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ORIGINAL ARTICLE

An Energy Management Strategy for Power-split Hybrid Electric Vehicle using Model Predictive Control

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ABSTRACT – To fulfil future demand for energy and to control pollution, a power-split hybrid electric vehicle is a promising solution combining attributes of a conventional vehicle and an electric vehicle. Since energy is available from two subsystems i.e, engine and battery, there is the freedom to manage it optimally. In this work, model predictive control strategy, that has the constraint handling which makes it a better choice over other strategies for efficient energy management of hybrid electric vehicles. A detailed mathematical model of the power split configured hybrid electric vehicle is developed that encompasses the engine, planetary gear, motor/generator, inverter, and battery. An interior-point optimizer based-nonlinear model predictive control strategy is applied to the developed model by incorporation of operational constraints and cost function. The objective is to curtail fuel consumption while the battery's state of charge should be maintained within predefined limits. The complete developed model was simulated in MATLAB for motor, generator, engine speed, and battery SoC. Computed specific fuel consumption from the proposed MPC during the NEDC and the HWFET cycles are 4.356liters/100km and 2.474 litres/100 km, respectively. These findings are validated by the rule-based strategy of ADVISOR 2003 that provides 4.900 litres/100 km and 3.600 litres/100 km over the NEDC and the HWFET cycles, respectively. This indicates that the proposed MPC shows 11.11 % and 31.26 % improvement in specific fuel consumption in the NEDC and HWFET drive cycles respectively.

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INTRODUCTION

Some important considerations for the world to cater to the future demand of humankind include environmental protection, sustainability of fossil fuels, and economic resources of fuels. Fossil fuels are depleting, whereas the population of the world is increasing, which will cause the prices of fuels to rise. Residuals of burnt fossil fuels contaminate the environment. Pollution control is a big challenge for the world. Different governments are imposing stringent regulations to control the rate of pollution. One major source of pollution is the transportation sector. Buses and cars running on internal combustion engines are directly affecting the cities concerning pollution. To overcome these issues, researchers and scientists are working on alternative means of communication or innovation in its structure.

The electric vehicle is an eco-friendly solution with zero pollution due to zero emissions. But its limitations are causing the world to find some intermediate solutions that can be found in the usage of hybrid electric vehicles (HEV). A promising solution in the current situation is hybrid electric transportation which has advantages of both sides, i.e., attributes of pure internal combustion engine-based vehicles and pure electrified vehicles. Attributes of internal combustion engine-based vehicles may include; refuelling of petrol/diesel in a few minutes with high energy densities, the well-established infrastructure of petrol and diesel pumps, a light container for carrying the extra fuel for the remote area but prone to pollution. The best attribute of electric vehicles is zero pollution. Another benefit of this is engine can be operated within an efficient zone at specific rpm and power.

Since HEV consists of two power sources; a downsized internal combustion engine and a battery. Hence, power can be extracted from either of the two means, creating freedom in choosing a power source. This point makes it possible to operate the engine in its efficient zone irrespective of the road condition. This gap makes room for energy management strategies for HEV. Various energy management strategies (EMS) for hybrid electric vehicles (HEVs) have been conducted in the last decade. EMS can be categorised into rule-based and optimisation-based strategies. Rule-based strategies (thermostats and power followers) are governed by a set of rules that are devised following intuition, human proficiency, or mathematical models and, usually, without knowing driving information a priori. Optimisation-based strategies are further divided into offline optimisation (global) and online optimisation (instantaneous) strategies. Offline optimisation strategies include dynamic programming (DP), linear programming (LP), Pontryagin's minimum principle (PMP) algorithm, and Genetic algorithm (GA). They require complete knowledge of the driving cycle a priori, and thus a globally optimal solution is found. Moreover, DP is a good benchmark for other optimisation strategies. Online optimisation strategies include equivalent consumption minimisation strategy (ECMS), model predictive control (MPC), Robust and Intelligent control strategies. ECMS strategy is based on a co-state / equivalent factor which is very sensitive

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to given driving cycle and road conditions. MPC has great capabilities of constraints handling. Learning-based strategies are also evolving with the current researches. MPC has some advantages over other management strategies. One of its finest features is constraints handling. Constraints can be imposed on input and output. MPC is equally suited for single input, single output control systems (SISO) and also for multiple input, multiple output control systems (MIMO). The receding horizon in MPC accommodates for any disturbance and can also be implemented online [1-3].

There are three configurations for HEV, including parallel hybrid electric vehicles, series hybrid electric vehicles, Power split/series-parallel configured hybrid electric vehicles. The configuration in which power is provided to wheels by engine or battery in parallel fashion is classified as parallel hybrid electric vehicle because engine and battery power flow paths are mechanical and electric paths, respectively. In other words, the battery and engine can work independently or cooperatively in this configuration. Some examples of parallel HEVs are Honda models of Insight and Civic, Chevrolet Malibu, and Greenline series. In the series configuration of hybrid electric vehicles, the engine's function is to charge the battery. Hence, the engine is connected to a generator. Since engine operation is independent of wheel speed, so, it operates at its peak efficiency. Opel Flextreme and Fisher Karma are some prominent names for the series configuration of hybrid electric vehicles. As indicated by the name, power split / series-parallel HEV configuration is proficient to function in a series fashion or in a parallel way. It consists of a downsized ICE, battery, motor/generator set 1 (which mostly works as a generator), motor/generator set 2 (which primarily works as a motor), and a planetary gear set. The capture of regenerative energy is also done by the motor. Planetary gear works as a continuously variable transmission CVT and makes engine speed independent of road load. Thus, make it possible to operate the engine in its most efficient zone of operation.

A planetary gear set is a combination of different gears which are called sun gear in the middle of the assembly, planet gears which revolve around and meshed with the central sun gear, and an outer gear named ring gear. From one side, three or four planet gears mesh with the sun gear. Another side of planet gears meshes with the outer ring gear. Three or four planet gears are rigidly connected with a common carrier. Motor/generator set 1 is attached with sun gear. The internal combustion (IC) engine is attached with a rigid carrier. Motor/generator set 2 is attached with the outer ring gear. This ring gear is also attached with the vehicle wheels through a differential gear having desired gear ratio. Examples of power split configured hybrid electric vehicles include Toyota Prius, Chevrolet Volt, Toyota Camry, Lexus RX400h, Lexus NX300h, and Toyota Ford Fusion [4]. In [5], the authors had conducted a study to investigate the performance of different configurations, including series HEV, parallel HEV, and power-split HEVs, and had concluded that power-split configuration had the best fuel economy over other configurations.

Energy is managed and reported by [6] for a parallel hybrid vehicle using stochastic model predictive control. In this paper, the author has proposed a control approach based on a predicted stochastic model of driver behaviour for not only fuel consumption minimisation but also for emission. Authors in [7] have proposed optimising gear ratio and torque split of parallel HEV having continuous varying transmission (CVT) using model predictive control. Researchers in [8] have optimised the energy split between the IC engine and electric motor of a parallel hybrid electric vehicle by using Pontryagin's Maximum Principle. Optimal energy control in series HEV using dynamic programming is studied in [9]. Authors in the article [10] have proposed an improved dynamic programming algorithm by the inclusion of the brake recovering rule and implemented over the series configuration of hybrid electric vehicles. The energy management method of combining fuzzy logic control and threshold control has been done in [11] and also been concluded that a modified quantum genetic algorithm is superior to the genetic algorithm when implemented on a series hybrid electric vehicle. As a result, many kinds of research have been made on energy management of parallel HEV and series HEV configurations. In [12], a study has been carried out about the modification in Stochastic MPC by ECMS for a parallel hybrid electric bus to minimise fuel consumption. Energy recuperation by regenerative braking of a parallel hybrid electric vehicle is studied in [13]. Comparison of rule-based and dynamic programming over model predictive control is performed in [14] on series hybrid electric tracked bulldozers for improvement of fuel economy. Energy management of series plug-in HEV is proposed in [15] for multiple energy storage systems using MPC. Authors [16] had studied the energy optimisation on parallel HEV and had concluded that Pontryagin's minimum principle (PMP) algorithm is superior to rule-based strategy. But as discussed PMP algorithm can only be used for offline optimisation.

Instantaneous optimal control of power-split HEV with planetary gear set was investigated by implementing adaptive equivalent consumption minimisation strategy (A-ECMS), and its preeminence was revealed over engine optimal operation line (OOL) strategy, which is a kind of rule-based strategy but ECMS is sensitive to driving cycle and cannot deal with constraints [17]. In [18], series plug-in HEV configuration is used to study the supremacy of the teaching-learning based optimisation (TLBO) algorithm over a rule-based strategy of ADVISOR software. In [19] a simple HEV with engine and motor is considered to compare and to show the dominant performance of the particle swarm optimisation (PSO) algorithm and two of its modifications over the rule-based strategy, but the PSO algorithm may have a low convergence rate. Besides, a parallel plug-in HEV configuration was considered for the sack of comparison of the interior-point based-MPC method with the alternating direction method of multipliers (ADMM) algorithm [20]. The interior-point method found more converging than that of ADMM algorithm while both of the algorithms are computationally much faster than general optimisational convex algorithms. Keeping in view of research articles discussed above, until now, the interior-point based-MPC algorithm has not been applied on power-split configurations of HEVs which is a fast converging algorithm. Hence, the objectives of this research article are to optimise energy sources and minimise fuel consumption for power-split HEV configuration and compare the results of MPC strategy with the results of rule-based strategy over standard cycles such as the New European Driving Cycle (NEDC) and the Highway Fuel Economy (HWFET) cycle. This paper is arranged in sections. A simplified control modelling of power split configured hybrid electric vehicle is presented in section 2. A brief description of model predicted control and MPC problem formulation is in section 3. The simulation-based results are discussed in section 4.

MATHEMATICAL MODELLING OF POWER SPLIT HEVS

A sketch of power split configured hybrid electric vehicle is shown in Figure 1. This type of hybrid electric vehicle is composed of the following important parts, including a downsized efficient IC engine, generator motor set 1 (generator), generator motor set 2 (motor) also works as traction motor, battery, and planetary gear set.

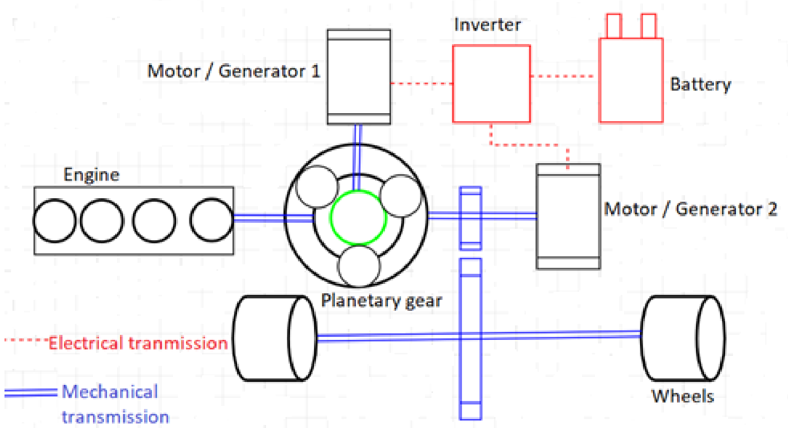


Figure 1. Schematic diagram of power-split configured hybrid electric vehicle.

The key component is a planetary gear device which can be subdivided into a sun gear, three or four planet gears affixed on a common carrier, and a ring gear. Sun gear is linked with generator motor set 1, a common carrier is linked with the engine, and the ring gear is linked with generator motor set 2 and also connected with vehicle wheels through a gear ratio. For simplicity, generator motor set 1 will be recalled as a generator, and generator motor set 2 be recalled as a motor. The masses of the engine, motor, and generator and their respective shafts are assumed lumped with the carrier, ring, and sun gears. The dynamics of power split configured HEV can be reduced to vehicle dynamics and battery dynamics [22-25].

As a result, the dynamics of the vehicle can be written as [22, 23]:

$$I_g \frac{d\omega_g}{dt} = T_g + F \times S \tag{1}$$

$$I_e \frac{d\omega_e}{dt} = T_e - F \times (S + R) \tag{2}$$

$$I_m \frac{d\omega_m}{dt} = T_m - \frac{T_d}{g_f} + F \times R \tag{3}$$

$$m \frac{dv}{dt} = \frac{T_d}{r_w} - \frac{1}{2}\rho A_f C_d v^2 - mg\sin(\theta) - \mu mg\cos(\theta) \tag{4}$$

where I_g, I_e, I_m, I_w are the inertia of the generator, engine, motor, and wheel, S, R are the number of teeth of the central sun and outer ring gears respectively, T_e, T_g, T_m are symbols for torques of IC engine, generator, motor respectively, and T_d denotes desired driver torque according to driving cycle, ω_e, ω_g, ω_m are the speeds of the engine, generator, and motor respectively, v is the speed, m is the mass, A_f is the frontal area and r_w is the wheel radius of the hybrid electric vehicle, μ, C_d, ρ, g_f are coefficient of rolling resistance, drag coefficient, air density, and final drive ratio g, θ, F are gravitational acceleration, road grade that is zero in this study and interaction force acting on different parts of the planetary gear. Planetary gear speeds are constraint by the following equation [22, 23].

$$R\omega_m + S\omega_g = (S + R)\omega_e \tag{5}$$

Motor rotational speed can be obtained from information of data points of the driving cycle and is expressed in the form of the following equation [23].

$$\omega_m = \frac{g_f v}{r_w} \tag{6}$$

Battery dynamics can be summarised in the form of state of charge as below [22, 23].

$$\frac{dSoC}{dt} = -\frac{V_{oc} - \sqrt{V_{oc}^2 - 4R_b P_b}}{2R_b Q_b} \tag{7}$$

$$P_b = T_m \omega_m \eta_m + \frac{T_g \omega_g}{\eta_g} \tag{8}$$

where SoC, V_{oc}, R_b, P_b, and Q_b denote the charging state of the battery, the open-circuit voltage, the internal resistance, power of the battery, and the capacity of the battery respectively. Also, the efficiency (η_m, η_g) for motor and generator are taken as 90% and 85% respectively. For the control-oriented model of a vehicle, engine behaviour related to the mass flow rate of fuel can be approximated as [23].

$$\dot{m}_f \approx c_f(P_{req} - P_b) \tag{9}$$

where P_{req} is the required power at the wheels at a specific time, which is obtained by knowing the required torque and angular speed at wheels, P_b is battery power at a particular time and c_f is a constant. When the speed is known from the driving cycle such as the standard New European Driving Cycle (NEDC), the above dynamics can be simplified to the following nonlinear model [22, 23].

$$\dot{x} = f(x,u) \tag{10}$$

$$x^T = [m_f, soc, \omega_e] \tag{11}$$

$$u^T = [T_e, T_g, T_m, \omega_g] \tag{12}$$

$$f(x,u) = \begin{bmatrix} c_f\left(P_{req} - P_b\right) \\ -\dfrac{V_{oc} - \sqrt{V_{oc}^2 - 4R_b P_b}}{2R_b Q_b} \\ A_1 T_e + A_2 T_m + \left(A_1 \dfrac{S+R}{S} - A_2 \dfrac{R}{S}\right) T_g - \dfrac{A_2}{g_f} T_d \end{bmatrix} \tag{13}$$

where

$$\begin{bmatrix} A_1 & A_2 \\ A_2 & A_3 \end{bmatrix} = \begin{bmatrix} I_e + (\dfrac{S+R}{S})^2 I_g & -\dfrac{R(S+R)}{S^2} I_g \\ -\dfrac{R(S+R)}{S^2} I_g & I_m + \dfrac{I_w}{g_f^2} + \dfrac{R}{S} I_g + m\dfrac{r_w^2}{g_f^2} \end{bmatrix}^{-1} \tag{14}$$

subject to the following constraints:

$$\dot{m}_{f\min} \le \dot{m}_f \le \dot{m}_{f\max}$$
$$soc_{\min} \le soc \le soc_{\max}$$
$$\omega_{e\min} \le \omega_e \le \omega_{e\max}$$
$$T_{e\min} \le T_e \le T_{e\max}$$
$$T_{g\min} \le T_g \le T_{g\max}$$
$$T_{m\min} \le T_m \le T_{m\max}$$
$$\omega_{g\min} \le \omega_g \le \omega_{g\max}$$

MODEL PREDICTIVE CONTROL AND PROBLEM FORMULATION

Many developments have been made in model predictive control (MPC) since the late seventies. MPC is not a specific control algorithm rather it adds a wide range of control methods that makes use of a model for minimising a cost function and hence, obtaining corresponding control inputs. The basic idea is; use a model to predict future output for a prediction horizon, then calculate a sequence of control inputs by minimising a cost function and then at the next instant, moving the prediction horizon and repeating the whole procedure, and so on. The basic concept of MPC is presented in Figure 2. Based on current and past values of inputs, outputs, and the proposed sequence of future control inputs, a prediction model is exercised to predict future outputs. Optimiser calculates this sequence of future control inputs by taking into account a cost function as well as the constraints. The cost function considers future tracking errors. Linear MPC is used very frequently where the plant can be modelled as a linear model. It gives reasonable results where the plant is to be operated at a steady-state or in the vicinity of a steady-state. But there are many applications where it is necessary to consider the dynamics behaviour, such as when the plant experiences continuous transition. Such applications need nonlinear MPC. Therefore, the main advantage of nonlinear MPC is that it can handle the nonlinear dynamics of a system [26].

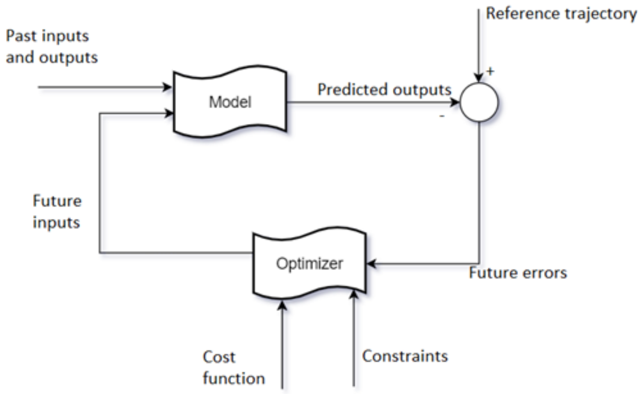


Figure 2. The basic structure of model predictive control.

In other words, MPC is an approach in which an objective function of the problem is solved optimally while keeping the constraints into considerations. In the cost function, values of output from the prediction model are compared with the desired set values, and a weight is assigned to deviation, which is to be minimised. Another weight is assigned for inputs. Manipulated variables are control inputs [27]. Mathematically, the general form of a nonlinear system can be represented as follows by omitting (t) for the sack of simplicity:

$$\dot{x} = f(x,u)$$

Where

$$x \in R^n$$
$$u \in R^m$$

The objective is to minimise a cost function defined by:

$$\underset{\min u}{J} = \int_{t}^{t+T} l(x_e,u_e)d\tau$$

The running cost is referred to as:

$$l(x_e,u_e) = (x_e^T Q x_e + u_e^T R u_e)$$

where x_e and u_e are the errors of outputs and inputs from respective reference trajectory and weight matrices are denoted by Q and R. Hence,

$$\underset{\min u}{J} = \int_{t}^{t+T} \{(x - x_r)^T Q (x - x_r) + u^T R u\} d\tau$$

So, the optimisation problem for nonlinear MPC is summarised as below:

$$\min_u J(x_e,u_e)$$

subject to:

$$\dot{x}(\tau) = f(x(\tau),u(\tau))$$
$$u(\tau) \in U, (\tau \in [t, t+T])$$
$$x_e(t+T) = 0$$

$x_e(t+T) = 0$ is used to define the terminal output equality constraints, which is a guarantee of the stability of the algorithm [21]. The optimal decision variable is computed at every time step t, by solving the above optimal control problems during the specified prediction horizon T. Only the first element of the optimal control sequence is applied that is done by using a shift function in the code. The prediction horizon travels forward and the same procedure is repeated again for the next time step.

SIMULATION SETUP AND RESULTS

Simulation Procedure

Simulations are performed over CasADi and ADVISOR 2003 software. CasADi software is open-source software that facilitates linear or nonlinear optimisations. It runs on MATLAB. It does not solve model predictive control problems rather facilitates solving nonlinear problems. Mehrez has formulated nonlinear MPC in a CasADi environment to track a fixed position in a bounded region for a robot. To perform this task, the numerical implementation of MPC in CasADi is required. Hence, problems of optimal control are transformed to linear or nonlinear problems to facilitate in implementing MPC numerically. There are different methods for this purpose, including single shooting methods, multiple shooting techniques, and collocation methods. The multiple shooting scheme is computationally much faster than a single shooting scheme. In multiple shooting schemes, the control inputs and states both are treated as decision variables. This is done by imposing equality constraints in which the upper and lower bounds are zero [21].

The proposed strategy of MPC, runs in CasADi software, is compared with the rule base strategy of ADVISOR 2003 software. ADVISOR 2003 is open-source software for the analysis of different vehicles. It runs in the environment of MATLAB®. It was established by National Renewable Energy Laboratory (NREL) to assist studies of hybrid propulsion systems program by the U. S. Department of Energy. The goals of ADVISOR 2003 were to provide a better understanding of multidimensional parametric studies and optimisation, flexible analysis of powertrain systems, easiness in utilising without in-depth knowledge of vehicle modelling. Since ADVISOR 2003 can use a backward-facing approach i.e., assuming that the vehicle is running at its desired speed, so, it does not require modelling of driver behaviour. The speed is directly computed from the driving cycle in a backwards-facing approach. The graphical user interface (GUI) allows the user to modify the selection of vehicles, different configurations of hybrid electric vehicles, different components, driving cycles, and mass of the vehicle with ease. There are three GUI pages, including the vehicle input page, simulation setup page, and result page. The result page of the software can show plots consisting of driving cycle, State of Charge (SoC), emissions, etc. Fuel economy is also shown on the result page. The user can also obtain driving cycles, motor, generator, and engine efficiency maps from the database of the software [28]. This software is developed to perform in a MATLAB environment with the capability of interacting with script files, graphical user interface (GUI), and block diagrams. Hence, it is a user-friendly software built to facilitate academia and industry for understanding multidisciplinary interactions of hybrid electric vehicles and to accelerate research in this field of study.

The procedure to perform simulation on ADVISOR 2003 is quite easy as it allows users to study the effect of changing components such as motors and generators on fuel economy. There are many configurations and types of vehicles that are provided for analysis. Similarly, there is the freedom to choose one type among various available components using a dropdown menu mentioned on the input page of ADVISOR 2003 against components such as a vehicle, fuel converter, exhaust after treat, motor and generator, etc. Selection of configuration of a hybrid electric vehicle, driving cycles, and the number of cycles of each driving cycle is much easy. Then, the second page is the simulation setup window in which. The third page is the result window which shows the summary of results. The result page shows the total distance travelled corresponding to the driving cycle used and total fuel consumption in litres per hundred kilometres [28, 29].

The flow chart of the algorithm for the proposed MPC is presented in Figure 3. A simplified mathematical model of power-split HEV is used in simulations, as discussed earlier in section 2. The procedure to perform simulations of the proposed MPC strategy is based on the algorithm developed as shown in Figure 3. Initial values of output states and control inputs are set to zero for the very first data point of the driving cycle, then from a second data point of the driving cycle to the last data point, every previous value of control inputs are utilised to initialize for next data point of driving cycle. While performing simulation for a single data point of the driving cycle, a control and prediction horizon of 5 seconds is chosen. The sampling time is set equal to 0.2 seconds. The future planned control input moves corresponding to predicted output state trajectory are obtained, but only the first moves of control inputs are implemented, neglecting the rest of the control moves, and then the same procedure is repeated with moving the prediction horizon. The code of the proposed MPC strategy is written in a free software CasADi, which can be imported in MATLAB. It (CasADi) has some salient features, including dealing with symbolic expressions, the ability to work with different solvers such as "Ipopt", and computer-readable fancy stuff by plugging in inputs. In this paper, Ipopt solver (interior-point optimiser solver) is used. Ipopt is an open-source nonlinear optimisation solver developed by Andreas Wächter and freely distributed under Eclipse Public License (EPL). This solver written in C++ language is much efficient in terms of computation as compared with the built-in 'fmincon' solver of MATLAB. In this solver, 'g' is for general nonlinear constraints with normally having lower bounds and upper bounds. Equality constraints can be applied by making upper bounds and lower bounds to zero. The parameters used in these simulations are obtained from the database of ADVISOR 2003 and presented in Table 1.

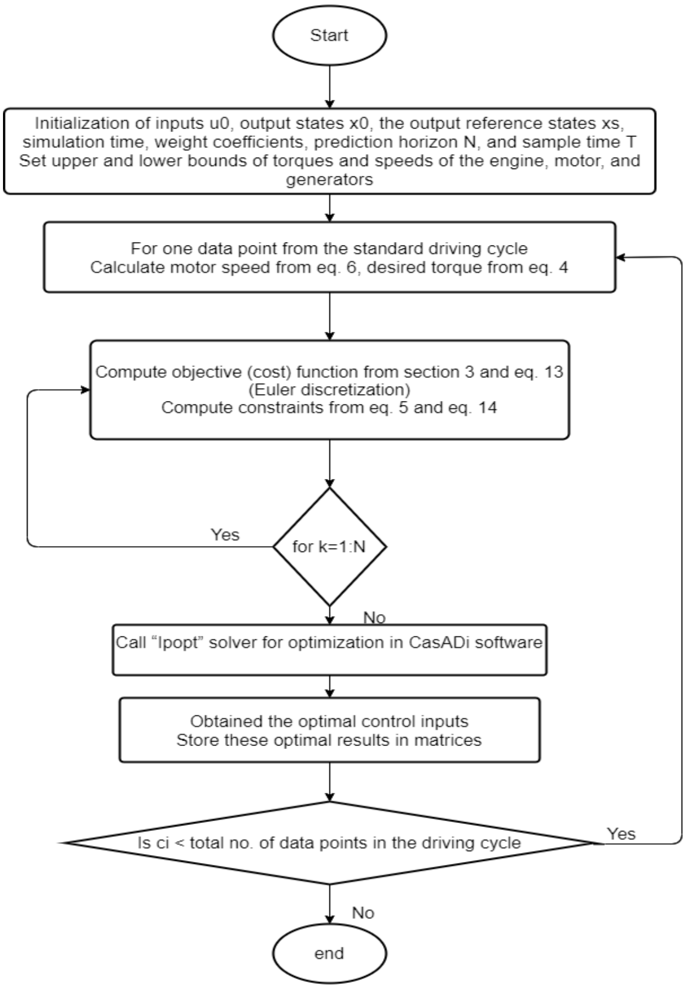


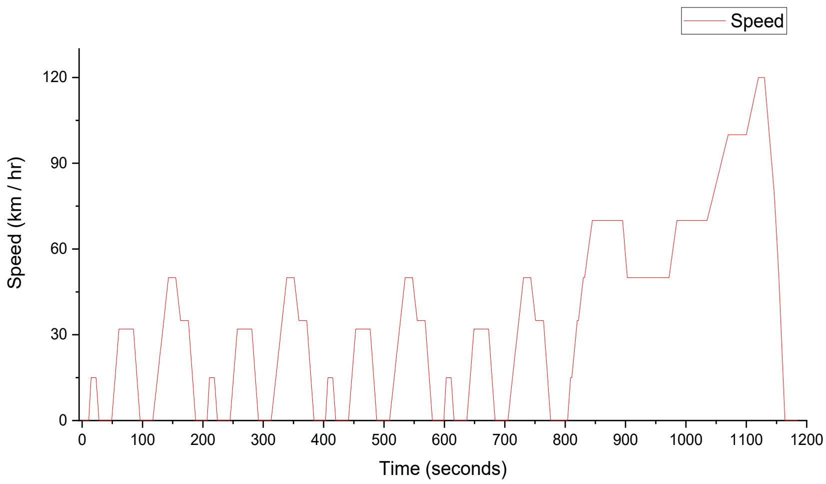
Figure 3. Algorithm of the proposed MPC strategy.

Table 1. Different parameters used in the simulation for Toyota Prius [24, 28].

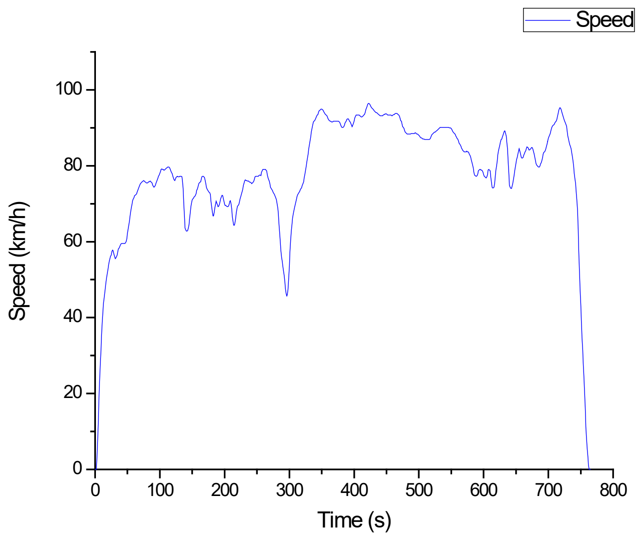
Symbols	Value	Symbols	Value
m	1504 (kg)	V_{oc}	307.85 (V)
A_f	1.746 (m^2)	R_b	1.004 (Ω)
C_d	0.3	Q_b	6.5 (Ah)
ρ	1.23 (kg/m^3)	soc_{max}	0.76
μ	0.015	soc_r	0.7
r_w	0.287 (m)	soc_{min}	0.68
g	9.81 (m/s^2)	c_f	0.0874
I_e	1.746 (kgm^2)	R	78
I_m	0.0226 (kgm^2)	S	30
I_g	0.0226 (kgm^2)	$T_{g\,max}$	55 (Nm)
I_w	3.3807 (kgm^2)	$T_{g\,min}$	-55 (Nm)
$\omega_{g\,max}$	575.9587 (rad/s)	$T_{m\,max}$	305 (Nm)
$\omega_{g\,min}$	-575.9587 (rad/s)	$T_{m\,min}$	-305 (Nm)
$\omega_{m\,max}$	628.3185 (rad/s)	$T_{e\,max}$	115 (Nm)
$\omega_{m\,min}$	-628.3185 (rad/s)	g_f	3.93
$\omega_{e\,max}$	418.8790 (rad/s)	$\dot{m}_{f\min}$	0 (g/s)
$\dot{m}_{f\,r}$	0 (g/s)	$\dot{m}_{f\max}$	0.4 (g/s)

The standard driving cycle used in both of the simulations are the New European Driving Cycle NEDC and the Highway Fuel Economy Cycle HWFET and are presented in Figure 4(a) and 4(b). Comparison of both of the driving cycles depicts that in the NEDC cycle, the time taken is 1184 s, the distance covered is 10.93 km, the average speed is 33.21 km/h, the maximum speed is 120 km/h, the maximum acceleration is 1.06 m/s^2, maximum deceleration is -1.39

m/s^2, the average acceleration is 0.54 m/s^2 and average deceleration is -0.79 m/s^2 whereas in the HWFET cycle the time taken is 765 s, the distance covered is 16.51 km, the average speed is 77.58 km/h, the maximum speed is 96.4 km/h, the maximum acceleration is 1.43 m/s^2, maximum deceleration is -1.48 m/s^2, the average acceleration is 0.19 m/s^2 and average deceleration is -0.22 m/s^2.



(a)

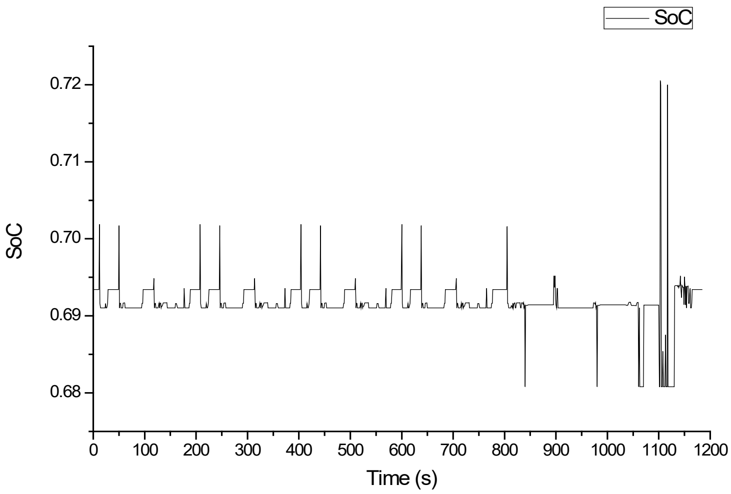


(b)

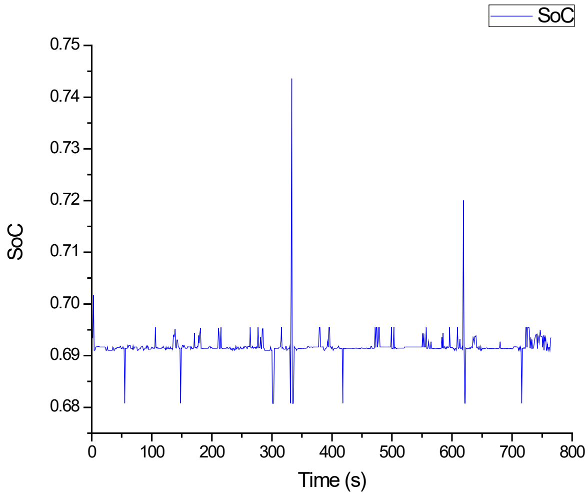
Figure 4. The (a) NEDC and (b) HWFET driving cycle.

Simulation Results

The results of the proposed MPC strategy and ADVISOR 2003 for battery state of charge are presented in Figure 5(a) and Figure 6(a) for the NEDC cycle and are presented in Figure 5(b) and Figure 6(b) for the HWFET cycle. Comparison of results over the NEDC and HWFET cycles show that, in propped MPC strategy, state of charge (SoC) of the battery remains closer to the reference value of 0.7 than that of the rule-based strategy of ADVISOR as shown in Table 2. This closeness of state of charge with the reference value is achieved by imposing appropriate bounds. The algorithm presented in this work does not violate the constraints imposed either on inputs or outputs. Hence, it can be truly said that the successfulness of code written for the proposed MPC strategy is that, the torques and the rotational speeds of thrice of the components, motor, generator, and engine are within their allowable lower and upper bounds for both of the cycles as depicted by Figure 7 and Figure 8.

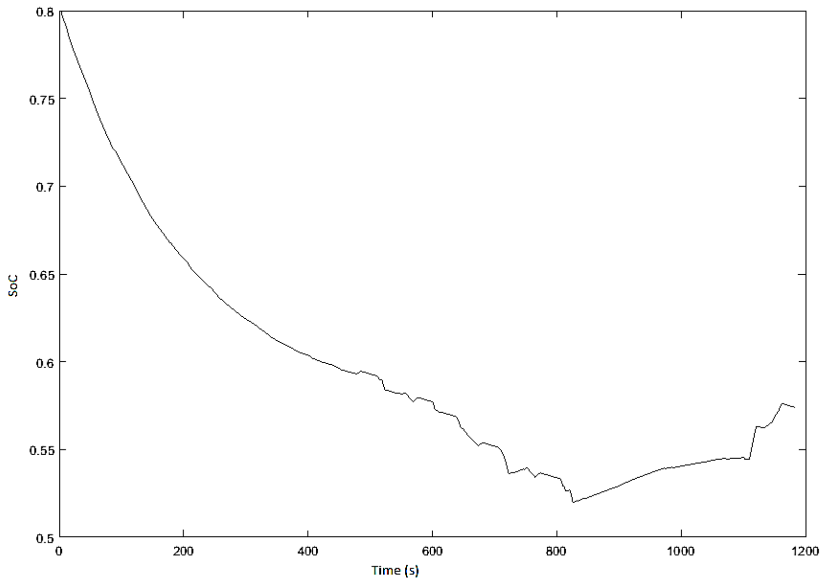


(a)

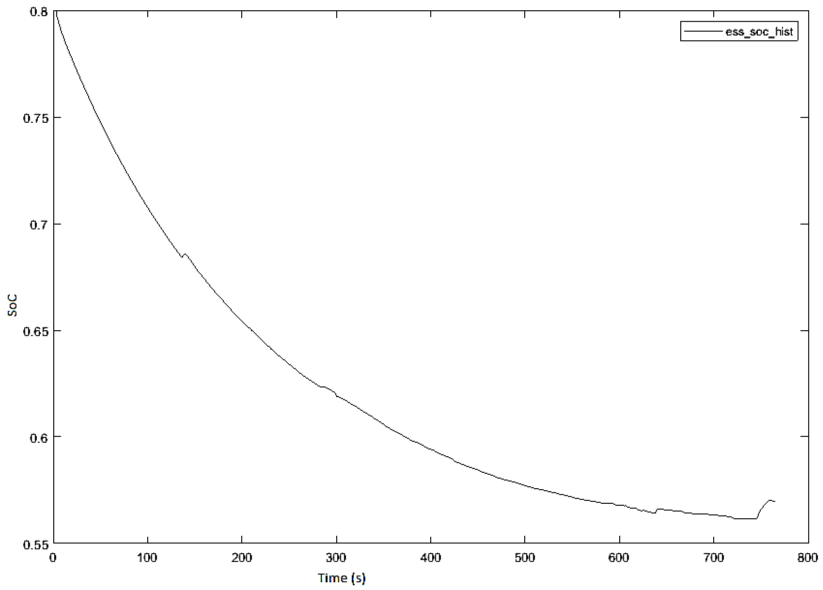


(b)

Figure 5. Battery state of charge using the proposed MPC over (a) NEDC and (b) HWFET cycles.

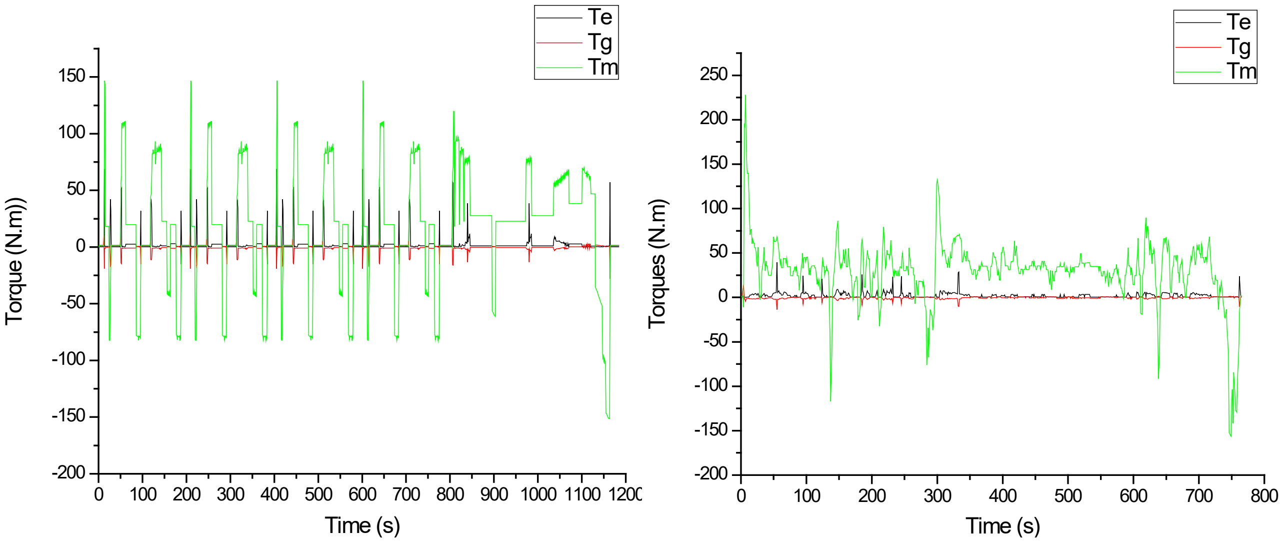


(a)



(b)

Figure 6. Battery state of charge using ADVISOR 2003 over (a) NEDC and (b) HWFET cycles.

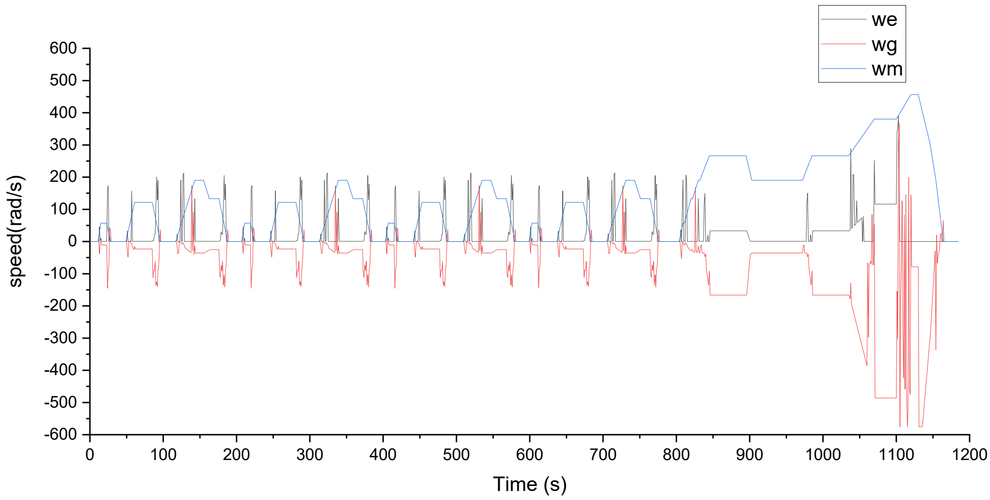


(a) (b)

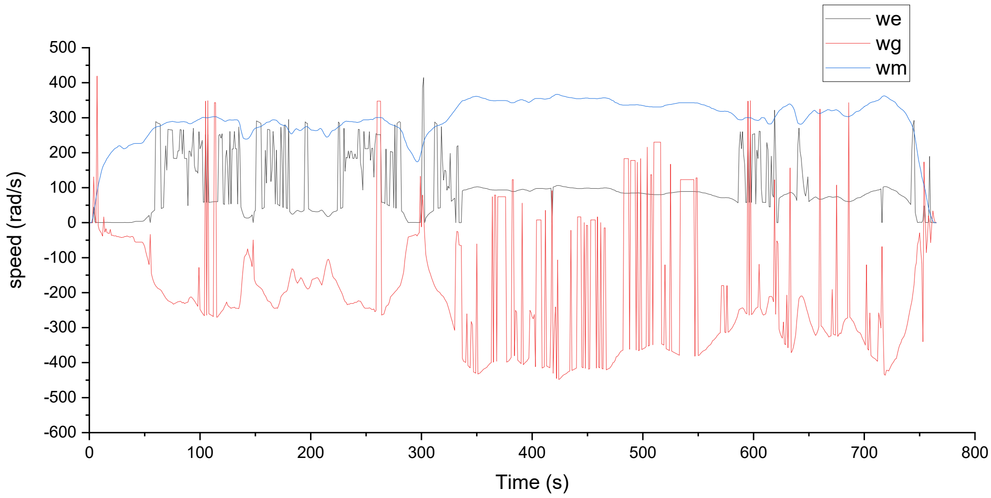
Figure 7. Torques of motor, generator and engine using the proposed MPC over (a) NEDC and (b) HWFET cycles.

The fuel economy of the model predictive control strategy is dependent on the simplified model of fuel mass flow rate of IC engine operating on gasoline. Here, an appropriate upper bound is imposed on the fuel mass flow rate for model predictive control problem formulation. As there are much fewer stops, less peak speed, and less average acceleration in the HWFET cycles than those of the NEDC cycles, so less power was consumed in HWFET than that of the NEDC cycle, and the same trend can be expected in the torques. Since it is necessary to provide peak speed in the NEDC cycle, which is greater than its counterpart in the HWFET cycle, there is less space for optimisation in the NEDC cycle while implementing the proposed model MPC. Moreover, the rule-based strategy works according to set rules. Thus, much energy is consumed in the rule-based strategy as compared with the proposed MPC strategy over the HWFET cycle.

There are some limitations of the study due to the main focus of study of optimising the energy of power-split HEV by implementing a fast algorithm based on interior-point optimisation but for further research following aspects can be considered which includes; (i) road slope as a function of data points of the driving cycle, (ii) environmental factors such as temperature, wind, and road terrain may be included in the analysis, (iii) although MPC has potential for real-time implementation but here it was supposed that the driving cycle is known a priori and only the successfulness of the proposed MPC is investigated as the driving habits are difficult to characterise and in the actual scenario, the relationship between the driver's demand and traffic environment is random and uncertain, (iv) dynamics losses of motor and generator may be included in the analysis by modifying the simulation code and, (v) battery ageing and thermal effects on battery may be considered.



(a)



(b)

Figure 8. Speeds of motor, generator and engine using the proposed MPC over (a) NEDC and (b) HWFET cycles.

A comparison of fuel economy is presented in Table 2. Simulations run on ADVISOR 2003 show that rule-based strategy gives a fuel economy of 4.900 liters per hundred kilometers over the New European Driving Cycle (NEDC) and a fuel economy of 3.600 liters per hundred kilometers over the HWFET cycle which are depicted in the result windows of ADVISOR 2003. On the other hand, the simulations of the proposed model predictive control (MPC) show a fuel consumption of 4.356 liters per hundred kilometers of the NEDC cycle whereas a fuel consumption of 2.474 liters per hundred kilometers of the HWFET cycle. As a result, fuel economy is improved by 11.11 % and 31.26 % using the proposed MPC strategy over the NEDC and the HWFET cycles. Hence, the results obtained are following the discussion made above. It can be inferred that this proposed strategy has the potential for real-time implementation.

Table 2. The fuel economy comparison.

Driving cycle	Method	Initial SoC	Final SoC	Fuel economy
NEDC	ADVISOR	0.8	0.570	4.900 litres /100km
	Proposed MPC	0.8	0.693	4.356 litres/100km (+11.11 %)
HWFET	ADVISOR	0.8	0.565	3.600 litres /100km
	Proposed MPC	0.8	0.6934	2.474 litres/100km (+31.26 %)

CONCLUSION

To overcome issues of pollution, depletion of fossil fuels and future energy demands, electric vehicle is an important part of the solution but it prone to; taking several hours to charge the batteries and limited available charging stations. So, power split configured hybrid electric vehicle is a fantastic alternative to both IC engine vehicles and full electric vehicles (EV) with all the benefits of series and parallel architectures. The objective of this study was to lessen fuel consumption of power-split hybrid electric vehicles by managing power coming from two different sources, i.e. ICE and batteries using an interior point optimiser based-nonlinear model predictive control approach. Model predictive control approach can benefit us over other strategies because of its finest feature of keeping constraints on inputs and outputs into its consideration. Moreover, it can optimise the use of engine and batteries. Therefore, nonlinear model predictive control is implemented over a simplified control model of power split configured hybrid electric vehicle. The results of MPC are compared with the rule-based approach of ADVISOR 2003 for fuel economy over the standard cycles of NEDC and HWFET cycles. The results depict the enhancement of fuel economy by 11.11 % and 31.26% using a nonlinear model predictive control strategy. The battery charge state (SoC) remains within its limit during the whole driving cycle and in the end, it is close to its reference value of 0.7. The results depict that the proposed NMPC strategy has the potential for implementation in real-time. In the future, a combination of rule-based strategy and model predictive control strategy will be implemented on power split configured hybrid electric vehicles using different standard driving cycles and also with road grade effects.

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