

## ORIGINAL ARTICLE

## Squeal Noise Analysis Using A Combination of Nonlinear Friction Contact Model

M. Rusli\*, M. H. Fesa, H. Dahlan and M. Bur

Mechanical Engineering Department, Faculty of Engineering, Andalas University, Kampus Limau Manis, Padang 25163, Indonesia  
Phone: +6275172586; Fax: +6275172566

**ABSTRACT** – Squeal noise is generated by an unstable friction-induced vibration in a mechanical structure with friction load. Nonlinear mechanisms like sprag-slip, stick-slip, and negative frictions damping are believed in contributing to generate this kind of noise. However, the prediction of its occurrence still counts on the analysis of complex-linear eigenvalue, which may underpredict the number of unstable vibration modes. The structure also is found to seem to generate squeal noise randomly. In this paper, nonlinear analysis of a squeal noise was investigated. The study was conducted numerically by a simple two-degree of freedom model and an experimental observation using a circular and slider plate with a friction contact interface. The friction force is modelled as a function cubic nonlinear contact stiffness and nonlinear negative velocity function of friction coefficient. It is found that mode coupling instability will occur if the normal contact stiffness and friction coefficient exceed the bifurcation point to generate a couple-complex conjugate eigenvalue and eigenvector. However, when the system is stated linearly stable, instability still can appear because of increasing the nonlinear contact stiffness and coefficient of friction. The instability is affected significantly by relative velocity and pressing force. Both parameters dynamically change depending on the vibration response of the structure. Furthermore, it is also found the stick-slip phenomenon interacted with mode coupling instability to generate squeal noise. It contributes to supply energy to increase the response caused by instability of mode coupling.

### ARTICLE HISTORY

Revised: 22<sup>nd</sup> July 2020

Accepted: 11<sup>th</sup> Sept 2020

### KEYWORDS

*Squeal noise;*  
*Nonlinear contact stiffness;*  
*Friction coefficient;*  
*Relative velocity;*  
*Normal force*

## INTRODUCTION

Squeal noise has become a big concern for automotive and railway vehicle engineers since forty years ago. This noise is generated by unstable vibration with friction contact. Three mechanical systems that are frequently found to emit this type of noise are automotive brake system, train-wheels when moving on a curved track [1-3], and wiper blades of automobiles [4, 5]. Some theories have been developed to explain the squeal mechanisms and to predict their generation. Numerous studies have tried with varied success in applying them. However, satisfying results in its prediction and suppression have not significantly been achieved because the mechanism is still not understood well [6, 7]. Many factors in both macro and microscopic levels contribute to affect squeal noise generation. Micro-level consists of a contact parameter such as coefficient of friction, normal or tangential contact stiffness, and surface roughness [8]. The macro-level is considered structural analysis and large nonlinear displacement of elastic flexural contact [5]. These factors that contribute and correlate among them to generate squeal noise are still challenging research.

In recent years, nonlinear vibration phenomena have been attracting more attention from researchers. Parametric and nonparametric techniques have been intensively studied. The friction has nonlinear contact parameters that increase the possibility of an unstable response generation. Some contact models and mechanisms show nonlinearity characteristics in its analysis such as flat and rough surface [8, 9], contact angle and spin creepage in wheel-curved rail contact [10], multiple contacts in discs and pad brake [11], and a positive friction characteristic [12]. In general, these nonlinear problems can be solved by some techniques like the method of harmonic balance, the perturbation methods, the Galerkin's method, or numerical integration [1, 13].

Some mechanisms are known to excite squeal noise, such as the negative gradient velocity-friction coefficient gradient, sprag-slip, stick-slip, and couple mode [2,3,14]. These mechanisms may contribute to the squeal generation singly or by multiple combinations. The most popular approaches to analyse and to predict unstable vibration modes in the squeal incident are complex eigenvalue (CEA) or mode coupling analysis, asymmetric eigenvalue formulation, which is a linear stability analysis. By this analysis, the unstable frequencies prediction shows a good agreement with the experimental result [15-17]. However, the CEA is possibly under-predictive, and linearisation is not able to predict all instabilities so that the results of linear CEA are not sufficient in predicting brake squeal [18, 19]. Linear CEA can not predict well how the unstable response occurs in a specific range of relative velocity and normal force.

Therefore, some researchers tried to model and predict the squealing noise using nonlinear parameters. Brunetti developed a method to predict unstable mode selection by modal absorption index from CEA [20]. Ding developed the assessment of the unstable vibration by mode coupling and nonlinear falling friction mechanism [21]. Oberst also modelled the brake pad contact by pressure-dependend material properties of lining [22], and Tison had the integration of

the contact interface, complex eigenvalue, probabilistic analysis, and a robustness criterion [22].

This paper presents the application of the nonlinear stability analysis for a simple two degree of freedom system with friction load. Some basic concepts of nonlinear contact and friction coefficient were applied to observe the vibration response of the model in generating unstable squeal noise. The contact stiffness is modelled as a cubic nonlinearity and friction coefficient as a nonlinear and negative function of velocity. This paper would observe how the relative speed friction and normal force affects the possibility of squeal generation. Then an experimental analysis of squeal noise in a circular plate is conducted. The friction force and the vibration response and their relation were analysed. The results from the stability analysis is presented.

## METHODOLOGY

### Numerical Model

Interfacial characteristics of friction contact, including stiffness of contact surface and friction coefficient, are critical parameters in analysing the squeal noise generation and dynamic behaviour of a structure with friction contact. Well-understanding of these parameters become essential to model and to understand squeal noise incident in many cases. The contact stiffness between two surfaces generally is modelled by Hertzian for flat surface [22]. On the other hand, when contact occurs between two rough surfaces, the contact establishes at some asperities with random shapes and dimensions. The deformation of the asperities can be elastic or elastic-plastic, and the normal contact stiffness can be modelled by polynomial equation [9,24]. When both surfaces slide each other, the asperities adhesion and other surface interaction mechanisms generate friction force. In this paper, a cubical nonlinearity of normal contact stiffness ( $k_{nl}$ ) is added to the linear contact stiffness ( $k_n$ ) as a function of contact deformation in the numerical model, like shown in Eq. (1).

$$k = k_n y + k_{nl} y^3 \tag{1}$$

Moreover, in the dynamics system literature, the most prominent friction model is given by a velocity function that assumed to be decreasing with increasing relative velocity. Generally, a sliding friction coefficient is modelled as a functional relationship to the velocity and three independent friction parameters [25] as defined by:

$$\mu = \mu_0 + \mu_1 \exp(-\alpha |V_{rel}|) \tag{2}$$

The parameter  $\mu_0$  controls the high relative velocity behaviour,  $\mu_1$  governs the low-velocity behaviour, and  $\alpha > 0$  controls the rate of change of friction with changes in relative velocity. Equation (2) also accommodates the negative-velocity function of the friction coefficient in the dynamics problem.

In this section, a numerical simulation is carried out to investigate the instability of the vibration response of a particle mass model having two degrees of freedom, as shown in Figure 1. This model is developed as the simplest model of a brake pad system. Two linear springs support the particle mass M. The parameters of c and k denote damping coefficient and spring stiffness, respectively. The stroke line of the spring  $k_2$  leans toward 45° from the normal direction. The contact stiffness between the mass and a rigid moving plane is modelled using a nonlinear spring  $k_n$  in the normal direction. Pressing force F is applied to this simple brake pad,  $F_n$  and  $F_f$  represent the normal reaction force and the friction force that are working at point O.

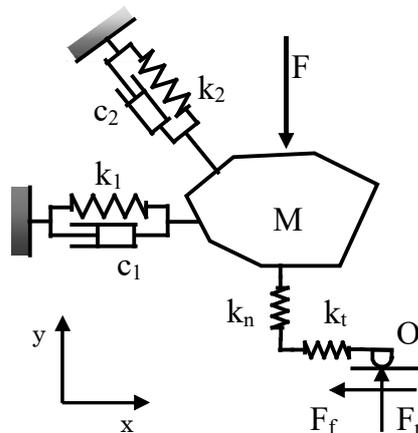


Figure 1. Two-degree of freedom model [6].

When the relative displacement of the mass M to the point O is neglected, the motion equation of the mass without damping can be defined as follows:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} k_1 + 0.5 k_2 + k_t & -0.5 k_2 \\ -0.5 k_2 & 0.5 k_2 + k_n \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} -F_f \\ F \end{Bmatrix} \tag{3}$$

When the normal contact stiffness is modelled as cubic nonlinearity in Eq. (1), then the friction force can be represented by Eq. (4).

$$F_f = \mu(k_n y + k_{nl} y^3) \quad (4)$$

where the friction coefficient is modelled in Eq. (2). Hence, the friction force can be modelled as:

$$F_f = (\mu_0 + \mu_1 \exp(-\alpha|V_0 - \dot{x}_1|))(k_n y_1 + k_{nl} y_1^3) \quad (5)$$

or

$$F_f = \mu_0 k_n y_1 + \mu_0 k_{nl} y_1^3 + \mu_1 \exp(-\alpha|V_0 - \dot{x}_1|) k_n y_1 + \mu_1 \exp(-\alpha|V_0 - \dot{x}_1|) k_{nl} y_1^3 \quad (6)$$

Friction force then can be distinguished into the linear and nonlinear faction.

$$F_f = F_f \text{ linear} + F_f \text{ nonlinear} \quad (8)$$

## Experimental Model

For the experimental study, a circular plate made of steel is connected to a stiff round bar and fixed supported to the clamp. The dimensions of the disc are 150 mm diameter and 1 mm thickness, as shown in Figure 2. The lowest part of the disc is contacted to a slider plate that can move forward and backwards controlled by the mechanism. The slider plate dimensions are 200 mm length, 120 mm in width, and 10 mm thickness. The slider bar moves back and forward following the sinusoidal like motion. Therefore, the motion speed is changed in a function of time. This experimental rig is a simple model of the train wheel that moving in curve track where the friction force acts out-plane of the disc.



**Figure 2.** A circular plate model for experimental study.

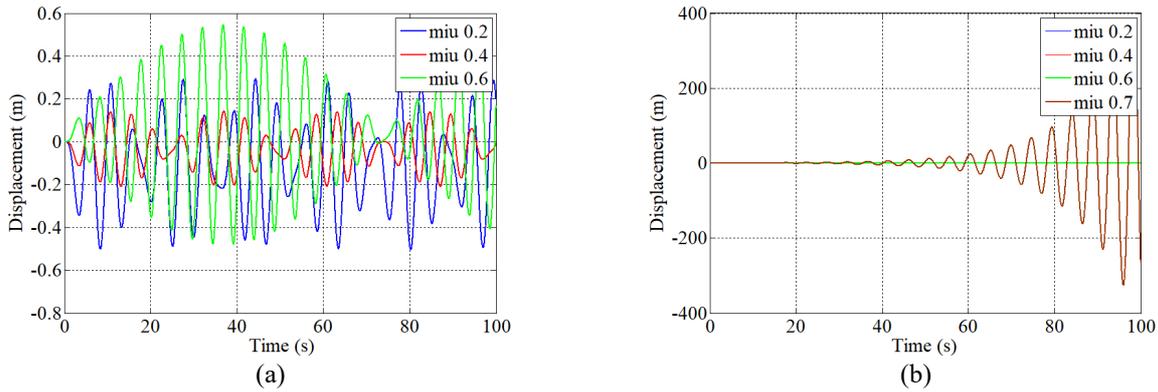
The friction force is measured using a load cell connecting the slider plate and driver mechanism. Then, the vibration response by an accelerometer allocated on the disc plate at the contact point. The friction force and vibration response were observed to know the mechanism of squealing noise generation.

## RESULT AND DISCUSSION

### Numerical Analysis

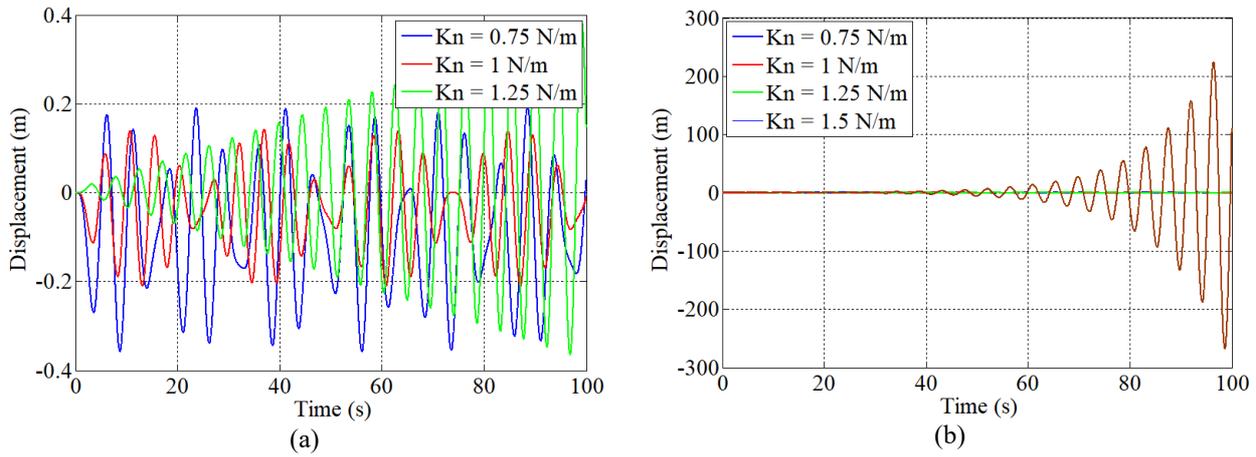
By setting  $M = 1$  kg,  $k_1 = k_2 = 1$  N/m, numerical simulations are conducted using the model to investigate the effect of the cubic normal contact stiffness and the friction coefficient to the instability of the system. First, the complex eigenvalue analysis (CAE) is carried out to predict the instability caused by the coupling mode. Then the stability response because of nonlinearity will be investigated using cubic normal contact stiffness and friction coefficient as a slip velocity function.

Figure 3 shows the vibration response of mass  $M$  in the horizontal direction with various values of a coefficient of friction. In this case, the normal contact stiffness is set constant of 1 N/m, and friction coefficient ( $\mu$ ) is set linear with different values. It is observed that when the friction coefficient is less than 0.6, the system is stable. A higher friction coefficient increased the vibration response, as in Figure 3(a). On the other hand, when the friction coefficient is 0.7 or more as in Figure 3(b), the response becomes unstable. It is obvious in complex eigenvalue analysis that the possibility of unstable vibration increased by raising the friction coefficient [9]. The bifurcation point for the friction coefficient is about 0.65. If the friction coefficient more than 0.65, the system was unstable.



**Figure 3.** Vibration response in the x-direction with various friction coefficient; (a) stable vibration response and, (b) unstable vibration response.

Moreover, the vibration response in the horizontal direction of mass  $M$  with various values of the normal contact stiffness is depicted in Figure 4. In this case, the coefficient of friction is set constant of 0.4, and contact stiffness is set linear with various values. It is observed that when the stiffness is less than 1.25 N/m, the system is stable. A higher friction coefficient increased the vibration response, shown in Figure 4(a). On the other hand, when the stiffness is 1.5 or more, the response becomes unstable, as shown in Figure 4 (b). It is also obvious in complex eigenvalue analysis that the possibility of unstable vibration will increase by raising the contact stiffness. The bifurcation point for contact stiffness by this condition is estimated at 1.3 N/m, and unstable vibration was generated when contact stiffness is 1.3 N/m or more.



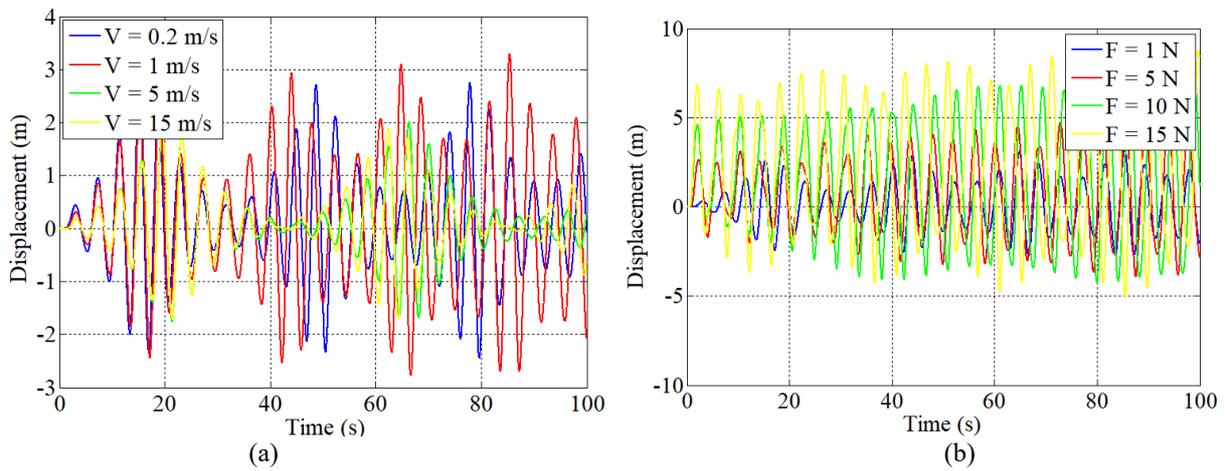
**Figure 4.** Vibration response in x-direction with various normal contact stiffness; (a) stable vibration response and, (b) unstable vibration response.

Next, the effect of the relative velocity of the motion and pressing force on the mass  $M$  was observed. Figure 5(a) shows the vibration response in various relative velocity. The normal contact stiffness is modelled by Eq. (1), where  $k_n$  and  $k_{nl}$  are equal to 1 N/m, the friction coefficient is modelled by Eq. (5), where  $\mu_o$ ,  $\mu_l$ , and  $\alpha$  are set at 0.4, 0.2, and 1, respectively. By these parameters, as explained previously, the system is stable. Figure 5(a) shows that the velocity affects the vibration response. Lower speed tends to have a higher vibration response. Decreasing velocity raised the friction coefficient that increases the system instability.

Similarly, the effect of pressing force to the vibration response is illustrated in Figure 5(b). The normal contact stiffness is modelled by Eq. (1), where  $k_n$  and  $k_{nl}$  are equal to 1 N/m, the friction coefficient is modelled by Eq. (5). Where  $\mu_o$ ,  $\mu_l$ ,  $\alpha$  and relative velocity ( $V$ ) are set 0.4, 0.2, 1, and 1 m/s, respectively. By these parameters, as explained previously, the system is stable. Figure 5(b) shows that the pressing force will affect the vibration response. Higher velocity tends to have a higher vibration response. Decreasing speed raised the friction coefficient that increases the system instability.

Therefore, if the effect of increasing pressing force and reducing the relative velocity raised the possibility of unstable vibration that generates squeal noise. The general fact shows that the squealing noise is generated randomly in a specific condition and then disappear in other ones. By previous simple analysis, it can be explained that squeal noise is generated when the normal contact stiffness and friction coefficient beyond its bifurcation points. These parameters are affected significantly by velocity and pressing force.

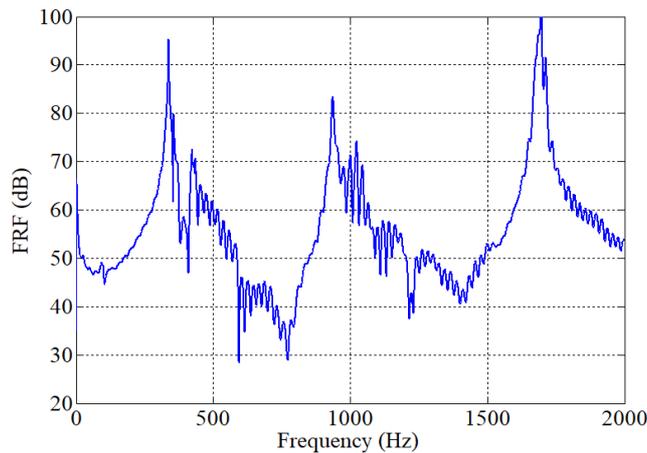
This nonlinear analysis has explained better how the squeal noise generation affected not only by contact stiffness and friction coefficient but also by other parameters like normal force and relative velocity of friction. Linear complex-conjugate eigenvalue analysis explained well in which natural frequencies are prone to emit the squeal [15-17]. However, it could not show why a mechanical structure emits the squeal in a limited range of velocity and normal force and disappears within other range.



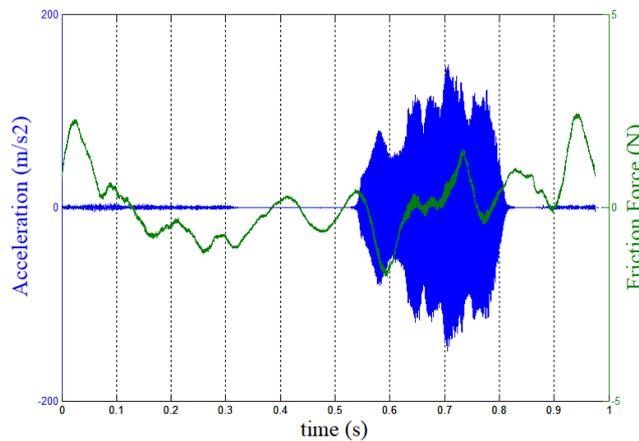
**Figure 5.** Vibration response in the x-direction with various (a) relative velocity and, (b) pressing force.

**Experimental Analysis**

Experimental modal analysis has become a powerful method in building a real condition and model of a mechanical structure [26, 27], including for predicting squeal noise generation [15-17]. In this analysis, first, the natural frequencies of the circular plate are observed experimentally by impact testing (Figure 6). The lowest three natural frequencies are 330 Hz, 936 Hz, and 1697 Hz respectively. While the slider plate is moving back and forth by a maximum speed of 5 m/s sinusoidally, and friction occurs between the circular and slider plates. Figure 7 shows the friction force and acceleration of the circular plate near the friction contact. The squealing noise occurs within 0.55 to 0.8 seconds of measurement time indicated by the high vibration response of the system. This squeal happens at a position when the slider plate just after passes the turning point where the speed is lower, although the friction force is around at zero values.

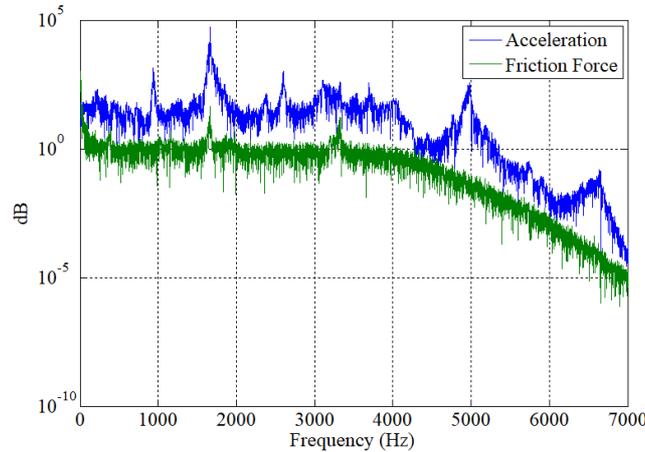


**Figure 6.** Frequency response function (FRF) of a circular plate.



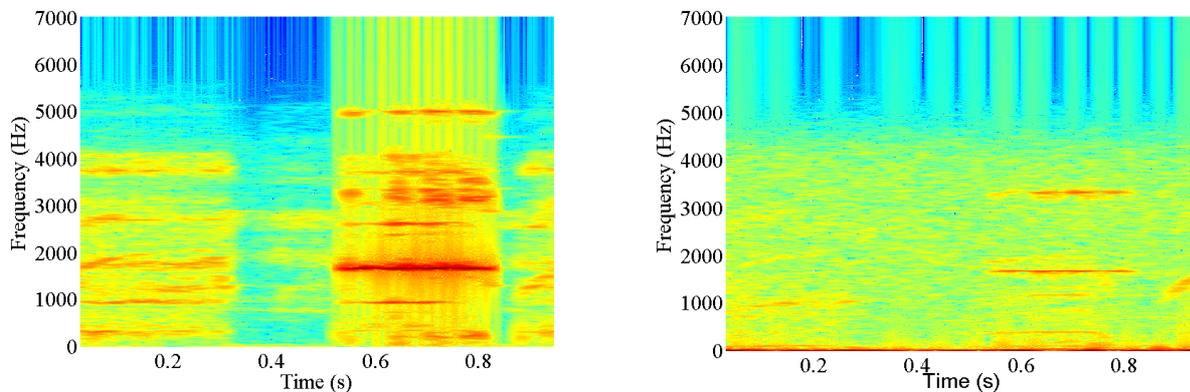
**Figure 7.** Measured acceleration and friction force.

The frequency spectrum of friction force and acceleration, while squealing is depicted in Figure 8. It has shown that the friction force and the vibration response have a different number of peaks. In general, friction generates force spectrum only at two frequency peaks, i.e., at 1661 Hz and its harmonics at 3332 Hz while the vibration response has more frequency peaks at 936 Hz, 1661 Hz, 2600 Hz, and their harmonics at higher frequency spectrum.



**Figure 8.** The measured frequency spectrum of acceleration and friction force.

Clearer indication also can be observed by spectrogram of vibration response and friction force like depicted in Figure 9. The vibration response has more frequency, especially at higher harmonics. The main frequencies response at 936 Hz, 1661 Hz, 2600 Hz are the second, the third, and the fourth natural frequency. It means the instability or squealing occurs at three natural frequencies at the same time. On the other hand, the friction force has fewer number of peaks, especially only at 1660 Hz and its harmonics at 3332 Hz. It also shows the possibility of stick-slip or sprag-slip mechanism occurs during the friction. During sticking, the circular plate follows the slider plate, and while slipping, it vibrates freely at other natural frequencies.



**Figure 9.** Spectrograms of (a) acceleration and (b) friction force.

These experimental phenomena confirm the numerical analysis result. In general, the circular and slider plate with friction contact is a stable system. The squealing noise from unstable vibration only occurs within a limited region. The reciprocating motion of the slider plate makes the relative velocity changes to follow the sinusoidal function. Therefore, at relatively low speed around the turning point, the friction coefficient increases. Moreover, the normal contact force also changed by increasing acceleration to make higher nonlinear contact stiffness. The increasing friction coefficient and normal contact force at the same time may pass the bifurcation point to have instability in vibration response.

Besides, a stick-slip mechanism also occurs in this contact during relative motion. It is shown by the different frequency responses of friction force and vibration response in higher frequency. This analysis confirmed that mode coupling and stick-slip mechanism are interacted with others to make instability to the system to generate squeal noise.

## CONCLUSION

Numerical and experimental analyses have been conducted to observe the mechanism of how the squealing noise generated. A simple two-degree of freedom model and circular plate with friction contact is used in this analysis. The friction force is modelled as a function cubic nonlinear contact stiffness and nonlinear negative velocity function of friction coefficient. It is previously discussed that in linear analysis, the frequency of squeal can be predicted well, but how the squeal seems to emit in the random incident.

It is found that mode coupling instability occurs if the normal contact stiffness and friction coefficient exceed the bifurcation point to make a complex-conjugate eigenvalue and eigenvector. However, when the system is stated linearly stable, instability still can appear because of increasing the nonlinear contact stiffness and coefficient of friction. Therefore, the instability is affected significantly by relative velocity and pressing force. Lower speed and higher pressing force increased the incidence of squeal noise. Both parameters dynamically change depending on the vibration response of the structure. Furthermore, it is also found the stick-slip phenomenon interacted with mode coupling instability to generate squeal noise. Stick-slip contributes to supply energy to increase the response caused by instability of mode coupling.

## ACKNOWLEDGEMENT

The authors gratefully acknowledge to Faculty of Engineering Andalas University that funds this research by publication grand by contract number 038/UN.16.09.D/PL/2020.

## REFERENCES

- [1] Zhang Z, Oberst S, Lai JCS. Instability prediction of brake squeal by nonlinear stability analysis. *Inter-noise* 2014;1-7.
- [2] Kinkaid NM, O'Reilly OM, Papadopoulos P. Review automotive disc brake squeal. *Journal of Sound and Vibration* 2003; 267; 105-166.
- [3] Ibrahim RA. Friction-induced vibration, chatter, squeal, and chaos, Part II: Dynamics and modeling. *ASME Applied Mechanic Review* 1994; 47; 227-253
- [4] Mohamad SM, Othman N, Sharif I, Zain MZMD, Abu Bakar AR. Dynamic behaviour and characteristics of rubber blade car performance. *International Journal of Automotive and Mechanical Engineering*. 2019; 16;1; 6437-6452
- [5] Lancioni G, Lenci S, Galvanetto U. Dynamics of wind screen wiper blade: Squeal noise, reversal noise, and chattering. *International Journal of Nonlinear Mechanics* 2016; 80; 132-143.
- [6] Rusli M, Bur M, Son L. Application of dynamics vibration absorber for squeal noise suppression in simple model structure. *International Journal of Structural Stability and Dynamics* 2015; 15; 5; 1450778
- [7] Rusli M, Bur M, Son L, Lahuddin M. Experimental verification of squeal noise suppression by mass addition in mechanical structure with friction. *ARPN Journal of Engineering and Applied Sciences* 2017; 12; 5; 876-881.
- [8] Wang AY, Mo JL, Wang XC, Zhu MH, Zhou JR. Effect of surface roughness on friction-induced noise: Exploring the generation of squeal at sliding friction interface. *Wear* 2018; 402–403; 80–90.
- [9] Rusli M, and Okuma M. Effect of surface topography on mode-coupling model of dry contact sliding systems. *Journal of Sound and Vibration* 2007; 308; 721-734.
- [10] Zenzerovic I, Kropp W, Prieringer A. Influence of spin creepage and contact angle on curve squeal: A numerical approach. *Journal of Sound and Vibration* 2018; 418; 268-280.
- [11] Denimal E, Nacivet S, Nechak L, Sinou JJ. On the influence of multiple contact conditions on brake squeal. *Procedia Engineering* 2017; 199; 3260–3265.
- [12] Liu X, Meehan PA, Investigation of squeal noise under positive friction characteristics condition provided friction modifier. *Journal of Sound and Vibration* 2016; 371; 393-405.
- [13] Sinou JJ, Thouverez F, and Jezequel L. Application of a nonlinear modal instability approach to brake systems. *ASME Journal of Vibration and Acoustics*. 2004; 126; 101-107.
- [14] Ouyang H, Mottershead JE, Cartmell MP, and Friswell MI. Friction-induced parametric resonance in discs: effect of a negative friction-velocity relationship. *Journal of Sound and Vibration* 1998; 209; 2; 251-264
- [15] Cao Q, Ouyang H, Friswell MI, Mottershead JE. Linear eigenvalue analysis of the disc-brake squeal problem. *Internasional Journal For Numerical Methods in Engineering* 2004; 61; 1546-1563.
- [16] Ouyang H, Cao Q, Mottershead JE, Treyde TR. Vibration and squeal of a disc brake: modelling and experimental results. *Proceeding of IMechE Part D: Journal of Automobile Engineering* 2003; 217; 867-875
- [17] Rusli M, Okuma M. Squeal noise prediction in dry contact sliding systems by means of experimental spatial matrix identification. *Journal of System Design and Dynamics* 2008; 2 ;3; 585-595
- [18] Zhang Z, Oberst S, Lai JCS. On the potential of uncertainty analysis for prediction of brake squeal propensity. *Journal of Sound and Vibration* 2016; 377: 123–132
- [19] Brunetti J, Massi F, D'Ambrogio W, Berthier Y. A new instability index for unstable mode selection in squeal prediction by complex eigenvalue analysis. *Journal of Sound and Vibration* 2016; 377; 106–122
- [20] Magaswaran K, Phuman Singh, AS, Hassan, MZ. New method in the identification of noise and vibration characteristics of automotive disk brakes in the low frequency domain. *International Journal of Automotive and Mechanical Engineering* 2014; 9; 1554-1577.
- [21] Ding B, Squicciarini G, Thompson D, Corradi R. An Assessment of mode-coupling and falling-friction mechanism in railway curve squeal through a simplified approach. *Journal of Sound and Vibration* 2018; 423;126-140.
- [22] Oberst S, Lai JCS. Squeal noise in simple numerical brakes models. *Journal of Sound and Vibration* 2015; 352; 129-141.

- [23] Tison T, Heussaff A, Massa F, Turpin I, Nunes RF. Improvement in the predictivity of squeal simulations: Uncertainty and robustness. *Journal of Sound and Vibration* 2014; 333; 3394-3412.
- [24] Shi X, Polycarpou AA. Measurement and modeling of normal contact stiffness and contact damping at the meso scale. *ASME Journal of Vibration and Acoustic* 2005; 127; 53-60.
- [25] Berger EJ. Friction modeling for dynamic system simulation. *ASME Applied Mechanic Reviews* 2002; 55; 6; 535-577.
- [26] Mohd Zin MS, Abdul Rani MN, Yunus MA, Ayub MA, Sani MSM, Aziz Shah MAS. Modal based updating for the dynamic behavior of a car trunk lid. *AIP Conference Proceedings* 2019; 2059; 020001
- [27] Wan Iskandar Mirza WII, Abdul Rani MN, Yunus MA, Athikary B, Sani MSM. Correlating finite element model of a car spot-welded front-end module in the light of modal testing data. *International Journal of Automotive and Mechanical Engineering* 2020; 17;2; 7974-7984.