

ORIGINAL ARTICLE

Vibration Control of Vehicle by Active Suspension with LQG Algorithm

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ABSTRACT – In this paper, the ride performance of a vehicle with active suspension and Linear Quadratic Gaussian (LQG) controller has been studied and is compared to the performances of a traditional passive suspension system. The study includes variables that are related to a passenger's comfort: vertical position, vertical velocity, pitch angle, pitch velocity, roll angle, and roll velocity. The performances of the two systems are evaluated by maximum values and root mean square (RMS) of the variables when riding on a sinusoidal road profile. The simulation results show that the vehicle with active suspension and LQG controller performs better than passive suspension system where the maximum values decrease by 85.77%, 92.73%, 50.31% 86.83%, 89.41%, 43.28%, and RMS values decrease by 88.59%, 92.36%, 42.99%, 87.61%, 90.85%, and 42.79% for vertical position, vertical velocity, pitch angle, pitch velocity, roll angle, and roll velocity, respectively.

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INTRODUCTION

The suspension is necessary to reduce vibration tendency that occurred in different road conditions. Generic suspension system contains three main sections namely; vehicle structure supporter structure attached to the main components, a coil spring to transform kinetic energy to potential energy, and absorber that converts kinetic energy [1]. The vehicle suspension systems can be classified into three groups which are passive-type, semi-active type, and fully active type suspension [2,3]. The passive suspension (PS) system consists of springs and conventional oil shock absorbers that provide design simplicity and cost-effectiveness. However, the performance must be traded-off between driving comfort and vehicle stability. The fully active suspension (AS) can provide a high-performance system but require many expensive sensors, actuators, a controller that results in high power consumption [4]. The semi-active suspension system (SASS) is a compromise between passive and fully AS systems which produces desirable performance with moderate cost increase and power consumption [5].

A mathematical car model is necessary for evaluating the suspension performance of a vehicle. Quarter-car model is the simplest mathematical model for simulation of the vehicle dynamics. The model consists of a wheel, a sprung mass, an unsprung mass, and suspension components [6, 7]. The output responses of the quarter-car model are only in a vertical direction which cannot represent all the feeling of passengers. However, because of its simplicity, the quarter-car model is most frequently used for studying suspension system performance [2, 6, 8-11].

On the other hand, the full-car model considers full dynamic responses and is a more accurate modelling system [12]. The model consists of four quarter-car models that are connected to the sprung masses with the solid rods. The pitching and rolling at the centre of gravity of the vehicle can be evaluated by the motion of four sprung masses [7]. Several methods for a control algorithm have been proposed and applied to AS systems. These algorithms include sliding mode control [12, 13], adaptive control [14], H_∞ control [15], fuzzy control [16], neural network control [17], LQG control [18, 19], and PID control [20]. The control algorithms are applied to quarter-car, half-car, or full-car models. All articles mentioned above report improvements performances of suspension systems. Katalin et al. proposed the application of LQG control method for the non-linear system through the EKF algorithm, which was very effective compared with UKF algorithm [21]. In addition to automotive design, LQG was also developed for medical research known as Electromyographic (sEMG) signal relationships. In this case, the LQG control algorithm offered a real-time performance of 92% mean correlation and 9% mean relative error having a standard deviation of ± 1.4 and ± 1.3 [22]. Rene et al. developed a genetic type algorithm used to generate data for a variable speed wind turbine [23]. Nguyen Duy Cuong suggested the addition of a learning feed-forward component to the LQG control system, which was found to be successful for electromechanical applications [24].

In this study, a Linear Quadratic Gaussian (LQG) algorithm, a control technique based on optimal control theory is applied to a semi-active suspension system with a full-car model. The performances of a vehicle have been investigated by simulation using MATLAB software. The vibration of a vehicle with an active suspension and an LQG controller is compared to a traditional passive suspension system when the road profile is sinusoidal.

System and Plant Model

The system block diagram for an LQG controller is given in Figure 1. The disturbance signal is the profile of road conditions. The plant is modelled by a full car model whose parameters, as shown in Figure 2.

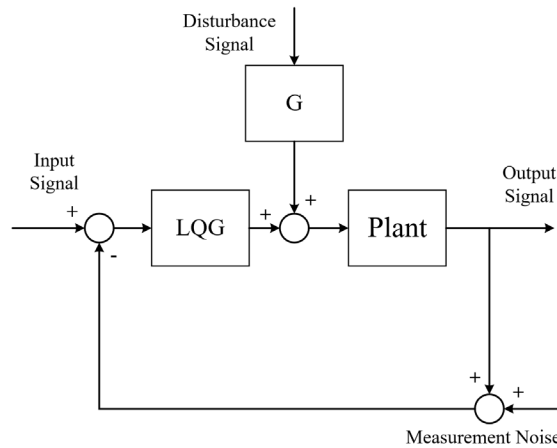


Figure 1. The system block diagram with an LQG controller.

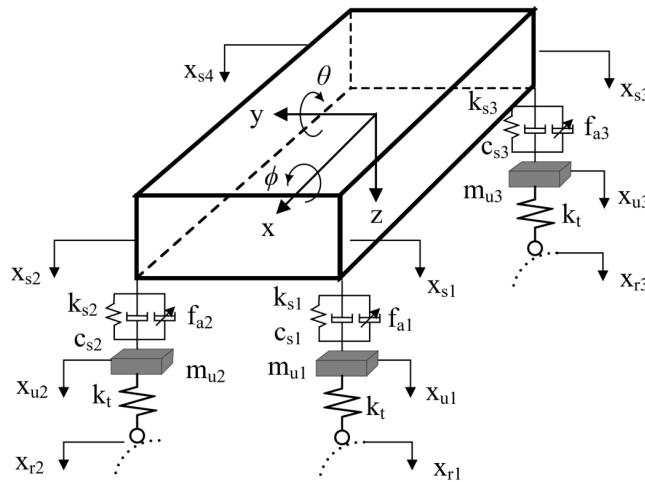


Figure 2. Parameters of a full car model.

The state-space equations of active suspension of the plant are given by [28].

$$\dot{X} = AX + BU + GW \tag{1}$$

$$Y = CX + V \tag{2}$$

Where A is the state matrix, B is the control matrix, G is the disturbance matrix, C is the output matrix, X is the state vector, U is the control signal, W is the disturbance process noise, Y is the output signal, and V is the measurement noise. To evaluate the passenger ride comfort, the output parameters of the vehicle are measured at the midsection of the car and the relationships are modelled by the following equations.

$$X_C = RX_S \tag{3}$$

$$R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\frac{t_f}{2} & \frac{t_f}{2} & -\frac{t_r}{2} & \frac{t_r}{2} \\ -l_f & -l_f & l_r & l_r \end{bmatrix} \tag{4}$$

$$X_C = [z \quad \varphi \quad \theta]^T \tag{5}$$

$$X_S = [X_{S1} \quad X_{S2} \quad X_{S3} \quad X_{S4}]^T \tag{6}$$

where X_{s1} , X_{s2} , X_{s3} , and X_{s4} are the vertical displacements of the car's body at wheel locations. z , ϕ , θ are the vertical displacement, pitch angle, and roll angle, respectively, at the centre of the vehicle. t_f and t_r are the front and rear track width. l_f and l_r are the front and rear wheelbase [29, 30].

Linear Quadratic Regulator Design

Designing the LQG controller consists of two major parts which are Linear Quadratic Regulator (LQR) design and optimal state estimation design [28]. For this system, the state-equation and control signal are given by the following equations [29].

$$X(k + 1) = A \cdot X(k) + B \cdot U(k) \tag{7}$$

$$U(k) = K_{lqr} X(k) \tag{8}$$

where K_{lqr} is a state feedback gain matrix. The performance index of LQR can be defined as:

$$J = \frac{1}{2} \sum_{k=0}^n (X^T(k) \cdot Q_{lqr} \cdot X(k) + U^T(k) \cdot R_{lqr} \cdot U(k)) \tag{9}$$

Value of the optimal state feedback gain is computed by:

$$K_{lqr} = (R + B^T P B)^{-1} B^T P A \tag{10}$$

Finally, the value of P can be obtained by solving the steady-state algebraic Riccati equation (ARE).

$$0 = A^T P A - P + Q - A^T P B (R + B^T P B)^{-1} B^T P A \tag{11}$$

Optimal State Estimation Design

The systems are defined by the following equation:

$$X(k + 1) = A \cdot X(k) + G \cdot W(k) \tag{12}$$

$$Y(k) = C \cdot X(k) + V(k) \tag{13}$$

where $W(k)$ is the disturbance process noise, $V(k)$ is the measurement noise.

$$Q_{kf} = E\{W(k) \cdot W^T(k)\} \tag{14}$$

$$R_{kf} = E\{V(k) \cdot V^T(k)\} \tag{15}$$

$$E\{W(k) \cdot V^T(k)\} = 0 \tag{16}$$

For the stable conditions, the value of the error covariance matrix converges to:

$$P \equiv P(k + 1) = P(k) \tag{17}$$

The steady-state value of the optimal state gain matrix is obtained from the following equation:

$$K_{kf} = P \cdot C^T (C \cdot P \cdot C^T + R_{kf})^{-1} \tag{18}$$

where the value of P is obtained by solving another steady-state algebraic Riccati equation (ARE):

$$P = A \cdot [P - P \cdot C^T (C \cdot P \cdot C^T + R_{kf})^{-1} C \cdot P] A^T + G \cdot Q_{kf} \cdot G^T \tag{19}$$

Finally, the optimum estimated state value is given by:

$$\hat{X}(k + 1) = A \cdot \hat{X}(k) + B \cdot U(k) + A \cdot K_{kf} (Y(k) - C \cdot \hat{X}(k)) \tag{20}$$

COMPUTER SIMULATION RESULTS

The dynamic responses of a vehicle are obtained by a set of computer program simulations. The parameters of the vehicle model are listed in Table 1 [25, 30]. For the sake of simplicity, the road profile is assumed to be a pure sinusoidal [31] with amplitude 0.1 m, as shown in Figure 3. Simulations are performed for two cases, namely a vehicle with a passive

suspension, and a vehicle with an active suspension and an LQG controller. Performances of the systems are evaluated by comparing the results measured for the two systems. The simulation results for vertical displacement, pitch angle, roll angle, vertical velocity, pitch velocity, and roll velocity at the centre of the vehicle for the two cases are plotted and shown in Figure 4 to Figure 9.

Table 1. Parameters of vehicle [32,33].

Parameters	Values
Sprung mass	1011 kg
Stiffness of front suspension	10950 N/m
Stiffness of rear suspension	14360 N/m
Damping coefficient of front	530 N.s/m
Damping coefficient of rear	910 N.s/m
Stiffness of tires	190000 N/m
Front track width	1.481 m
Rear track width	1.493 m
Front-rear wheelbase	2.73 m

According to the automobile industry, ride comfort is defined as the ability of the suspension to damp and isolate road vibration due to obstruction in the three important frequency range. The frequency interval included ordinary ride (0-3 Hz), intermediate ride (3-8 Hz) and highly obstacle ride condition (8-100 Hz).

Figure 4(a) and 4(b) shows the vertical displacements of a passive suspension system and an active suspension system with an LQG controller, respectively. The result indicates that the active suspension system decreases the maximum vertical displacement from 0.1533 m to 0.0218 m and the RMS value from 0.1021 m to 0.0116 m. Figure 5(a) and 5(b) show the pitch angle of a passive suspension system and an active suspension system with an LQG controller, respectively. The result shows that the active suspension system decreases the maximum pitch angle from 0.2031 rad to 0.0148 rad and the RMS value from 0.1358 rad to 0.0104 rad.

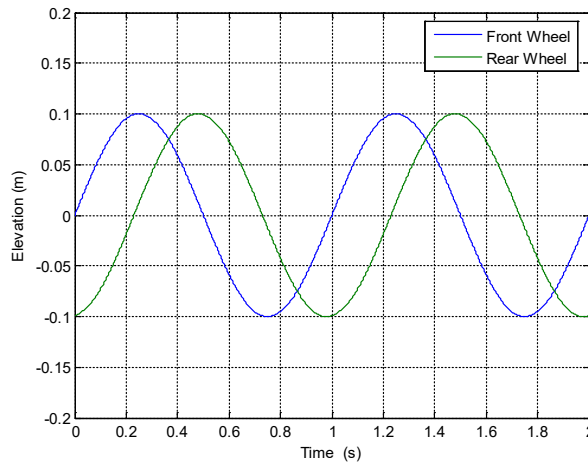


Figure 3. Sinusoidal road profile.

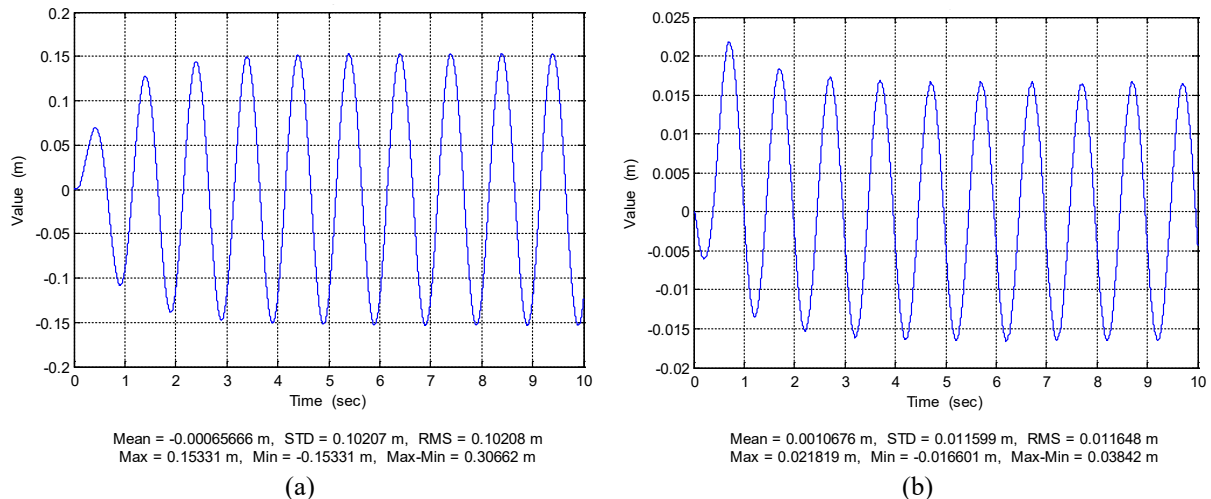


Figure 4. Vertical displacement of a (a) passive suspension system, and (b) an active suspension system with LQG.

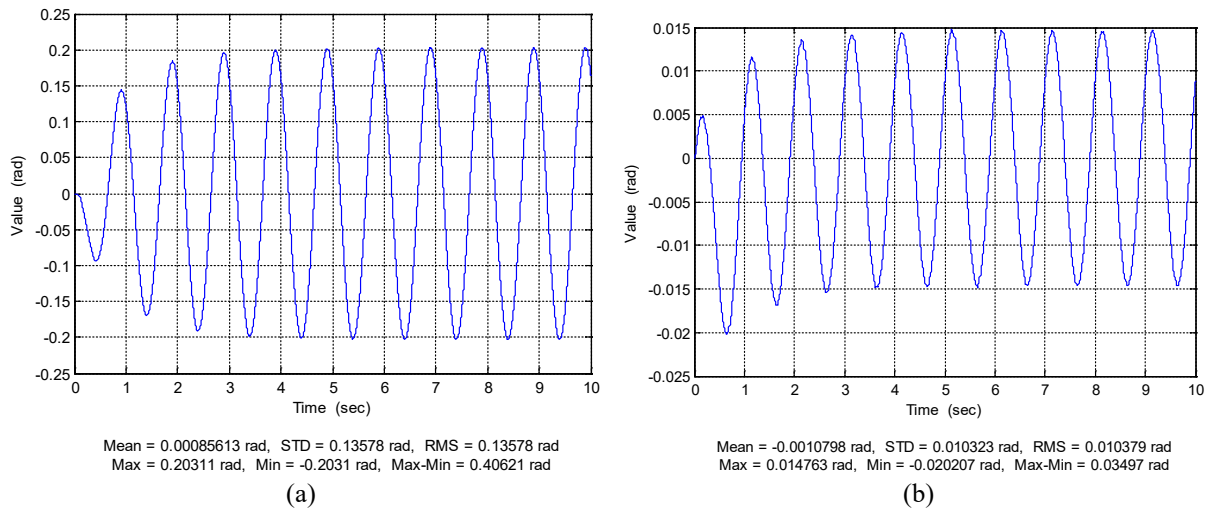


Figure 5. Pitch angle of a (a) passive suspension system and (b) an active suspension system with LQG.

Figure 6(a) and 6(b) show the roll angle of a passive suspension system and an active suspension with an LQG controller, respectively. The result indicates that the active suspension decreases the maximum roll angle from 0.0398 rad to 0.0198 rad and the RMS value from 0.0251 rad to 0.0143 rad. Figure 7(a) and 7(b) show the vertical velocity of a passive suspension system and an active suspension system with an LQG controller, respectively. The result shows that the active suspension decreases the maximum vertical velocity from 0.9632 m/s to 0.1269 m/s and the RMS value from 0.6355 m/s to 0.0787 m/s. The simulated data of the AS system assisted by LQG controller demonstrated useful features from the obtained vibration signals. These data can be developed further to create an accurate health monitoring system for the suspension system. [1].

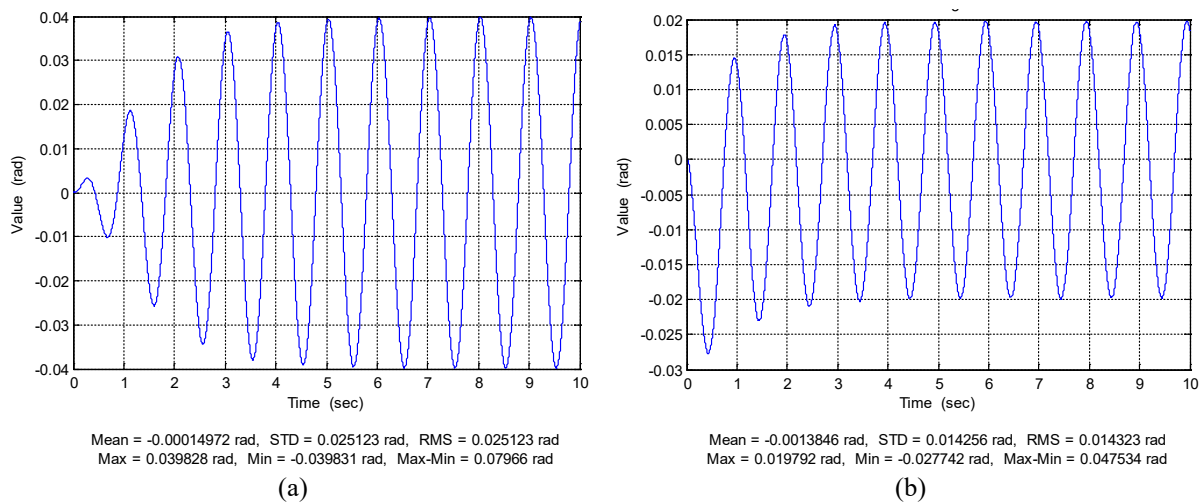


Figure 6. Roll angle of (a) a passive suspension system and (b) an active suspension with LQG.

Figure 8(a) and 8(b) show the pitch velocity of a passive suspension system and an active suspension system with a LQG controller, respectively. The result show that the active suspension system decreases the maximum pitch velocity from 1.2927 rad/s to 0.1369 rad/s and the RMS value from 0.8454rad/s to 0.0774rad/s. The shift from active to passive suspension system can help reduce the cost of production of the suspension system. This is due to the decrease in required power from the low bandwidth of the system of lower than 4 kW peak. In addition to power requirement, it was also pointed out that active component also required damper fluid solution, such as Magneto-rheological fluid. Matlab and simulink programs were used to develop new algorithm for fast converging and lower-cost active suspension.

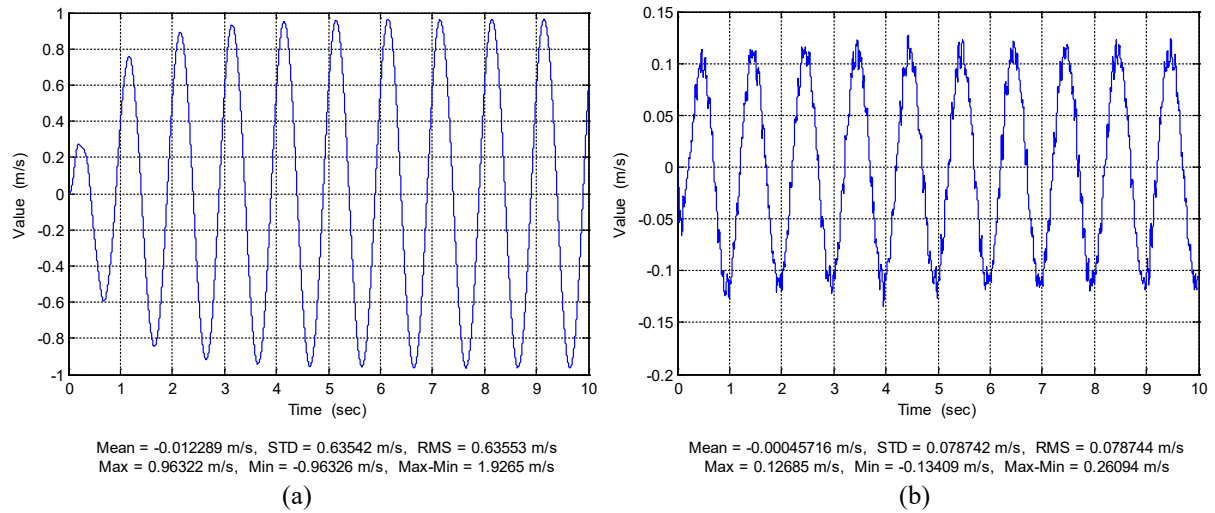


Figure 7. Vertical velocity of (a) a passive suspension system and (b) an active suspension system with LQG.

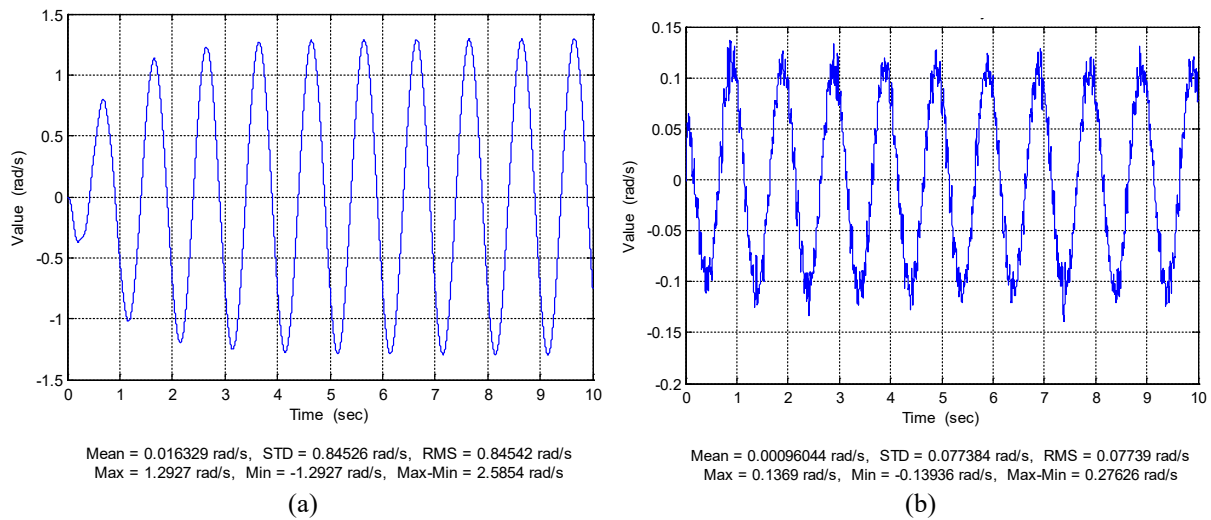


Figure 8. Pitch velocity of (a) a passive suspension system and (b) an active suspension system with LQG.

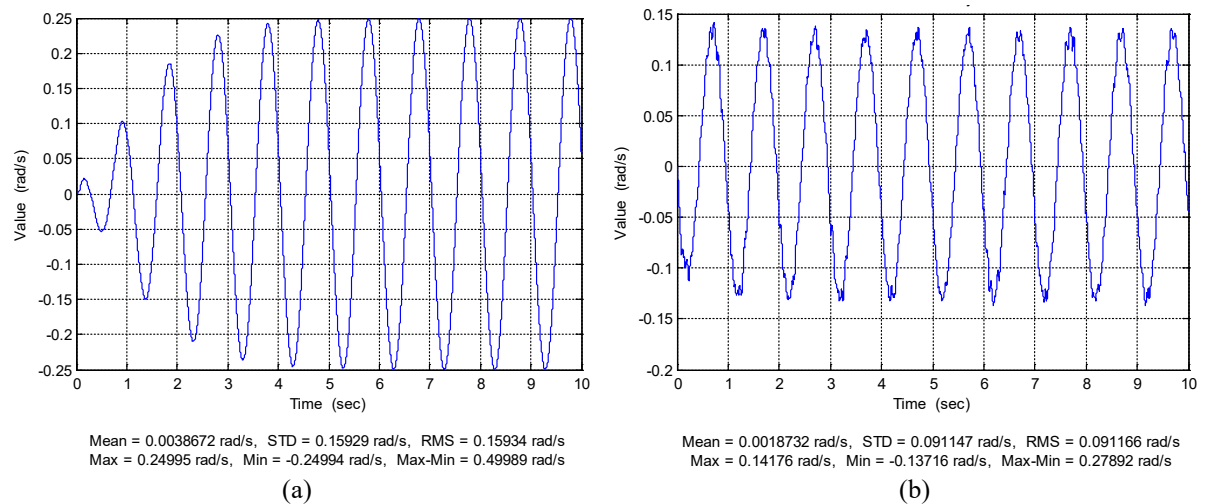


Figure 9. Roll velocity of (a) a passive suspension system and (b) an active suspension system with LQG.

Figure 9(a) and 9(b) show the roll velocity of a passive suspension system and an active suspension system with an LQG controller, respectively. The result shows that the active suspension system decreases the maximum roll velocity from 0.25 rad/s to 0.1418 rad/s and RMS value from 0.1593 rad/s to 0.0912 rad/s. Table 2 and 3 demonstrated the output variables for both passive and active suspension system. The major restriction of both types of the feedback controller is the sampling frequency. According to the current requirement by the automotive industry, it is important to reconstruct at least two periods of signal in order to predict the motion frequency.

Table 2. Compare the maximum values of output variables between passive and active systems.

Output variables	Passive system	Active system	Decrease (Value)	Decrease (%)
Vertical disp.	0.1533	0.0218	0.1315	85.77%
Pitch angle	0.2031	0.0148	0.1883	92.73%
Roll angle	0.0398	0.0198	0.0200	50.31%
Vertical velo.	0.9632	0.1269	0.8364	86.83%
Pitch velocity	1.2927	0.1369	1.1558	89.41%
Roll velocity	0.2500	0.1418	0.1082	43.28%

Table 3. Compare the RMS values of output variables between passive and active systems.

Output variables	Passive system	Active system	Decrease (Value)	Decrease (%)
Vertical disp.	0.1021	0.0116	0.0904	88.59%
Pitch angle	0.1358	0.0104	0.1254	92.36%
Roll angle	0.0251	0.0143	0.0108	42.99%
Vertical velo.	0.6355	0.0787	0.5568	87.61%
Pitch velocity	0.8454	0.0774	0.7680	90.85%
Roll velocity	0.1593	0.0912	0.0682	42.79%

CONCLUSION

By solving the Linear Quadratic Regulator equation established by drift and diffusion coefficients of the optimal state estimation design, the steady-state PDFs of stochastic response systematic, amplitude energy and velocity are obtained. The simulation results on a sinusoidal road profile of the active suspension system with an LQG controller clearly demonstrates that it increases ride comfort and stability of the vehicle by decreasing value of the output variables that affect the comfort of passengers. The maximum RMS and the decrease in the values of the output variables are provided in Table 2 and Table 3 for the vehicles with a passive suspension system and the active suspension system with an LQG controller. These output values included vertical displacement, pitch angle, roll angle, vertical velocity, pitch velocity, and roll velocity. Although active component can only operate with magneto-rheological fluid, it is still a better option compared with passive suspension. This is because active suspension reduced the power required to maintain the automotive chassis in a stable configuration. All of the analytical results are verified by digital simulations.

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