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RESEARCH ARTICLE

Optimal Control of an Active Suspension System Applying Distributed Parameter Simulation Test

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ABSTRACT - Automotive is one of the most important industries in the world, which lead to a need for continuous improvement of vehicles and their internal systems. Suspension systems have been improved for better vehicle performance and passenger comfort, keeping the tire in contact with the road surface. Active suspensions require optimal control to modulate the flow of energy and generate the control force by implementing active actuators able to provide negative damping and a wider range of forces and velocities. This article aims the design of an active suspension system based on LQG and LQR controller evaluating its performance in a distributed parameters simulation using COMSOL Multiphysics® and MATLAB®. The quarter car model is proposed and is linearized to design the optimal control (LQG and LQR) respectively. An early mathematical simulation is developed in MATLAB (R) software to verify and compare the open and closed loop results. Finally, the full system model is implemented in COMSOL Multiphysics (R) software considering rigid materials and the controller to analyze the distributed parameters simulation results.

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1.0 NOMENCLATURE

FEM	Finite Element Method
DPS	Distributed Parameter System
MBDS	Multi Body Dynamic System
LQR	Linear Quadratic Regulator
LQG	Linear Quadratic Gaussian
DOF	Degree of Freedom
PID	Proportional Integrative Derivative

2.0 INTRODUCTION

For the past years, the automotive industry has grown due to the impact of vehicles in people's daily life, creating the necessity for car systems improvement. One of the most important vehicle systems is the suspension system which usually is composed of a damper and a spring, providing comfort for passengers and the same sense, improving road-handling performance. Also, the suspension system plays an important role in supporting the vehicle weight, offering effective isolation from road excitations and keeping the tire in contact with the road surface [1].

A typical classification of the suspension systems can be: passive, semi-active and active, which present some nonlinear characteristics that depend on the car type [2]. Especially conventional passive suspension systems have reached the limits of their performance, suggesting the need for improvement with active actuators able to vary the damper characteristics along with the road profile. Then, an active shock absorber could provide negative damping and a wider range of forces at low velocities increasing the system's performance [3]. On the other hand, active suspensions can continually supply and modulate the flow of energy and generate the control force. This aspect allows to increase passenger comfort and vehicle performance [4]. The difference between passive and active quarter car suspension can be observed in Figure 1.

In addition, active suspension controllers have been extensively accepted and demonstrated more effectiveness at improving suspension performance in comparison with conventional passive and semi-active suspension systems [1]. The literature review describes linear and nonlinear techniques to control vehicles with active suspension systems. Some cases of practical cars have been reported [1-3]. Firstly, some approaches consisted of linear control strategies based on linear physical car models consisting of lumped masses, linear springs and dampers, and an active shock absorber modelled as an ideal force source. However, real car dynamics and active shock absorbers have more complex nonlinear dynamic behavior. Consequently, active suspension system controllers should be designed and tested in a more realistic simulated environment[3]. Besides, the major studies are based on linear models using adaptive control, the linear quadratic regulator (LQG) and H∞ [4-6]. Despite the springs and dampers of the suspension system's nonlinearities, they are usually modeled as linear elements [2].

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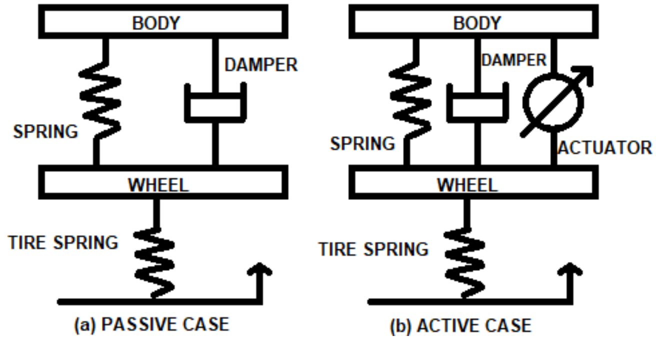


Figure 1. Suspension systems in quarter car model [2]

Another point is that several systems from science and engineering are Distributed Parameter Systems (DPS), which are modeled by sets of partial differential equations, boundary conditions and initial conditions, which describe the evolution of the state variables in several independent coordinates. Most distributed parameter models are derived from the first principles allowing a model structure to be defined. In the same sense, degrees of freedom are usually left for model parameterization, and unknown parameters must be estimated from experimental data. Therefore, experimental design, sensor configuration and error calculations are important issues which must be considered in order to ensure parameter identifiability. Once a distributed parameter model has been obtained, a system simulator can be implemented. Due to the complexity and nonlinearity of the model equations, an analytical solution is not achievable, and it is necessary to resort to a numerical procedure. A wide range of numerical algorithms are available, either for spatial approximation or one of them is Finite Element Analysis (FEA).

For control implementation purposes, it is required to select input and output variables and to define the associated equations. So, process disturbances are assumed to be known or to be modeled by additional equations. Most partial differential algebraic equations models are given in a state space representation which is the basis for system analysis and control model reduction techniques. In order that, simplify assumptions regarding the problem physics, dimensionality and geometry, as well as several techniques that include parameter sensitivity analysis and singular perturbations, which turn out to be a good alternative to derive a model suitable for model-based control.

Particularly, with the aim to achieve and organize procedures to the work, the Design Science Research methodology was selected [7]. The steps of the methodology are evidenced in Figure 2. Also, this methodology looks for the development of an artifact. This artifact could be an algorithm, a model, and it is possibly an actual prototype. As a result, for our study case, the main artifact is the distributed parameter model controlled by linear control techniques.

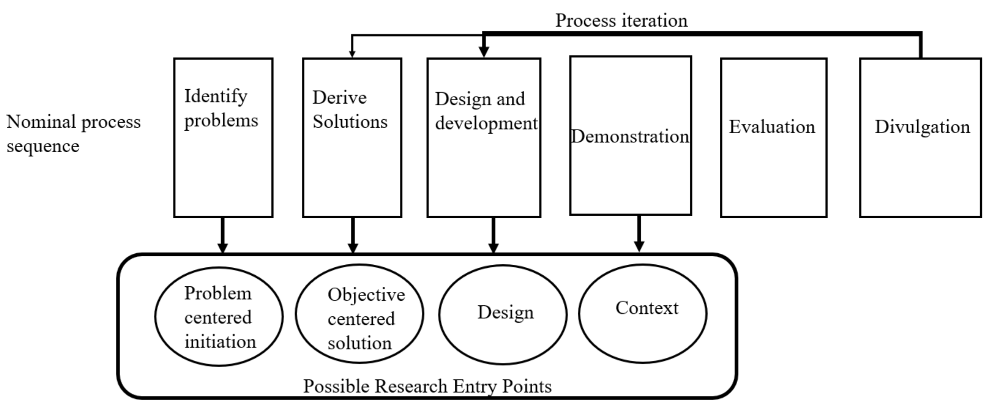


Figure 2. DSR (Design Science Research) methodology for artifacts development [7]

3.0 LITERATURE REVIEW

In the beginning, the framework has been proposed as a basis to check and review a series of scientific documents related to the subject in question in order to establish relevant authors that can provide accurate information, concepts and different innovative control methods for active and semi-active suspension systems were reviewed.

A fault-tolerant control based on a neural network to accommodate oil leaks in a magnetorheological suspension system based on a dynamic half-car model attracts attention [8]. This model consists of the body of the vehicle (spring mass) connected by the MR suspension system to two side wheels (mass not suspended). The semi-active suspension system is a four-state nonlinear model; can be written as a representation of the state space. So, the modeling and design of two control strategies for the semi-active suspension system were checked, allowing to establish comparison metrics between the techniques used and develop two laws of control, classic PID and Fuzzy Logic control law, with a simulation of the stability and performance properties of our controllers in various scenarios using analysis and simulation simultaneously. System performance is determined by computer simulation in MATLAB/Simulink [5].

Besides, an active suspension control approach combines a filtered feedback control scheme and an “input decoupling transformation” for a complete vehicle suspension system. In order to that, suspended mass movements (i.e. car body) above and below wheel frequency modes are mitigated by using active filtering of damping and spring coefficients through internal control loops (suspension controller) plus skyhook damping of lifting, pitching and rolling speeds through external control loops (attitude controller). Internal suspension control loops and external attitude loops are combined with the input decoupling transformation [6]. This technique conducted research on the coordinated control scheme of the anti-lock braking system (ABS) and the active suspension of the vehicle. The goal is to obtain maximum braking force on the road and minimize braking distance and, in the meantime, maintain the vehicle’s directional stability and maintain driving comfort. The controller was designed using fuzzy model control theory and was implemented in the MATLAB/Simulink software environment [10].

A four-degree linear model of freedom is used to represent a vehicle with different front and rear characteristics. Therefore, filtered road and acceleration inputs are applied to the model to simulate real-life use. Performance criteria are filtered to include frequency sensitivity and weighted according to a standard passive suspension system. The front and rear independent controllers are optimized with the genetic algorithm. The controller includes linear gains and frequency dependency to take advantage of these two different control methods [11]. On the other hand, an adaptive variable structure model reference controller (VS-MRAC) for active control of vehicle suspension was studied to consider a one-quarter DOF car model. The reference model is a vibratory system of a DOF with a skyhook shock absorber. The structure of the switching functions is designed according to the requirements of global exponential stability and shows perfect model tracking in finite time [12].

4.0 QUARTER CAR MODEL

The quarter car model is highly used in literature because it is a simple and useful system that implements a spring-damper configuration. This paper works with two different models of the quarter car suspension, a linear model with concentrated parameters and a nonlinear model with distributed parameters. The linear model developed has two masses, two springs, one viscous damper and an applied force between the two masses. This model neglects the coulomb friction, the vibration of the materials and the nonlinear displacements [13]. The model selected for this work is presented in Figure 3.

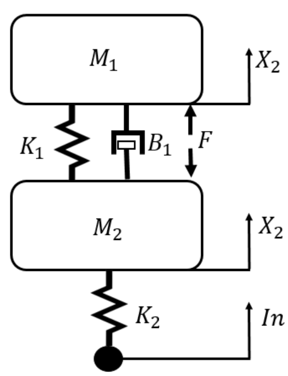


Figure 3. Sketch of the linear model of a quarter-car suspension

The two equations of the behavior were obtained using the second Newton´s law, Hooke’s law and the damping definition.

$$\boldsymbol{m_1}\ddot{x_1} + b_1(\dot{x_1} - \dot{x_2}) + k_1(x_1 - x_2) + \mathrm{F} = 0 \tag{1}$$

$$\boldsymbol{m_2}\ddot{x_2} + b_1(\dot{x_2} - \dot{x_1}) + k_1(x_2 - x_1) + k_2(x_2 - \mathrm{In}) - \mathrm{F} = 0 \tag{2}$$

where $\ddot{x}_2$, $\ddot{x}_1$, $\dot{x}_2$, $\dot{x}_1$, x_2, x_1 are the acceleration, velocity and position of the two masses.

The finite element method (FEM) was selected as solution to obtain the nonlinear modelling. This is because the FEM generates nonlinear behaviours inferred by the geometry and material properties. Consequently, these features could be extracted mathematically in nonlinear differential equations but, at the same time, requires parameters hard to estimate analytically; however, it is possible to obtain automatically and compute by the FEM model.

The simulation software selected was COMSOL Multiphysics; this software allows multi-body dynamics simulations (MBDS) with material deformation and stress calculations [12][13]. Due to FEM simulations increasing significantly the computational cost, some model simplifications were required. A simplified geometry of a conventional quarter car suspension is presented in Figure 4.

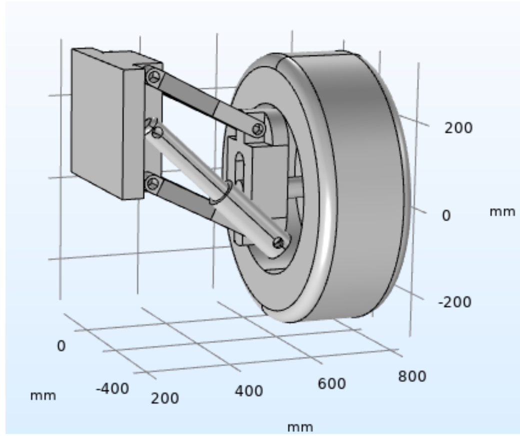


Figure 4. Simplified 3D geometry used for distributed parameters model

5.0 OPTIMAL CONTROLLER DESIGN

5.1 Open Loop Response

Based on the model section, the numeric representations of the linear state space matrixes are shown from Eqs. (3) to (6).

$$A = \begin{bmatrix} -3.5 & 3.5 & -30.9 & 30.9 \\ 18.1 & -18.1 & 158.5 & -2876.3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ -3.53 & 3.53 & -30.89 & 30.89 \end{bmatrix} \tag{5}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

The parameters that were used for this system are shown in Table 1:

Table 1. Model parameters

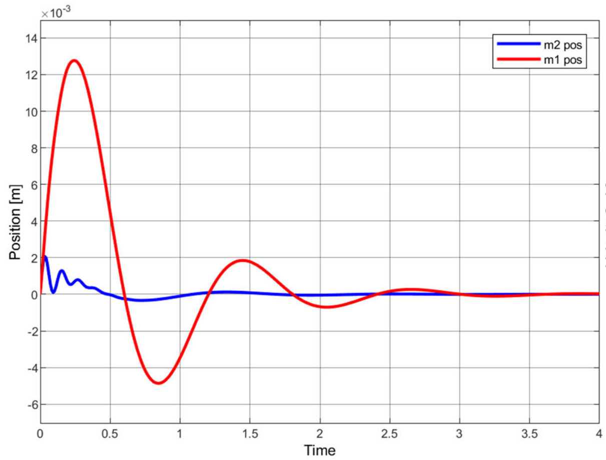
Parameter	Description	Value	Units
m1	Mass of body	226.55	Kg
m2	Mass of wheel	44.154	Kg
$\beta 1$	Viscous coefficient of damper	800	N*m*s
k2	Spring coefficient	120000	N/m
k1	Spring coefficient	7000	N/m

Following this, the linear numeric state space model presents the following eigenvalues shown in Table II:

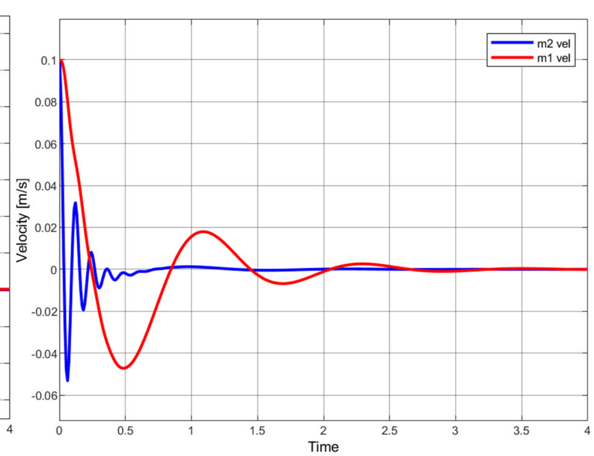
Table 2. Model Eigenvalues

Parameter	Value
s1	-1.6052+5.2173i
s2	-1.6052-5.2173i
s3	-9.2196+52.28i
s3	-9.2196-52.28i

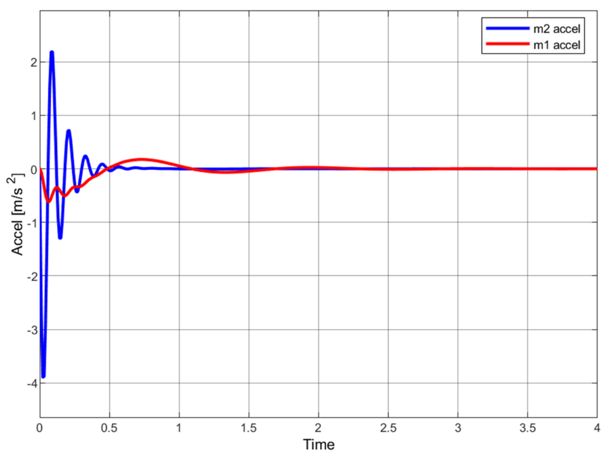
These values show an unstable system because of its positive poles. The linear model was implemented in Simulink® Software to obtain the open loop response, which is shown in Figures 5(a) to 5(c).



(a) Position vs Time



(b) Velocity vs Time



(c) Acceleration vs Time

Figure 5. Linear models

The procedure of the controllers design (LQR and LQG) based on the previous model is presented in this section. For the LQR development, it must be found the best input u(t), that allows to keep the system in the operating point in a specific time frame. The block diagram of the LQR controlled is shown in Figure 6.

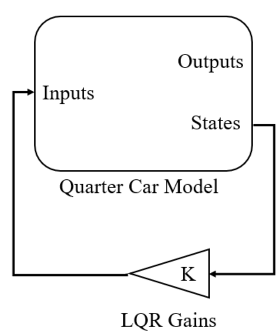


Figure 6. LQR block diagram [13]

This is equivalent to solve Eq. (7), with Eqs. (8) and (9) as restrictions:

$$J = \frac{1}{2}x(t_f)^T P_f x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}(x(t)^T Q x(t) + u(t)^T R u(t))dt \tag{7}$$

$$\dot{x} = Ax(t) + Bu(t) \tag{8}$$

$$u(t) = -K(t)x(t) \tag{9}$$

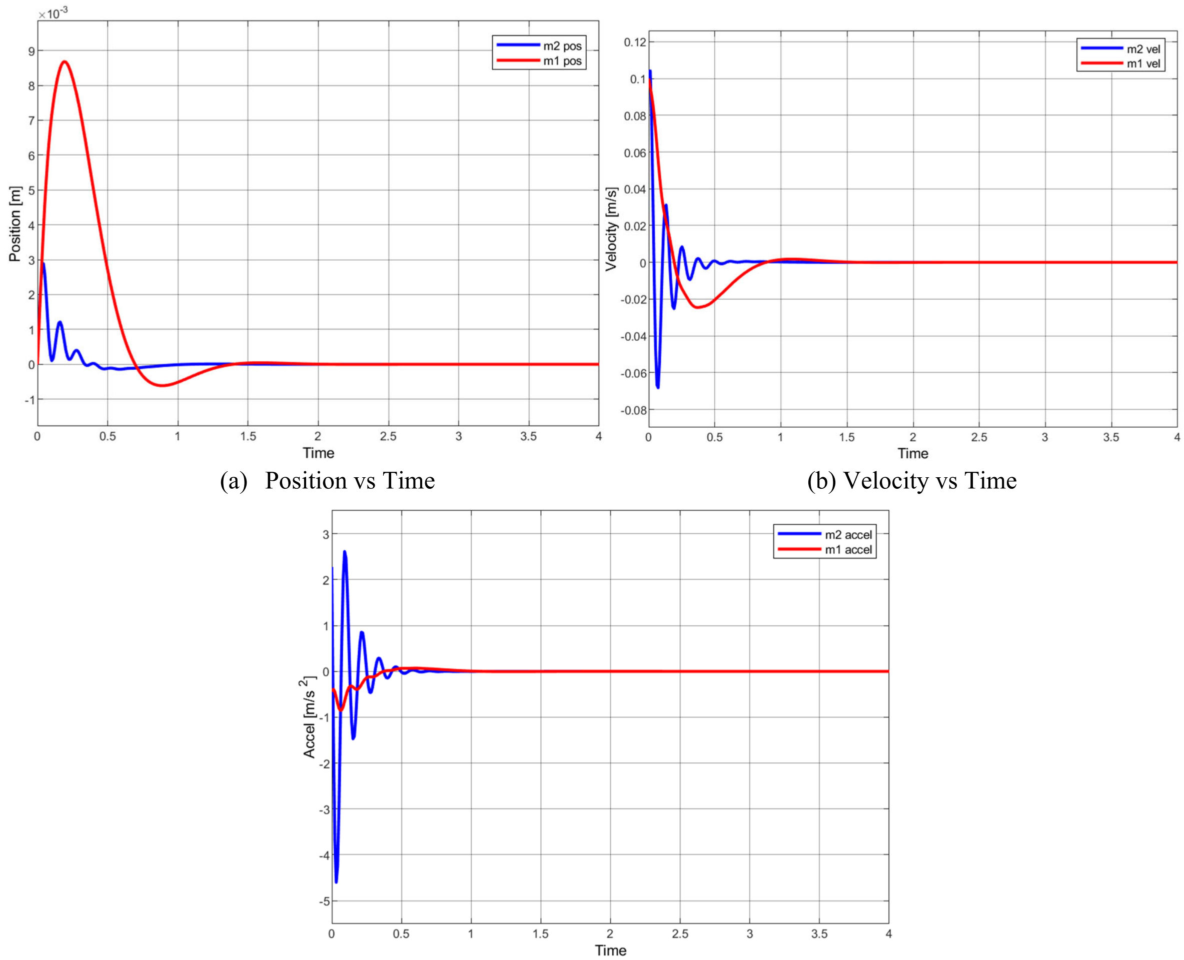
Q and R matrices are the weights of the cost function and are both defined as positive and are usually diagonal. Q matrix is the importance of the states and R matrix is the importance of the inputs. To solve this problem, it's developed Riccati's matrix equation (equation (10)) with a Q and R matrices defined in equations (11) and (12): [8]

$$\dot{P} = -PA - A^T P + PBR^{-1}B^T P - Q \tag{10}$$

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \tag{11}$$

$$R = 0.001 \tag{12}$$

with this, Simulink® software mathematical simulation is implemented with the closed loop to obtain the response of the LQR controller applying the states gains this could evidences in Figure 7(a) to 7(c).



(a) Position vs Time (b) Velocity vs Time

(c) Acceleration vs Time

Figure 7. LQR controller

On the other hand, the LQG controller is designed by implementing a Kalman filter to generate an observer, which is a suitable choice as a state estimator technique for the suspension system. The block diagram that evidence of how the estimator is added is presented in Figure 8.

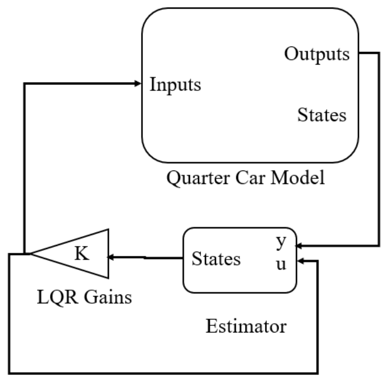


Figure 8. LQG block diagram [13]

The equation for the state estimator is in the following:

$$\dot{x}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t) - Du(t)) \tag{13}$$

where the L matrix is obtained using a matlab toolbox with their respective noise matrices. To improve the performance of the LQG controller, a different choice of weights for the cost function is selected, having the following Q and R matrixes:

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix} \tag{14}$$

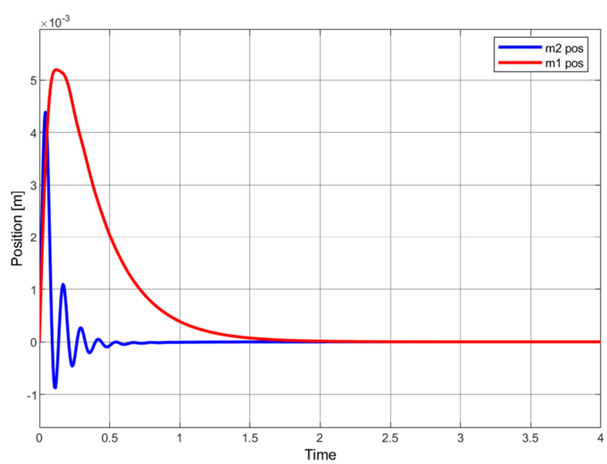
$$R = 0.001 \tag{15}$$

Also, the following noise levels are assumed for the system:

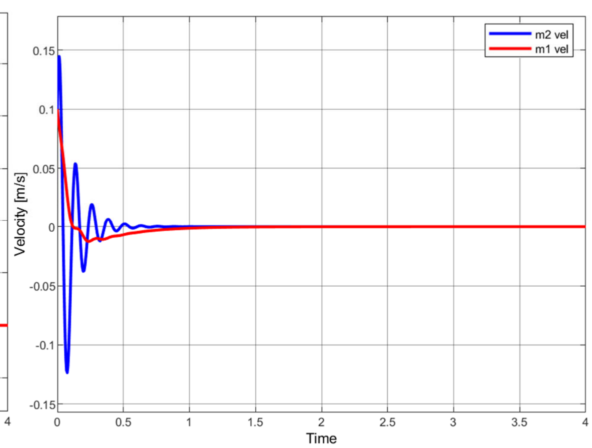
$$Actuator\ Noise = \ 0.001^2 \tag{16}$$

$$Sensor\ Noise = \begin{bmatrix} 0.001^2 & 0 & 0 \\ 0 & 0.001^2 & 0 \\ 0 & 0 & 0.001^2 \end{bmatrix} \tag{17}$$

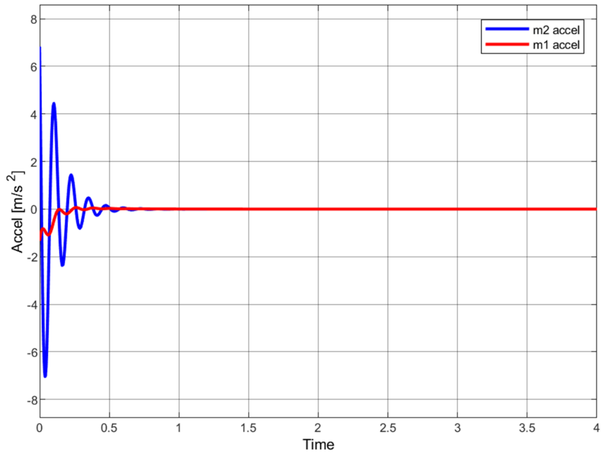
After obtaining the matrices of the controller, a Simulink simulation was made to obtain the closed-loop response for the LQG controller. And the results were observed in Figures 9(a) to 9(c).



(a) Position vs Time



(b) Velocity vs Time



(c) Acceleration vs Time

Figure 9. LQG controller

6.0 DISTRIBUTED PARAMETERS SIMULATION

The simulation of distributed parameters was developed following the following steps:

i. Define global variables
ii. Define Boundary conditions
iii. Mesh the domain
iv. Configure solver
v. Export and evaluate results

The variables needed in a distributed parameters simulation are those referred to as the material, gravity and behavior of the dampers. The variables used were the parameters of the structural steel and the variables previously mentioned. The boundary conditions specify the behavior of some parts of the model. These conditions used in the model are specified in the next table.

Table 3. Boundary conditions

Boundary Condition	Description
Hinge joint	Allows rotations only in one axis.
Prismatic joint	Allows displacement belong on axis.
Boundary load	Specify the force in a surface.
Damper	Specify spring and damping ubication.

The domain was meshed using tetrahedral elements and using Delaunay method obtaining the following mesh according to Figure 10. The solver is the core of the finite elements software's; this makes necessary that the configuration has to be the most suitable for the problem evaluated. The configuration used for the simulation is shown in Table 4.

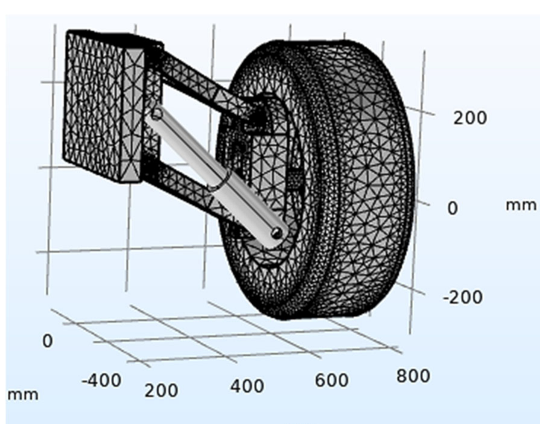
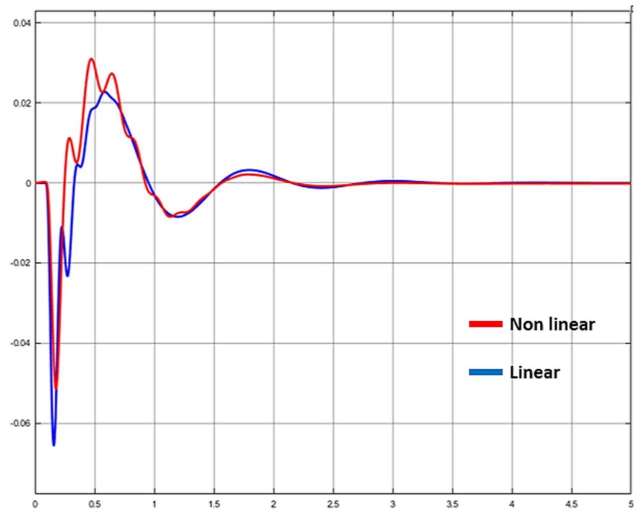


Figure 10. Domain meshed to be computed

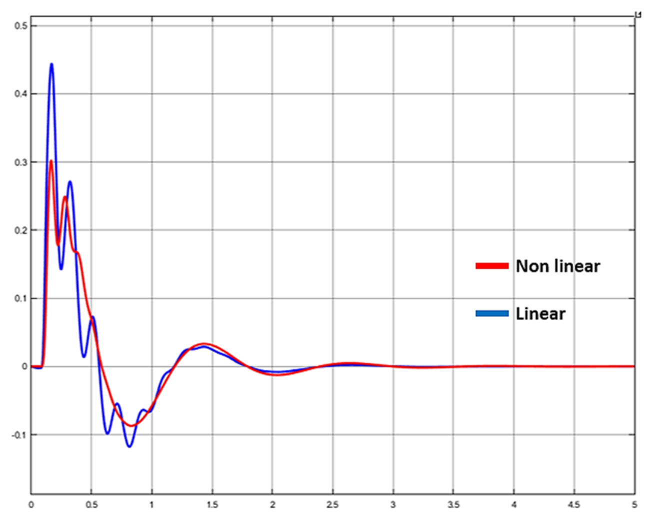
Table 4. Solver configuration

Description	Value
Solver	PARDISO
Relative tolerance	0,001
Absolute tolerance	0,001
Time-step	0,002 [seg]

The results obtained from the simulations were exported in a text file and compared with the simulation of the linear system. This comparison allows to evaluate the accuracy of the modeling and verify the concentrated parameters. According to Figures 11(a) to 11(c) is observed that the nonlinear model present peaks in the transient response but have a similar behaviour.



(a)



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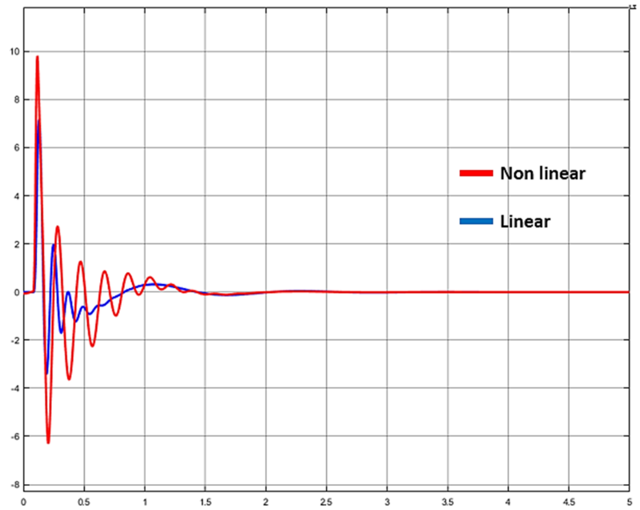
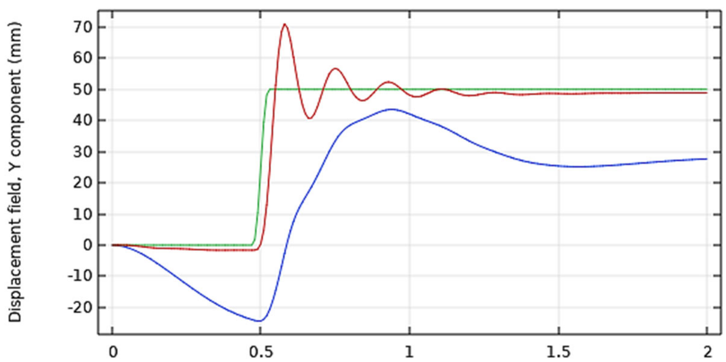


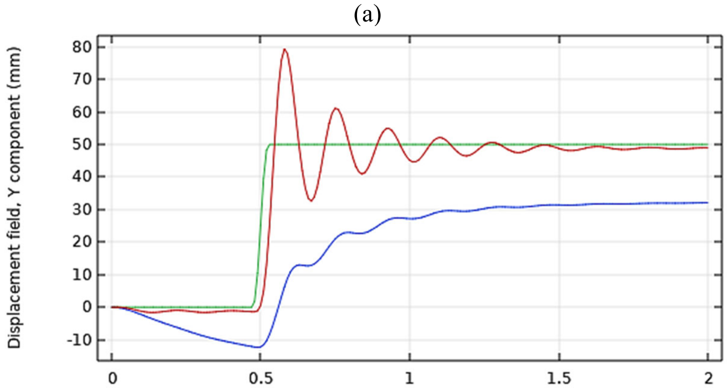
(c)

Figure 11. Suspended (a) mass position, (b) mass velocity and (c) mass acceleration obtained in two simulations

Thus, with the purpose of establishing the comparison of the two models and verifying the parameters, the controllers were implemented based on the distributed parameters simulation with external disturbance instead of the inner initial values. The controllers were applied using a boundary condition as an actuator and using the expanded equation of the state space model to implement the LQG. The obtained results are presented in Figure 12(a) to 12(c), where the blue line represents the suspended mass position, the red one represents the not suspended mass position, and the green is the floor elevation.



(a)



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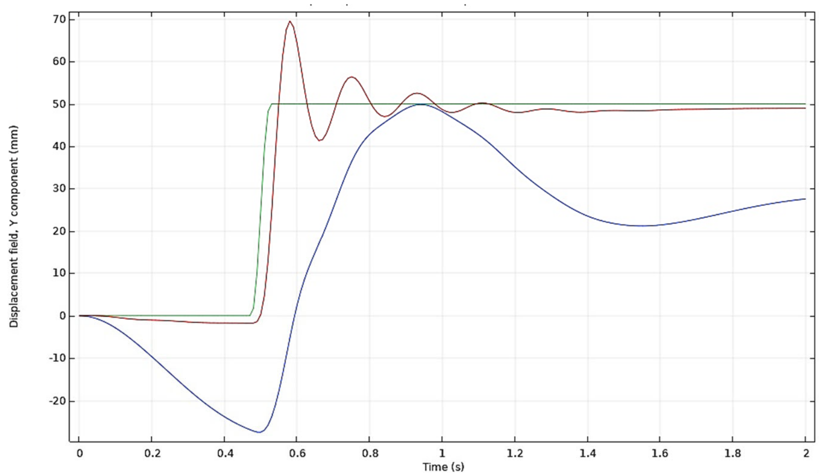

(c)

Figure 12. (a) LQG response, (b) LQR response and (c) open loop response in the distributed parameters simulation

7.0 CONCLUSIONS

The main contribution of this work is the implementation of a finite element model (FEM). This implementation of a FEM simulation brings the following advantages that can be explored:

i. The higher number degrees of freedom in FEM simulations imply evidence behaviors of non-modelled in the control design for the FEM model, which required 546.000 DOF, meanwhile the conventional quarter car model only model 2 DOF.
ii. Calculation of stress in dynamic controlled behavior implies taking into account counter forces discontinuities and the forces exerted by a fast change in the road.
iii. Due to COMSOL, a multiphysics environment allows to model and implement the electromagnetic or non-constant viscous dampers including the dynamic of multiple actuators.
iv. Parametrization of a real suspension obtaining stiffness constants and viscous coefficients reduces the risk over a real implementation.

The control tuning and linear modelling also bring remarkable facts as the differences in the response of the suspended and the non-suspended mass. In the modeling section, it was evidenced that the eigen-frequency for m_2 is ten times the eigen-frequency of m_1. Perhaps the interest variable for comfort design is the response of m_1 the differences in the frequency produce that for implementation in real applications or discrete control design, the sample time is ruled by m_2 characteristics. This is applicable in most suspension systems because the frequencies are defined by the stiffness of the degree of freedom, and a non-suspended one generally has higher stiffness because it depends on the wheel in comparison to the suspension that is defined by a spring.

Evaluating the performance of the LQR and LQG controller, the most suitable is the LQG because it has a smoother response in the sense of acceleration and position, although it has a slower response. This characteristic aligns with the comfort design criteria. Another advantage of the LQG that was not tested in the article because it is an inner characteristic of the Kalman filter is the robustness against noise.

Finally, some challenges were identified because, for this implementation, the feedback process uses as state variables the velocity and position; and the acceleration as an added output to estimate all the states. But for a real implementation the main idea is to reduce as possible the number of required sensors. But with only an acceleration sensor, which is one of the most common in vibrational problems, the system became unobservable; this leads to the possibility of future work to identify, based on a real active suspension, a configuration that reduces the instrumentation as possible allowing the implementation of state space techniques as LQR and LQG.

8.0 ACKNOWLEDGEMENT

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9.0 REFERENCES

[1] H. Li, J. Yu, C. Hilton, and H. Liu, "Adaptive sliding-mode control for nonlinear active suspension vehicle systems using T-S fuzzy approach," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 8, pp. 3328–3338, 2013.

[2] Y. Ando and M. Suzuki, "Control of active suspension systems using the singular perturbation method," *Control Engineering Practice*, vol. 4, no. 3, pp. 287–293, 1996.

[3] C. Lauwerys, J. Swevers, and P. Sas, "Robust linear control of an active suspension on a quarter car test-rig," *Control Engineering Practice*, vol. 13, no. 5, pp. 577–586, 2005.

[4] S. Liu, J. Zhong, H. Yamada, and Y. Suematsu, "H∞ control of an active suspension system," *Journal of Central South University of Technology*, vol. 5, pp. 134-137, 1998.

[5] J.G. Maradey Lazaro, H. Esteban Villegas, B. Ruiz, and A. Aldana, "Control design strategies for semi-active suspension system," In: ASME International Mechanical Engineering Congress and Exposition, 2019, p. V004T05A004.

[6] S. Ikenaga, F. L. Lewis, J. Campos, and L. Davis, "Active suspension control of ground vehicle based on a full-vehicle model," In Proceedings of the American Control Conference, 2000, pp. 4019–4024.

[7] A. Dresch, D.P. Lacerda, J.A.V. Antunes Jr, A. Dresch, D.P. Lacerda, J.A.V. Antunes, *Design Science Research*. New York: Springer International Publishing, 2005.

[8] S.A. Rueda Villanoba, and C. Borrás Pinilla, "Neural network based fault tolerant control for a semi active suspension," In: Proceedings of the ASME 2019 International Mechanical Engineering Congress and Exposition, 2019. p. V004T05A089.

[9] S.J. Liang, and X.K. Chen, "An investigation on vehicle active suspension and anti-lock braking system coordination control," In Proceedings of the ASME 2013 International Mechanical Engineering Congress and Exposition, 2013, p. V04AT04A010.

[10] M. Bouazara, S. Gosselin-Brisson, and M. J. Richard, "Design of an active suspension control for a vehicle model using a genetic algorithm," *Transactions of the Canadian Society for Mechanical Engineering*, vol. 31, no. 3, pp. 317–333, 2007.

[11] A.K. Mohammadi, "Vehicle active suspension control: Using aavariable structure model reference adaptive controller," In: ASME International Mechanical Engineering Congress and Exposition, 2005, pp. 611-616.

[12] A. Florin and P. Liliana, "Pasive suspension modeling using MATLAB, quarter car model, input signal step type," *TEHNOMUS - New Technologies and Products in Machine Manufacturing Technologies*, no. 1224–029x, pp. 258–263, 2013.

[13] K. Ogata, *Modern control engineering (Vol. 5)*. Upper Saddle River, NJ: Prentice Hall, 2010.