computational difficulties, 1000 samples were selected compared to 2000 samples or more.

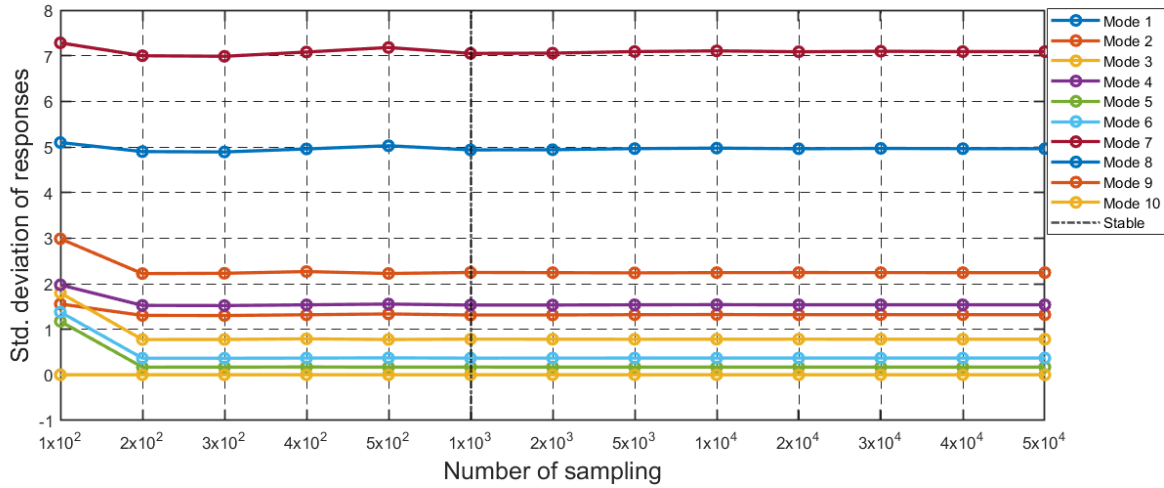


Figure 6. Convergence graph of the number of sampling.

RESULTS AND DISCUSSION

The main aim of this study was to identify and quantify highly random parameters of assembled bolted structures using stochastic model updating. There are four steps involved in this work, as explained before. Initially, the experimental data obtained were compared with the initial predicted data using nominal input parameters. The MAC values between the experiment model and the initial FE model were then calculated to identify the correlation level between both models. Subsequently, multi-dimensional analysis was conducted with two targets: to identify highly random parameters using analysis of variance and to identify an optimum starting distribution of the highly random parameters using lattice-based sampling. Lastly, stochastic model updating using perturbation theory was implemented to estimate the distribution of random parameters accurately. In the all mentioned steps, experimental data obtained from modal testing were used as a benchmark to evaluate the predicted data's accuracy. The results from the steps involved were discussed in the following subsections.

Multi-Dimensional Analysis using Lattice Sampling

The result of the dynamic behaviour obtained from the single experimental process, initial prediction, optimum prediction and distributions are tabulated in Table 3, Figure 7(a) and Figure 7(b). Table 3 describes the correlation in the natural frequencies, while Figure 6 describes the correlation of mode shapes using the modal assurance criterion (MAC) approach. In Table 3, the natural frequencies of the experimental and initial FE model recorded poor correlation with a total error of the first 10 modes was 103.4 %. The most significant discrepancies can be seen in the 1st mode, where 30.6 % was recorded. On the other hand, in order to confirm the accuracy of the predicted model, mode shapes of the initial prediction were compared with the experimental counterpart. In Figure 7(a), it can be seen that the initial prediction model experienced cross modes problems mainly on the set of 2nd – 3rd modes and the set of 8th – 9th modes of the initial model. Moreover, the obtained mode shapes' quality was in doubt as the overall MAC values of the initial prediction model were below 80 % except for the 4th mode, where the MAC value recorded was 90 %. At the same time, the poorest correlation of mode shape can be found on the 10th mode, where the MAC value recorded was 30 %.

Table 3. Natural frequencies of the initial model and optimum model compared with experimental counterpart.

Frequencies order (th)	Experimental (Hz)	Initial model (Hz)	Error %	Optimum model (Hz)	Error %
1	496.8	344.6	30.6	527.5	6.2
2	619.4	604.4	2.4	630.6	1.8
3	661.3	577.8	12.6	673.1	1.8
4	860.3	854.2	4.7	870.4	1.2
5	904.8	857.4	5.2	910.3	0.6
6	1063.9	1061.7	5.8	1066.7	0.3
7	1462.7	1302.9	10.9	1524.9	4.3
8	1552.3	1597.8	2.9	1579.4	1.7
9	1747.0	1532.1	12.3	1779.7	1.9
10	1939.8	1672.7	13.8	1979.9	2.1
			103.4		21.8

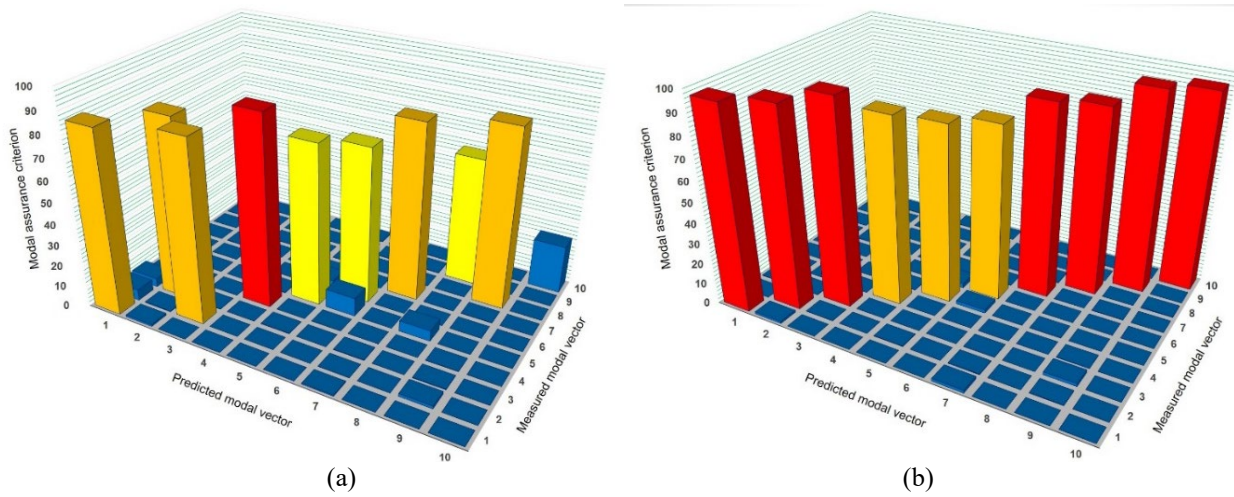
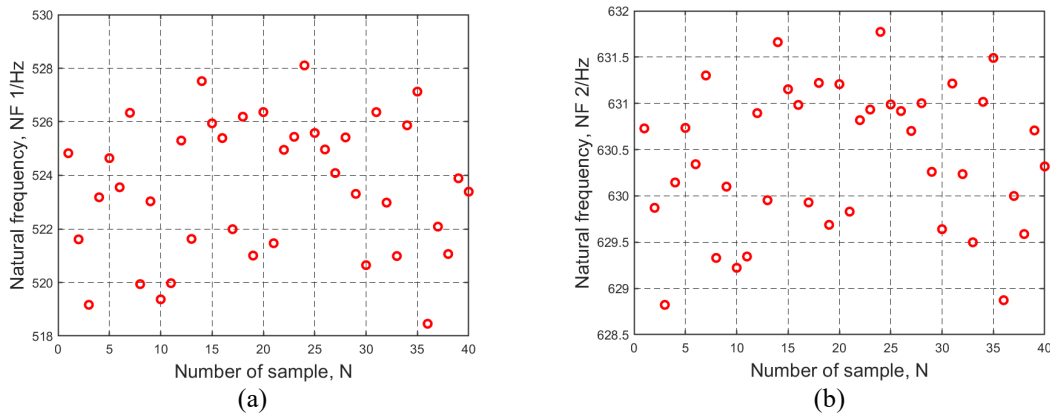


Figure 7. MAC value of (a) initial and (b) optimum FE model.

From the result, it can be seen that the initial model of the assembled structure was not suitable to be used in stochastic model updating, especially to identify and quantify the uncertain random parameters of the bolted joint structure. This is because, in stochastic model updating, the initial model used must be in minimum errors to avoid high uncertainties so that only minor changes are encountered [21]. However, in modelling a bolted joint structure consists of contact interfaces. To estimate the initial starting distribution of input parameters that are close to the actual value are problematic and cumbersome in which only high errors are exhibited. This is because initial assumptions of structural parameters, especially on the contact joint interfaces, are often unavailable due to insufficient information of the structural system [39], [40]. Furthermore, the variations in the surface roughness applied pressure, and contact dimensions often influence the overall stiffness of the contact interfaces. Therefore, identification of optimum initial starting distribution of random parameters is highly essential.

Table 3 and Figure 7(b) show the optimum model's result using the optimum initial starting distribution of random parameters while Figure 8(a) to Figure 8(d) show the distribution of mode 1, mode 2, mode 3 and mode 4 respectively using multi-dimensional analysis. In Table 3, it can be seen that the prediction of natural frequencies of assembled bolted joint structure was improved where the total error recorded was 21.8 %. Satisfactory achievement can also be found on the predicted mode shapes of the optimum model where all the MAC values were above 80 % with no cross modes were detected.

Furthermore, results from Figure 8(a) to Figure 8(c) shows that the MELS sampling scheme in multi-dimensional analysis managed to distribute equally the sampled predicted natural frequencies within the spaces. Meanwhile, Figure 9 shows the F-values of all input parameters from the initial model based on analysis of variance. The satisfactory results here were obtained by identified the highly uncertain parameter as illustrated in Figure 9. The F-value of Normal stiffness-Z, Tangential stiffness-yz and -xz were 0.8 and above. Therefore, these parameters were considered as highly random and the initial distribution of the parameters required to be tuned using lattice sampling coupled and evaluated using MAC correlation coefficient. The obtained optimum starting values are shown in Table 4. The result obtained here shows the capabilities of multi-dimensional analysis using systematic lattice sampling in obtaining the optimum initial distribution of random parameters at a satisfactory level of accuracy.



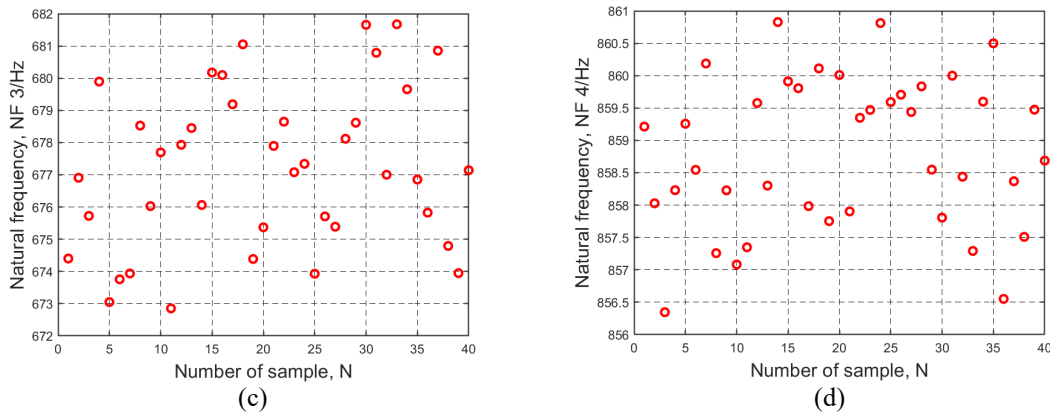


Figure 8. Distribution on (a) mode 1, (b) mode 2, (c) mode 3 and (d) mode 4 using multi-dimensional analysis.

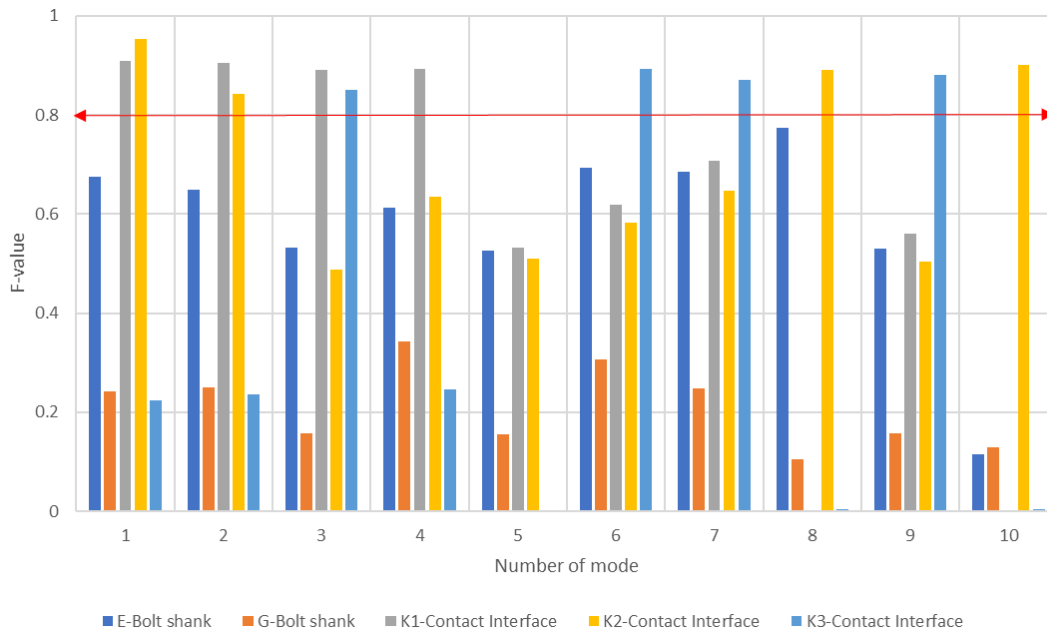


Figure 9. F-value for initial random parameters based on analysis of variance.

Table 4. Initial starting distribution and optimum starting distribution of contact interfaces.

Parameters	Initial distribution		Optimum distribution	
	Mean (MPa)	Std. deviation (MPa)	Mean (MPa)	Std. deviation (MPa)
Normal stiffness-z, K1	10	10×10^{-2}	800	800×10^{-2}
Tangential stiffness-yz, K2	10	10×10^{-2}	642	642×10^{-2}
Tangential stiffness-zx, K2	10	10×10^{-2}	716	716×10^{-2}

Stochastic Model Updating

After successfully acquiring the optimum initial distribution of random parameters, stochastic model updating was utilised to obtain the random parameters' accurate distribution. Initially, statistical properties of the optimum model were developed using systematic lattice sampling by sampling the random parameters using optimum initial distribution. In this work, the Gaussian distribution function was used to obtain the statistical properties. Meanwhile, the statistical properties of experimental data were obtained by assembling and reassembling the bolted joint. The variability of natural frequencies for the experimental and optimum model was recorded in Table 5. The result shows that the variability in natural frequencies of both models was not well correlated with total mean errors of 31.4 %. This is because the establishment of optimum initial distribution in random parameters alone was inadequate in producing an accurate prediction, especially for quantifying random parameters. Implementation of stochastic model updating on the stochastic-optimum model reduced the total mean errors significantly from 31.4 % to 12.5 %. Meanwhile, from the updated model, it can be found that the estimated distribution of random parameters was changed as tabulated in Table 6. These results show stochastic model updating capabilities in quantifying random parameters, especially for the assembled bolted structure focusing on contact interfaces problem.

Table 5. Mean natural frequencies of optimum model and updated model compared with experimental counterpart.

Frequencies order ^(th)	Mean experimental (Hz)	Mean optimum model (Hz)	Error %	Mean updated model (Hz)	Error %
1	489.9	526.6	7.5	498.7	1.8
2	617.4	629.1	1.9	613.1	0.7
3	647.0	672.1	3.9	659.8	2.0
4	861.2	872.7	1.3	876.7	1.8
5	900.8	910.3	1.1	908.4	0.8
6	1062.4	1089.2	2.5	1068.1	0.5
7	1435.6	1520.7	5.9	1458.8	1.6
8	1554.2	1577.1	1.5	1539.9	0.9
9	1711.6	1779.7	4.0	1744.7	1.9
10	1941.9	1978.0	1.9	1950.1	0.4
			31.4		12.5

Table 6. Updated distribution of contact interfaces.

Parameters	Optimum distribution		Updated distribution	
	Mean (MPa)	Std. deviation (MPa)	Mean (MPa)	Std. deviation (MPa)
Normal stiffness-z	800	800×10^{-2}	856.01	1235×10^{-2}
Tangential stiffness-yz	642	642×10^{-2}	567.93	983×10^{-2}
Tangential stiffness-zx	716	716×10^{-2}	771.47	1542×10^{-2}

CONCLUSION

A new scheme involving stochastic model updating to estimated random parameters in assembled structure is presented. Problems relating to significant errors and cross modes in the initial prediction due to the insufficient information on the structure system has been solved. The scheme is based on a combination of multi-dimensional approach and perturbation based stochastic model updating method. Implementing a multi-dimensional approach with the aid of analysis of variance and lattice sampling in minimising the large errors and cross modes in initial prediction managed to produce a significant achievement in estimating the distribution of stiffness terms in contact interfaces. The depreciation in the predicted mean of the assembled structure from 31.4 % to 12.5 % is reasonable since ten sets of responses are involved in the model updating solution.

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