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New Methodology for Inertial Identification of Low Mobility Mechanisms Considering Dynamic Contribution

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ABSTRACT

Knowledge of dynamic parameters of mechanical systems is required in different applications, particularly in the simulation and control problems. In this paper, the standard identification methods are discussed and a new methodology for identification of inertial parameters is raised when the closed chain has low mobility. The methodology includes formulating a symbolic model based on the transfer of inertial properties and a reduction using dynamic contribution indices based on CAD approximations. The new method is applied to the front suspension of an electrical vehicle. After applying the procedure, a model with few parameters that allows accurately reproducing the dynamic behaviour of the system is obtained. A novel methodology has been developed that allows the identification of dynamic parameters in low mobility mechanical systems.

Keywords: Identification; inertial parameters; low mobility mechanism; vehicle suspension

INTRODUCTION

Obtaining models that accurately represent the behaviour of the systems, which allow evaluating functional performance, requires the identification of a set of parameters included in those models. The identification of geometric parameters is a known technique in the field of robotics. However, dynamic parameters are difficult to determine because they require mostly the application of several experimental identification techniques. For example, to determine mass and mass-centres, techniques such as force measurement, suspension cables and balancing tables may be enough [1]. On the other hand, the experimental determination of inertia tensors implies more complex methods that are based on their dynamic response, such as the gravitational pendulum method and small angular motions [2-4]. Generally, these methods require special assemblies, high execution times and highly qualified personnel. So, if the mechanical system is large, in size and number of pieces, these methodologies are impractical.

The parameter identification by a dynamic model allows estimating a high number of dynamic parameters in a single experiment. In general, in an inertial parameters identification process, not all the individual parameter values are determined. The main purpose of the process is to determine an equivalent model that can predict with the minimum error the generalised forces for any movement of the system. This equivalent model is smaller (fewer parameters) than the original model. Considering that in

simulation and control tasks, dynamic models are computationally extensive, with high execution times, obtaining reduced models with a sufficiently accurate response is a job that is well worth it.

The dynamic parameter identification has been addressed until today, especially in robotics [6-9], [11-18]. However, for mechanical systems in general, few studies have been carried out [19-22]. Bearing in mind that almost all the mechanisms of machines are closed kinematic chains, with low mobility, it is essential to propose a methodology to identify dynamic parameters for this type of mechanisms. The term "low mobility" refers not only to the low number of degrees of freedom but also to the limited workspace and the limited relative movement between the elements of the chains. Some previous works have developed symbolic and numerical methodologies such as SVD decomposition and inertial transfer to address the identification of low mobility systems. However, they have experimented problems with the results reached due to the numerical characteristics of the determined models. These works were analysed in the following section.

The proposed identification methodology in this study requires the construction of a dynamic model written in a linear manner with respect to the parameters that will be identified. The elaboration of the model and the development of the required mathematical expressions are presented in the following section. Subsequently, the application of the standard method of identifying dynamic parameters for low mobility mechanisms (LMM) is analysed. The difficulties that were encountered after the application of this method are discussed and some actions are proposed, such as the use of symbolic procedures and the criteria of reduction of the model in the search to obtain a set of well-identified parameters. Finally, the proposed methodology is applied and evaluated on a front suspension of an electric vehicle.

DYNAMIC MODELLING FOR PARAMETER IDENTIFICATION

An important characteristic of the dynamic model of a mechanical system is its linearity with respect to inertial parameters. This linearity depends on the location of the reference systems. In order to obtain a dynamic model that depends linearly on all inertial parameters, it is necessary to use local reference systems outside the mass-centre of each element.

For a mechanical system composed of a link *i* and modelled by a set of independent generalised coordinates $\vec{q}$, and applying the virtual work principle, the dynamic model of a low mobility mechanism can be expressed as,

$$-\vec{Q}_{in_i} - \vec{Q}_{g_i} = \vec{Q}_{ex_i} \tag{1}$$

where $\vec{Q}_{in_i}, \vec{Q}_{ex_i}, \vec{Q}_{g_i}$ are inertial, external and gravitational generalised forces.

If the local reference systems of the links are located outside their centre of mass, the dynamic model will depend linearly on the inertial parameters and the dynamic model in Eq. (1) can be written as a linear model where K_i is a coefficient matrix and the vector $\vec{\Phi}_i$ is the inertial parameter vector,

$$\vec{Q}_{ex,i} = K_i \cdot \vec{\Phi}_i \tag{2}$$

The matrix K_i is obtained as,

$$K_i = \left[\underbrace{\left(\frac{\partial \vec{v}_{A_i}}{\partial \dot{\vec{q}}}\right)^T \cdot \left(\vec{a}_{A_i} + \vec{g}\right)}_{6\times 1} \;\middle|\; \underbrace{\left(\left(\frac{\partial \vec{v}_{A_i}}{\partial \dot{\vec{q}}}\right)^T \cdot (\tilde{\omega}_i \cdot \tilde{\omega}_i + \tilde{\alpha}_i) - \overline{\left(\vec{a}_{A_i} + \vec{g}\right)}\right) \cdot {}^0R_i}_{6\times 3} \;\middle|\; \ldots \right. \tag{3}$$

$$\left. \ldots \;\middle|\; \underbrace{\left(\frac{\partial \vec{\omega}_i}{\partial \dot{\vec{q}}}\right)^T \cdot {}^0R_i \cdot \left(\widehat{{}^i\alpha_i} + \widetilde{{}^i\omega_i} \cdot \widehat{{}^i\omega_i}\right)}_{6\times 6} \right]_{6\times 10}$$

and $\vec{\Phi}_i = \begin{bmatrix} m_i & m_i x_{Gi} & m_i x_{Gi} & m_i x_{Gi} & J_{xxi} & J_{xyi} & J_{xzi} & J_{yyi} & J_{yzi} & J_{zzi} \end{bmatrix}$.

In Eq. (3) $\vec{v}_{A_i}$ and $\vec{a}_{A_i}$ are the velocity and acceleration of the local reference, $\vec{\omega}_i$ y $\vec{\alpha}_i$ are the angular velocity and angular acceleration on link *i*. The other vectors are defined as,

$$\tilde{\omega}_i = \begin{bmatrix} 0 & -\omega_{z_i} & \omega_{y_i} \\ \omega_{z_i} & 0 & -\omega_{x_i} \\ -\omega_{y_i} & \omega_{x_i} & 0 \end{bmatrix}, \tilde{\alpha}_i = \begin{bmatrix} 0 & -\alpha_{z_i} & \alpha_{y_i} \\ \alpha_{z_i} & 0 & -\alpha_{x_i} \\ -\alpha_{y_i} & \alpha_{x_i} & 0 \end{bmatrix}, \bar{\bar{a}}_{A_i} = \left(\frac{\partial \vec{\omega}_i}{\partial \dot{\vec{q}}} \times \vec{a}_{A_i}\right) = \begin{bmatrix} 0_{3\times 3} \\ -\left(\tilde{a}_{A_i} \cdot G_i\right)^T \end{bmatrix},$$

$$\hat{\alpha}_i = \begin{bmatrix} \alpha_{x_i} & \alpha_{y_i} & \alpha_{z_i} & 0 & 0 & 0 \\ 0 & \alpha_{x_i} & 0 & \alpha_{y_i} & \alpha_{z_i} & 0 \\ 0 & 0 & \alpha_{x_i} & 0 & \alpha_{y_i} & \alpha_{z_i} \end{bmatrix} \text{ and } \hat{\omega}_i = \begin{bmatrix} \omega_{x_i} & \omega_{y_i} & \omega_{z_i} & 0 & 0 & 0 \\ 0 & \omega_{x_i} & 0 & \omega_{y_i} & \omega_{z_i} & 0 \\ 0 & 0 & \omega_{x_i} & 0 & \omega_{y_i} & \omega_{z_i} \end{bmatrix}$$

In order to complete a mechanism with closed kinematic chains, the restriction forces must be added for generating a dynamic model whose equation can be written as,

$$K \cdot \vec{\Phi} = \vec{Q}_{ex} - \left(C_q\right)^T \cdot \vec{\lambda} \tag{4}$$

where $\left(C_q\right)^T \cdot \vec{\lambda}$ corresponds internal generalised forces, C_q is the Jacobian matrix and $\vec{\lambda}$ is a vector that includes Lagrange multipliers.

It is not easy to obtain information about the internal forces that appear in the joints during the movement of the mechanism; hence it is convenient to eliminate those internal forces in Eq. (4), by applying any of the procedures proposed in the field of Multibody Dynamics. These procedures can be classified into two large groups: partition of coordinates and admissible movement subspace. In the latter, it is considered that the internal forces are orthogonal to the admissible movements in the kinematic joint so that an orthogonal complement or velocity projection matrix will be obtained. From Eq. (4), it is about obtaining a matrix R^* that verifies,

$$\dot{\vec{q}} = R^* \cdot \dot{\vec{q}}^i \tag{5}$$

where $\dot{\vec{q}}^i$ is a vector of independent velocities. The matrix R^* is characterised by,

$$C_q \cdot R^* = 0 \tag{6}$$

which serves to eliminate the internal forces in Eq. (4).
Multiplying both terms in Eq. (4) by R^*,

$$R^{*T} \cdot K \cdot \vec{\Phi} = R^{*T} \cdot \vec{Q}_{ex} - R^{*T} \cdot C_q{}^T \cdot \vec{\lambda} \tag{7}$$

What it is,

$$R^{*T} \cdot K \cdot \vec{\Phi} = R^{*T} \cdot \vec{Q}_{ex} \tag{8}$$

Since Eq. (8) has as many scalar equations as degrees of freedom. Equations corresponding to different positions of the mechanical system must be included to obtain a numerically over-determined system. The obtained matrix is called the observation matrix of the system and the dynamic model that is expressed in linear form with respect to the parameters:

$$W_{n\times m} \cdot \vec{\Phi}_{m\times 1} = \vec{\tau}_{n\times 1} \tag{9}$$

where m is the number of dynamic parameters and n is the number of configurations multiplied by the degrees of freedom of the mechanical system.

STANDARD PROCEDURES FOR DETERMINING BASE PARAMETERS

Due to the linearity of the inertial parameters, it is possible to perform the estimation of the parameter vector using numerical methods as the least mean square method LMS. The requirement for using LMS is to have an observation matrix W with complex rank. That is not met from Eq. (9) because some inertial parameters do not contribute to generalised forces, while other ones do as a linear combination. For it, numeric and symbolic methods are used to obtain the expressions of sets of base parameters.

Among the main referenced numerical methods in the identification processes are the decomposition in singular values (SVD) and QR decomposition [5]. Calafiore et al. [6], Farhat et al.[7] and Diaz-Rodriguez et al [8] used SVD to perform parameter estimation in serial robots and Parallel. Guegan et al. [9] realised identification of parameters on an Orthoglide robot through the QR Decomposition. In the field of mechanisms, Chen and Beale [10] used the SVD decomposition in the determination of the base parameters of a vehicle suspension and Venture et al. [11] applied QR decomposition in the identification of dynamic parameters of a commercial vehicle.

In the symbolic methods for determining base parameters, there is an iterative procedure that has been derived from the energy analysis. Several authors [12],[13] used it in identification processes of simple series robots, while Khalil [14] applied it in robots with a branched structure. Other authors as Mayeda et al. [15] grouped certain inertial parameters of the same element (predetermined grouping) according to the direction of the kinematic joint of the element in the topology of the manipulator.

In closed kinematic chains, the linear dependencies of parameters by simple inspection of the topology do not constitute a trivial task. In parallel robots, the work of Bennis et al. [16] presents a method to determine the symbolic base parameters through the analysis of the energy the system. Another method that can be applied to closed kinematic chains is developed by Iriarte [17] and Diaz-Rodriguez [18]; the method is

based on the dimensional analysis of the values provided by the numerical method and has been tested and validated for planar mechanisms [19], not so for spatial mechanisms.

An efficient method to obtain the symbolic relationships between parameters is the transfer of inertial properties. The concept of mass and inertia transfer is based on the virtual redistribution of the inertial properties of the elements in the mechanical system whenever the Lagrangian of the system only changes in a constant way. Chen et al. [20] present the development of this method in planar mechanisms and subsequently perform an approach to spatial mechanisms [21]. The weakness of the method lies in the limitation of transferring only the masses, possible only in some kinematic joints (revolute, spherical and universal). This difficulty is overcome by Ros et al. [22], who uses the so-called monopoles, dipoles and quadrupoles.

Regardless of the method applied to obtain a set of base parameters, a model constituted by an over-determined linear system is determined, in which the transmissibility of the errors in the data to the solution depends on the condition number, CN, of the W_{base} [23].

DRAWBACKS IN THE APPLICATION OF THE STANDARD PROCEDURE TO LOW MOBILITY MECHANISMS (LMM)

The described above standard numerical procedures for calculating the base parameters have several drawbacks when they are applied to low mobility systems, raising doubts about their use. Below, those drawbacks in standard identification procedures are presented.

Uncertainty in Determining the Rank

The first drawback observed after the application of a numerical procedure, such as SVD and QR decompositions, is the difficulty in determining the rank of the observation matrix; a magnitude that is usually determined from the analysis of the singular values of the matrix.

High Condition Number

If the condition number κ of the observation matrix W_{base} is very large, no matter how small the measurement error is, the deviation in the estimation of the parameters will be high. In open chain systems, low condition numbers in the observation matrices are assured, using trajectories that excite many dynamic parameters. However, in closed chain mechanisms, characterised for having a low mobility, due to the kinematic restrictions imposed, this condition is far from being guaranteed. Though in closed chains the optimisation of trajectories improves the conditioning, it does not allow obtaining sufficiently low values to carry out a process of parameter identification. Valero *et al.* [24] and Mejía et al. [25] report condition number values of $1.0 \cdot 10^{+11}$ y $5.7 \cdot 10^{+10}$ respectively, both for studies on vehicle suspensions with trajectory optimisation.

High Standard Deviation on Identified Parameters

Due to the persistence of the high conditioning of the W_{base} even after the optimisation, the parameters were not well calculated. This requires a further reduction of the dynamic model in the search for a considerable reduction in the conditioning of the observation

matrix. Different authors have already made these reductions mostly eliminating parameters of high standard deviation [18], [26], [27], [28] or with physically unfeasible values (e.g. negative mass values)[18], [29]. The new set of reduced parameters is called relevant parameters [30]. Khalil and Dombre [27] propose to reduce the model by iteratively eliminating the base parameters with relative deviations greater than 10 minimum deviations. However, in closed chain mechanisms, some authors [8], [30], [31] report high standard deviations of many parameters well above the criterion posed in [27].

SOLUTIONS IN THE IDENTIFICATION OF PARAMETERS IN LOW MOBILITY MECHANISMS (LMM)

In order to find a solution to the described drawbacks, it proposes a series of actions such as:

i. to obtain the rank of the observation matrix from the analysis of the linear dependencies obtained in the symbolic expressions, which are determined by the inertial transfer.
ii. to generate sets of base parameters from inertial transfers.
iii. to obtain relevant parameters from reductions based on the dynamic contribution of each identified parameter.

Determination of the Symbolic Expressions of Base Parameters through the Transfer of Inertial Properties

A general symbolic method that can be applied in closed chains and low mobility systems is the mass transfer concept proposed by Ros et al. [22]. This concept is based on the premise that the dynamics of a mechanical system will not be modified by the virtual mass transfers between rigid bodies when the variation of the Lagrangian is constant with respect to time. The concepts of monopoles, dipoles, quadrupoles and their transfer conditions are used for obtaining a set of base parameters.

The inertial parameters transferred between solids correspond to the linearly combined parameters. In order to obtain the exact rank, it is necessary to transfer as many parameters as possible in each kinematic joint. After making the transfers, the vector of independent parameters (they appear alone in each base parameter), the vector of dependent parameters, and the matrix that relates them is obtained,

$$\vec{\Phi}_{base} = \vec{\Phi}_1 + \beta \cdot \vec{\Phi}_2 \tag{10}$$

Obtaining Other Sets of Symbolic Base Parameters Derived from a Transfer of inertial parameters

From a set of expressions of symbolic base parameters, it is possible to determine different sets of parameters that also represent a valid set. The denomination of valid refers to a set of maximum rank, rank calculated from the symbolic expressions. Starting from Eq. (10), the matrix B formed for the identity matrix I and the matrix β, which contains the dependency relations for the initial inertia property transfer.

$$\vec{\Phi}_{base} = \vec{\Phi}_1 + \beta \cdot \vec{\Phi}_2 = \underbrace{\begin{bmatrix} I_{r\times r} & \beta_{r\times(m-r)} \end{bmatrix}}_{B} \cdot \begin{bmatrix} \vec{\Phi}_1 \\ \vec{\Phi}_2 \end{bmatrix} \tag{11}$$

For this transfer, the identity matrix is associated with the vector of independent parameters $\vec{\Phi}_1$. However, in *B* it is also possible to find other submatrices of maximum rank. It is then a matter of finding combinations of *r* columns of *B* that generate matrices of maximum rank so that their associated parameters correspond to a new independent vector. In this way, multiple sets of base parameters can be found without performing the inertial transfer procedure. In total there will be $m!/(m-r)!r!$ possible combinations. The more bodies the mechanical system possesses, the more computationally expensive will be the evaluation of all possible combinations. However, it is easier to determine if a combination of parameters corresponds to a set of valid base parameters, calculating the rank of the columns of *B* associated with these parameters. Analysing the mechanical system, a series of possible transfers of inertial parameters can be selected a priori, considering several convergence solids, so that the combinations of parameters associated with these transfers are evaluated exclusively. Once guaranteed that the combination of parameters is valid, a new vector is generated $\vec{\Phi}' = \begin{bmatrix} \vec{\Phi}_1' \\ \vec{\Phi}_2' \end{bmatrix}$, where $\vec{\Phi}_1'$ corresponds with the independent parameters for that combination. The columns of the matrix B and the parameter vector are exchanged in the same order through the matrix *P* so that

$$\vec{\Phi}_B = B \cdot \begin{bmatrix} \vec{\Phi}_1 \\ \vec{\Phi}_2 \end{bmatrix} = \left(B \cdot P^T\right)\left(P \cdot \begin{bmatrix} \vec{\Phi}_1 \\ \vec{\Phi}_2 \end{bmatrix}\right) = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \cdot \begin{bmatrix} \vec{\Phi}_1' \\ \vec{\Phi}_2' \end{bmatrix} \quad (12).$$

The partition of B is done so that B_1 is square. If B_1 is also full rank then the selection of $\vec{\Phi}_1'$ an independent parameter vector is valid. Pre-multiplying both sides of the equation by B_1^{-1}, the new base parameters are obtained,

$$\vec{\Phi}_B' = B_1^{-1} \cdot \vec{\Phi}_B = B_1^{-1} \cdot \begin{bmatrix} B_1 & B_2 \end{bmatrix} \cdot \begin{bmatrix} \vec{\Phi}_1' \\ \vec{\Phi}_2' \end{bmatrix} = \begin{bmatrix} I & B_1^{-1} \cdot B_2 \end{bmatrix} \cdot \begin{bmatrix} \vec{\Phi}_1' \\ \vec{\Phi}_2' \end{bmatrix} = B' \cdot \begin{bmatrix} \vec{\Phi}_1' \\ \vec{\Phi}_2' \end{bmatrix} \quad (13)$$

from which it follows that $\beta' = B_1^{-1} \cdot B_2$.

Model for Identification

The existing relationship between physical parameters $\vec{\Phi}$ and the partition of dependent and independent parameters for a specific transfer is given through a single permutation matrix,

$$P^T \cdot \vec{\Phi} = \begin{bmatrix} \vec{\Phi}_1 \\ \vec{\Phi}_2 \end{bmatrix} \quad (14)$$

Using Eq. (14), the model for identification can be written as,

$$W \cdot \vec{\Phi} = W \cdot P \cdot P^T \cdot \vec{\Phi} = \begin{bmatrix} W1 & W2 \end{bmatrix} \cdot \begin{bmatrix} \vec{\Phi}_1 \\ \vec{\Phi}_2 \end{bmatrix} = \begin{bmatrix} W1 & W1 \cdot \beta \end{bmatrix} \cdot \begin{bmatrix} \vec{\Phi}_1 \\ \vec{\Phi}_2 \end{bmatrix} \quad (15)$$

Thus,

$$W \cdot \vec{\Phi} = W1 \cdot \left[\vec{\Phi}_1 \quad \beta \cdot \vec{\Phi}_2 \right] = W_{base} \cdot \vec{\Phi}_{base} \tag{16}$$

where

$$W_{base} = W1 \tag{17}$$

Obtaining Relevant Parameters

The high conditioning in the W_{base} observation matrix in LMM does not allow obtaining well-identified base parameters. The decrease of the condition number in W_{base} is possible if some columns of the matrix are eliminated. To obtain this new set of parameters denominated relevant (the parameters not eliminated), a series of criteria is required to determine which and how many base parameters were eliminated. Due to the high conditioning of the observation matrix, the standard deviations were poorly calculated and do have a real meaning of the quality of the identified parameters. An alternative criterion is based on the elimination of those base parameters with few contributions in the generalised forces [8]. These contributions are determined through the dynamic contribution index of each parameter. Some authors [18], [32], [33] have used the dynamic contribution to determine which parameters are identifiable while others [24], [25] have used it to reduce the model. However, in low mobility systems, this index cannot be applied directly. To solve this problem, the dynamic contribution index is modified so that it is independent of the errors in the measured forces. For this, the symbolic expressions of the base parameters determined in the previous section must be used and evaluated with the values of an initial estimate from a CAD model. With these considerations, the dynamic contribution index in LMM is determined as,

$$\varsigma_i = \frac{\sqrt{\left(W_{base}(:,i) \cdot \Phi_{base\_CAD}(i)\right)^T \cdot \left(W_{base}(:,i) \cdot \Phi_{base\_CAD}(i)\right)}}{\sqrt{\vec{\tau}^T \cdot \vec{\tau}}} \cdot 100 \tag{18}$$

Once the parameter arrangement according to the dynamic contribution indices is determined, the base parameter of the lowest index is eliminated along with the W_{base} column associated with that parameter. If the condition number of the resulting $W_{relevant}$ matrix is low, the least-squares method is applied to determine the estimated value of the relevant parameters. If the conditioning of the observation matrix is not low enough, the elimination process is repeated for the next parameter with low index ζ. The flow diagram of the complete identification process for LMM is presented in Figure 1.

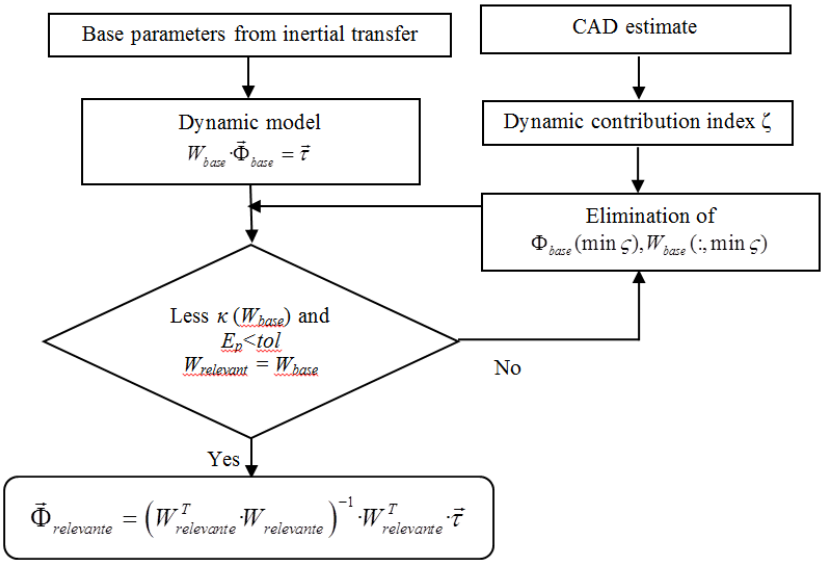


Figure 1. Flow chart of the identification process of parameters of LMM.

APPLICATION OF THE METHOD OF IDENTIFICATION OF DYNAMIC PARAMETERS TO A MECHANISM OF LOW MOBILITY. VEHICLE SUSPENSION

In this section, the procedure for identifying parameters is applied to an automotive front suspension system, composed of two articulated quadrilateral mechanisms (on the left and right wheels). Each side of the suspension mechanism, represented in Figure 2, is made up of: upright (3), lower (1) and upper (2) control arms, spring-damper module (6-7), wheel (5), steering rod (4), and a steering rack (8) that joins both sides of the suspension. Each kinematic joint is indicated by a letter, denoted by (R) revolute, (P) prismatic, (U) universal and (S) spherical joints.

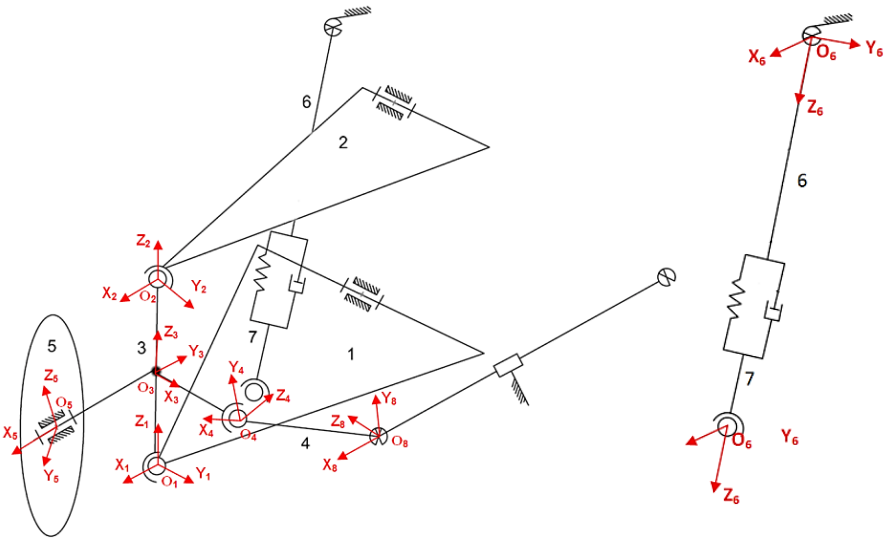


Figure 2. Kinematic diagram of the suspension with the location of the local reference systems.

The degrees of freedom of the mechanism corresponds to the vertical and angular movements of the wheels, and the lateral movement of the steering rod. To simulate conditions of the real operation, the measured forces are given errors associated with the measurement system itself.

Base Parameters

The mass transfer methodology requires the kinematic chain to be opened or branched, so it is necessary to cut through some kinematic joints. Figure 3(a) shows the cuts made in the revolute joints between control arms and chassis. After these cuts, there result in four branches around each wheel, Figure 3(b). For each branch, different mass or portion of the mass are transferred from the cutoffs and the ends of the mechanism to one or more solids, called convergence solids. For this first model, T1 model, the uprights are convergence solids.

The first branch starts from the upper control arm, attached to the mechanism through two kinematic joints (A_1 and D_1 in Figure 3(b)). In the spherical joint (D_1) only a single monopole can be transferred, while for the revolute joint (A_1) various types of poles can be transferred. Of these possible transfers, the quadrupole is chosen so that the second moment of inertia with respect to the axis of the joint (Iyy_2) is null after the transfer. To that end, part of the mass of the upper arm (m_{2A}) is removed and added to that of the chassis at the point A_1,

$$Iyy_2^{'} = Iyy_2 - m_{2A}\left(Ds_4^2 + z_{O_2A_1}^2\right) = 0 \tag{19}$$

where Ds_4, Ds_3 and $z_{O_2A_1}$ are the distances as presented in Figure 4. The value of m_{2A} that eliminates the parameter $Iyy_2^{'}$ in Eq. (19) is,

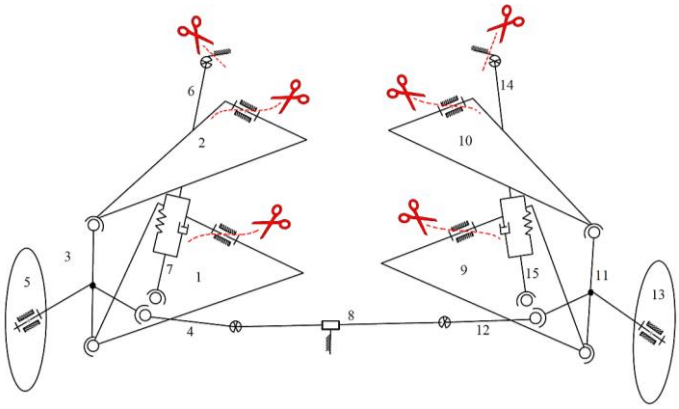
$$m_{2A} = \frac{Iyy_2}{Ds_4^2 + z_{O_2A_1}^2} \tag{20}$$

for $z_{O_2A_1} = 0$. Thus, the new parameters of the upper control arm are

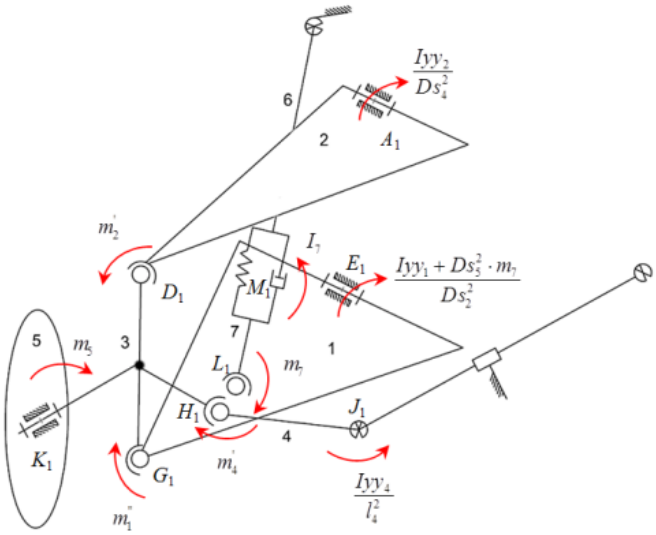
$$m_2^{'} = m_2 + m_{2A} = m_2 + \frac{Iyy_2}{Ds_4^2} \tag{21a}$$

$$mG_2^{'} = mG_2 - m_{2A}\begin{bmatrix} Ds_4 \\ Ds_3 \\ 0 \end{bmatrix} = \begin{bmatrix} mx_2 - \dfrac{Iyy_2}{Ds_4} & my_2 - \dfrac{Iyy_2 \cdot Ds_3}{Ds_4^2} & mz_2 \end{bmatrix}^T \tag{21b}$$

$$I_2^{'} = \begin{bmatrix} Ixx_2 - \dfrac{Iyy_2}{Ds_4^2}Ds_3^2 & Ixy_2 - \dfrac{Iyy_2}{Ds_4}\cdot Ds_3 & Ixz_2 & 0 & Iyz_2 & Izz_2 - \dfrac{Iyy_2\left(Ds_4^2 + Ds_3^2\right)}{Ds_4^2} \end{bmatrix} \tag{21c}$$



(a)



(b)

Figure 3. Branched-chain for the analysed suspension

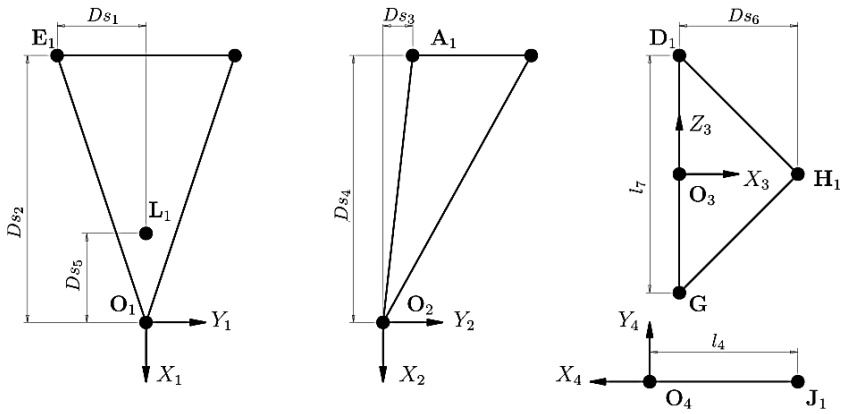


Figure 4. Dimensions of the links.

It is noted that Iyy_2 now appears as a combination in the new set of base parameters. Similarly, transferring the new total mass m_2' of the upper control arm to the upright 3 in the spherical joint, and given the fact that the origin of the local reference

frame of the link 2 coincides with the position of the kinematic connection ($x_{O_2D_1} = y_{O_2D_1} = 0$), monopole transfer does not affect the other inertial parameters of the body 2. After this transfer, the mass of the upright 3 depends on the mass and the second moment of inertia of the upper control arm 2,

$$m_2^{''} = 0, \;\; mG_2^{''} = mG_2^{'}, \;\; I_2^{''} = I_2^{'} \tag{22}$$

$$m_3^{'} = m_3 + m_2^{'} = m_3 + m_2 + \frac{Iyy_2}{Ds_4^2} \tag{23a}$$

$$mG_3^{'} = \begin{bmatrix} mx_3 & my_3 & mz_3 + m_2 \cdot \frac{l_7}{2} + \frac{Iyy_2 \cdot l_7}{2 \cdot Ds_4^2} \end{bmatrix}^T \tag{23b}$$

$$I_3^{'} = \begin{bmatrix} Ixx_3 + m_2 \cdot \frac{l_7^2}{4} + \frac{Iyy_2 \cdot l_7^2}{4 \cdot Ds_4^2} & Ixy_3 & Ixz_3 & Iyy_3 + \frac{m_2 l_7^2}{4} + \frac{Iyy_2 \cdot l_7^2}{4 \cdot Ds_4^2} & Iyz_3 & Izz_3 \end{bmatrix} \tag{23c}$$

From the branch of the steering rod (solid 4) two parameters can also be transferred, the first to the steering rack (solid 8) and the second to the upright. A monopole $m_{4J1}=Iyy_4/l_4^2$ corresponding to part of the mass of the tie rod is transferred to the link 8 in the universal joint J_1, to eliminate the second moment of inertia Iyy_4'. After this transfer the parameter is Iyy_4 linearly combined. Continuing along the same branch, now the new mass of the tie rod 4 is transferred to the upright 3. Given the location of the local reference of link 4, neither the first nor the second moments of inertia are modified.

In the joint K_1, connecting the wheel 5 with the upright 3, it can be transferred a monopole, a dipole in the direction of revolution, and a quadrupole in the same direction. So, the new base parameters for the wheel are

$$m_5^{'} = 0 \tag{24a}$$

$$mG_5^{'} = \begin{bmatrix} 0 & my_5 & mz_5 \end{bmatrix}^T \tag{24b}$$

$$I_5^{'} = \begin{bmatrix} Ixx_5 & Ixy_5 & Ixz_5 & Iyy_5 - Izz_5 & Iyz_5 & 0 \end{bmatrix} \tag{24c}$$

and the new base parameters for the upright are

$$m_3^{'''} = m_3 + m_2 + \frac{Iyy_2}{Ds_4^2} + m_4 - \frac{Iyy_4}{l_4^2} + m_5 \tag{25a}$$

$$mG_3^{'''} = \begin{bmatrix} mx_3 + m_4 \cdot Ds_6 - \frac{Iyy_4 \cdot Ds_6}{l_4^2} - m_5 \cdot DKx & my_3 - mx_5 - m_5 \cdot DKy & mz_3 + m_5 \cdot \left(\frac{l_7}{2} - DKz \right) \end{bmatrix}^T \tag{25b}$$

$$Ixy_3^{'''} = Ixy_3 - m_5 \cdot DKx \cdot DKy - mx_5 \cdot DKx, \tag{25c}$$

$$Ixx_3^{'''} = Ixx_3 + m_2 \cdot \frac{l_7^2}{4} + \frac{Iyy_2 \cdot l_7^2}{4 \cdot Ds_4^2} - 2mx_5 \cdot DKx + m_5 \left(DKy^2 + \left(\frac{l_7}{2} - DKz \right)^2 \right) + Izz_5,$$

$$Ixz_3^{'''} = Ixz_3 + m_5 \cdot DKx \cdot \left(\frac{l_7}{2} - DKz \right),$$

Moreover, because the origin of the local reference system of the lower strut 7 coincides with the spherical joint L_1, the transfer of its total mass to the lower control arm does not modify its other parameters. Additionally, its full inertia tensors may be combined with those of the upper strut, because they are coupled through a prismatic joint.

With respect to the lower control arm, two parameters can also be transferred. The first can be transferred to a monopole, a dipole and a quadrupole between lower control arm and chassis in E_1. However, the transfer of dipole and monopole eliminates parameters (Izz_1, my_1) not appearing in the dynamic equations, so that only a monopole with part of the mass of the lower arm is transferred to eliminate $Iyy_1^{''}$. In the second transfer, the new mass of lower arm $m_1^{''}$ is translated to the upright 3 in point G_1. After all transfers the complete set of base parameters is obtained.

The terms of all symbolic base parameters obtained can be regrouped in the form of Eq. (11),

$\vec{\Phi}_{base} = \vec{\Phi}_1 + \beta \cdot \vec{\Phi}_2 = \underbrace{\begin{bmatrix} I_{r\times r} & \beta_{r\times(m-r)} \end{bmatrix}}_{B} \cdot \begin{bmatrix} \vec{\Phi}_1 \\ \vec{\Phi}_2 \end{bmatrix}$ where B is the matrix containing an Identity Matrix associated with the independent parameters, at the time that β is the matrix containing the dependence relations between parameters.

Other valid models are obtained considering other convergence solids: Model T2 considers as convergence solids the lower control arms (1 and 9), which are the closest to the chassis. Model T3 takes as convergence solids the lower and upper control arms (1,2,8,9). In the model T4, one combination is selected randomly from a valid set. Finally, the T5 model is formed by taking the 32 dependent parameters randomly. For this model, the range (calculated numerically) of the observation matrix does not correspond to the range calculated symbolically by inertial property transfer models. So, this is the only model that is not valid.

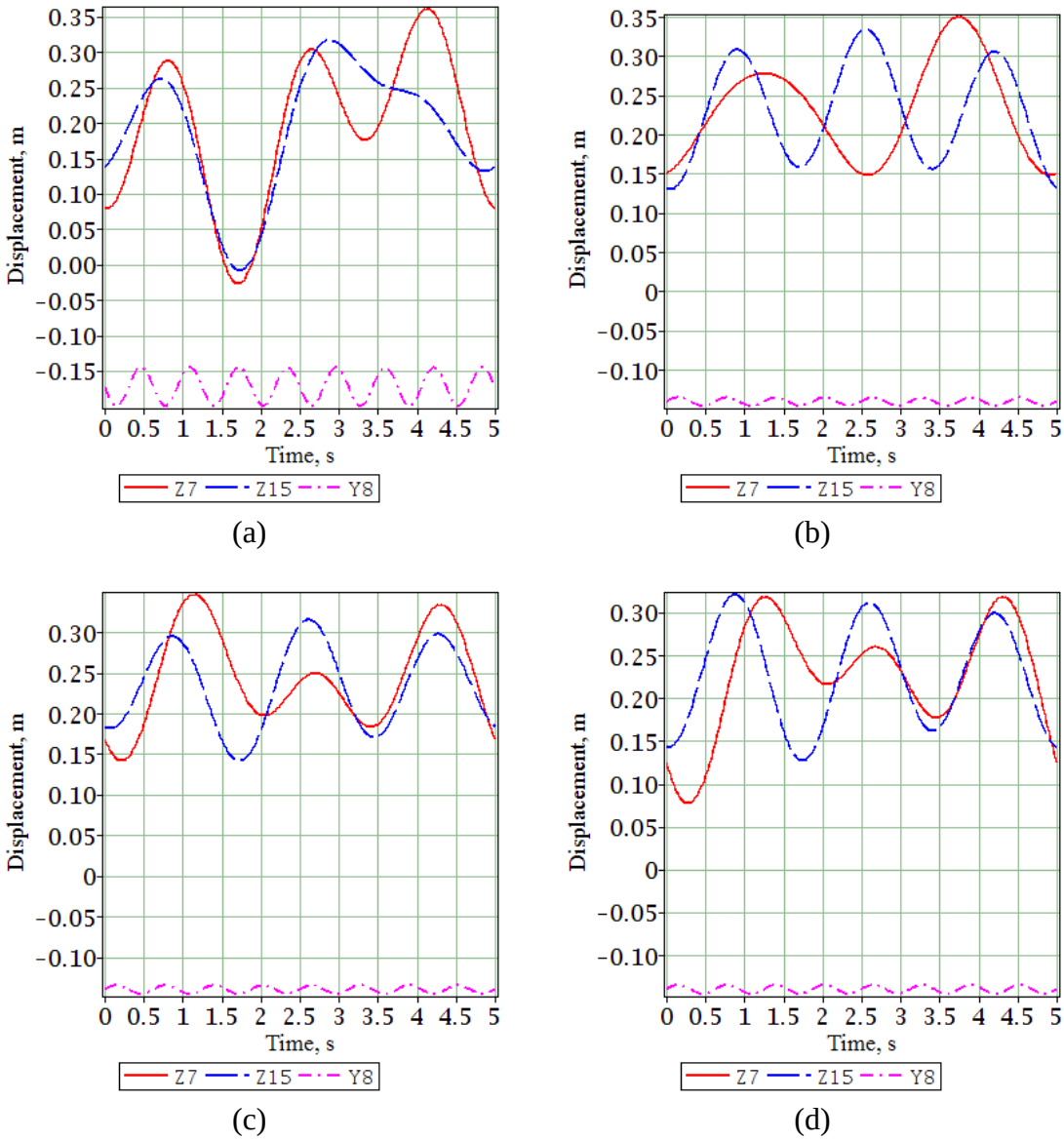
RESULTS AND DISCUSSION

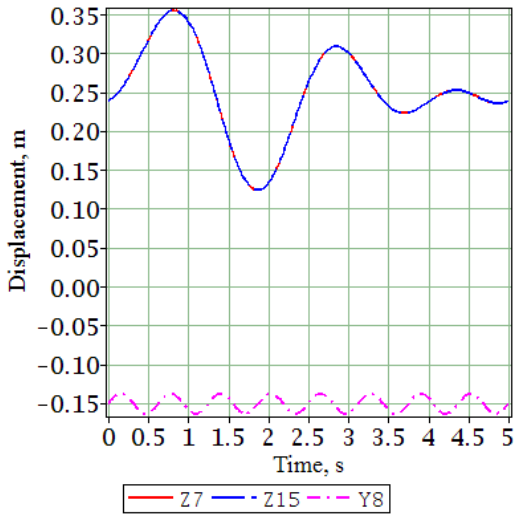
In Figure 5, the trajectories followed by the degrees of freedom of the mechanism are presented. These trajectories are obtained minimising the condition number in each model. The coordinates that control the rotation of both wheels ϕ_5 and ϕ_{13} are $\phi = 50 \cdot \sin(f \cdot \pi \cdot t)$. For these trajectories, the condition numbers reached by the W_{base} observation matrices of each model are found and presented in Table 1.

Table 1. Condition number of observation matrices.

Model	T1	T2	T3	T4	T5
κ	$1.13 \times 10^{+12}$	$1.11 \times 10^{+14}$	$9.55 \times 10^{+13}$	$1.42 \times 10^{+14}$	$1.21 \times 10^{+19}$

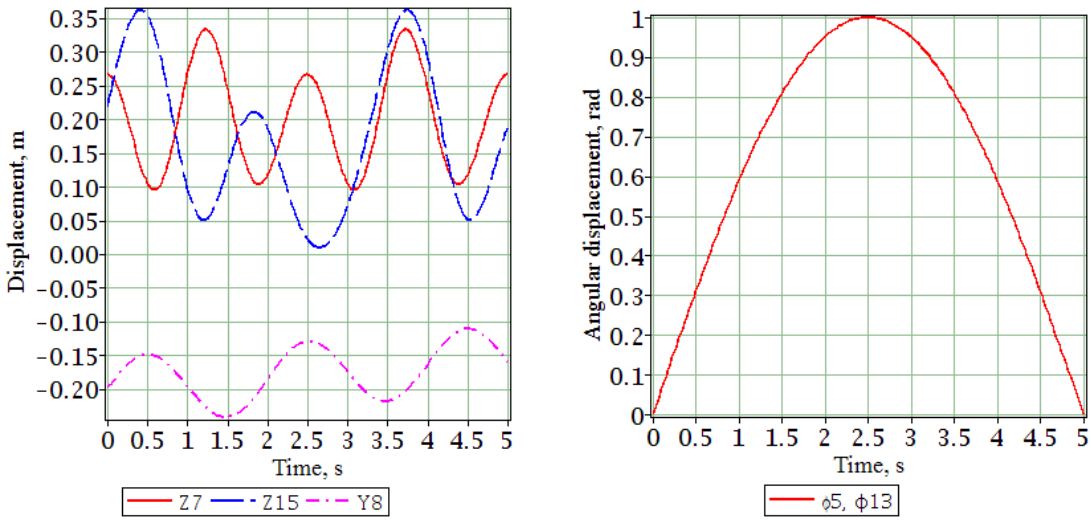
The numerical conditioning for the T5 model is several orders of magnitude greater than those obtained by transfer of inertial properties. Thus, despite the trajectory optimisations, the condition numbers of the observation matrices for all the models are still very high and, consequently, the identified parameters result very badly estimated when the forces used in the identification process include measurement errors. In order to verify this fact, a validation trajectory is considered that serves to validate the results obtained with the models T1-T5. The validation trajectory VT is presented in Figure 6.





(e)

Figure 5. Optimised trajectories of the observation matrix models: (a) T1, (b) T2, (c) T3, (d) T4 y (e) T5



(a) (b)

Figure 6. Validation trajectory (a) vertical displacements of the wheels and sides of the steering (b) turn of the wheels.

If the vector of generalised identification forces $\vec{\tau}$ used in the identification of the base parameters were free of errors, the model in base parameters adequately obtained would predict the generalised forces on other trajectories, regardless of the numerical conditioning of the observation matrix. When the parameters are identified in the presence of errors in the forces, which is inherent to the measurement process, the model in base parameters is not able to adequately reproduce the generalised forces on other trajectories.

From the above said, it follows that to predict the generalised forces under real conditions for any trajectory with low error, it is necessary to obtain a model with numerical conditioning much lower than that available in base parameters. By eliminating columns from the observation matrix and therefore eliminating base parameters, the

conditioning of the model is improved. The results of applying the reduction to the model in base parameters are presented below.

Reduction of the Model based on the Dynamic Contribution

The criterion proposed in this paper is the dynamic contribution index ζ, Eq. (18), by means of which the ordering used to eliminate base parameters can be determined. The least contributory parameters are eliminated from the set of base parameters until reaching the numerical conditioning levels of the W_{base} matrix that guarantee the identification process. Table 2 shows that with a 5% error in external forces, the values of ten of the base parameters identified for model T1 change greatly with respect to the theoretical base parameters determined from a CAD approximation.

Table 2. Variation of the base parameters identified with the addition of errors in the identification forces for model T1.

Φ_{base}	Theoretical CAD	Identified, error 5%	Φ_{base}	Theoretical CAD	Identified, error 5%
1	$-3.4\text{x}10^{-1}$	$-3.5\text{x}10^{+4}$	2	0	$-3.8\text{x}10^{+1}$
3	$-1.6\text{x}10^{-1}$	$4.9\text{x}10^{+1}$	4	0	$4.5\text{x}10^{+1}$
5	$3.5\text{x}10^{+1}$	$1.1\text{x}10^{+5}$	6	$-2.8\text{x}10^{-1}$	$-1.2\text{x}10^{0}$
7	$-1.4\text{x}10^{-1}$	$-2.4\text{x}10^{-1}$	8	$-1.4\text{x}10^{0}$	$-2.4\text{x}10^{+4}$
9	$5.9\text{x}10^{-1}$	$5.1\text{x}10^{+4}$	10	$-6.06\text{x}10^{-3}$	-0.12

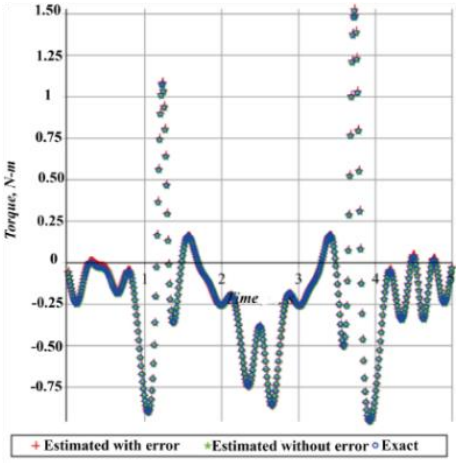
*Each parameter has units corresponding to kg, kg • m and kg • m^2.

Figure 7 shows the generalised forces predicted by the model T1, for the validation trajectory, when the identification was made without errors and with errors by 5%. It is seen how in the presence of errors of the identification forces, the estimated generalised force associated with the right vertical movement differs greatly from the theoretical generalised force, Figure 7(b). The estimated forces without error and the exact forces overlap in this figure, so that no differences are observed between them.

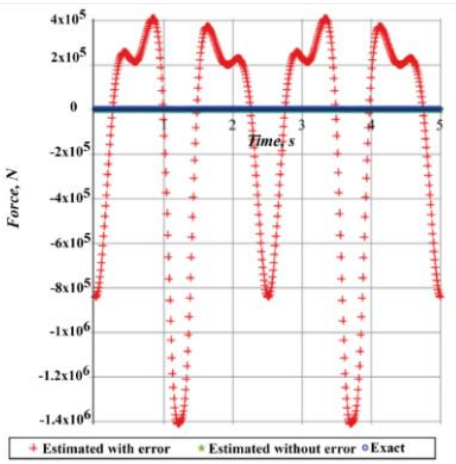
The increase in CAD estimation error did not generate an important difference in the order established as a criterion for reducing the models. This fact allows the use of high uncertainties in the approximation of the theoretical parameters, so that an insufficient refined CAD model will be acceptable to calculate the dynamic contribution indices.

Reduction to Relevant Parameters

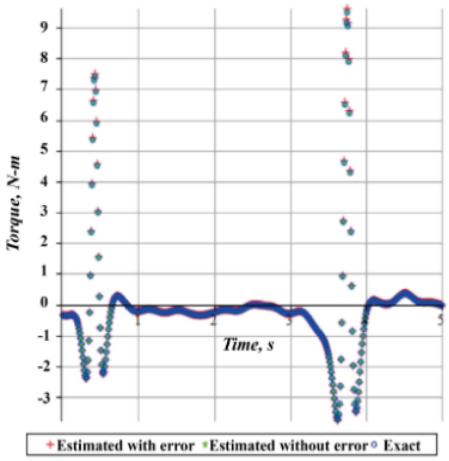
Following the methodology presented in the flow chart of the identification process of parameters of LMM, Figure 1, with the dynamic contribution indices of each base parameter an array is constructed with the parameters ordered from highest to lowest index in each model (T1 to T5). The parameter with the lowest index ζ is eliminated from the base parameter vector and the column of the W_{base} matrix associated with this parameter is also eliminated.



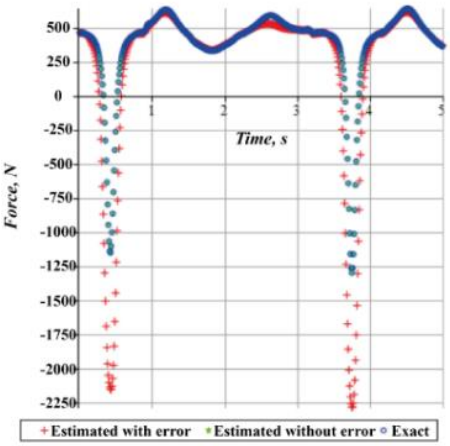
(a)



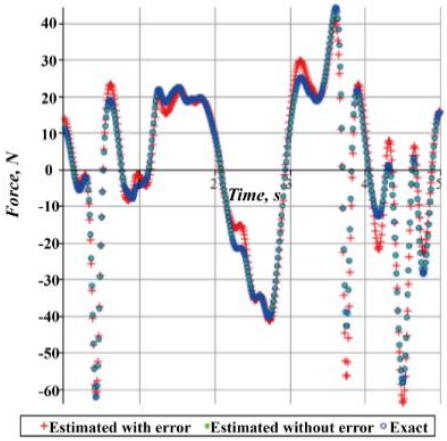
(b)



(c)



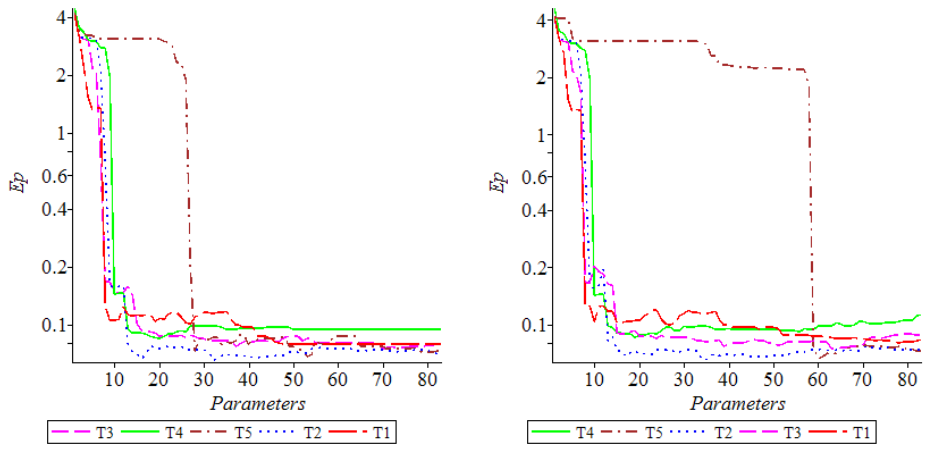
(d)



(e)

Figure 7. Generalised estimated and theoretical forces. (a) right wheel rotation, (b) right vertical movement of the lower strut, (c) left wheel rotation, (d) left vertical movement of the lower strut, (e) steering.

In Figure 8 the E_p errors of the models for two CAD estimation levels are presented. In models T1-T5, a limit of relevant parameters is observed, from which the prediction error remains nominally invariant.



(a) (b)

Figure 8. Error E_p. Forces used in identification with 5% standard deviation. (a) 0% error CAD estimate (b) 60% error CAD estimate.

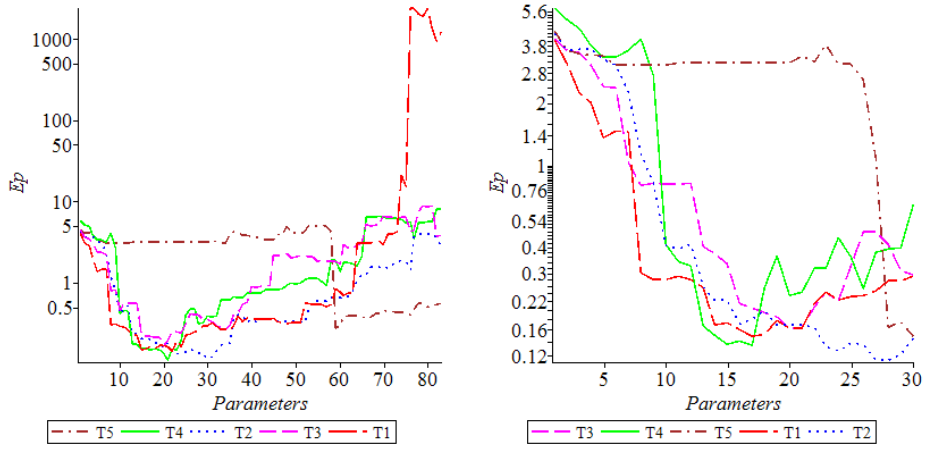
For a few relevant parameters (<10) the prediction error is above 300%. As the number of relevant parameters increases, the prediction error decreases to a value which remains almost constant. Therefore, it is possible to reduce the number of parameters until having a smaller size set of the relevant parameters for which the prediction error is unimportant. In models T2, T3 and T4 this value is close to 20 relevant parameters, in model T1 this limit is closer to 40 relevant parameters.

So far, the upper limit of the number of relevant parameters must also be considered, since increasing the number of parameters will imply increasing the condition number of the reduced model. From Figure 9, starting from a number of 40 relevant parameters, the condition number exceeds the value of 10^{+5}. Therefore, the maximum value of condition number can be used to limit the maximum number of relevant parameters. Furthermore, it should be considered that with less relevant parameters, a lower computational cost will be necessary to evaluate the model. For control actions and mechanism optimisation, this feature is important.

Taken into account that the errors with which the forces can be measured (with the actual commercial technology) are of the order of 10^{-2} (5%), estimations with prediction errors of the same order (≈7%), obtained with the model identified in relevant parameters, are satisfactory.

The prediction error E_p along with the condition number κ, as a function of the number of relevant parameters, allows obtaining the maximum and minimum limit values of the relevant parameters for each model. But given that each model is validated on its own identification trajectory, the previous graphs are not suitable to compare the different models among themselves. For this purpose, the model in identified base parameters is used to predict the generalised forces on the VT validation trajectory. In Figure 9, the errors E_p of each model validated for the same VT trajectory, considering 60% error in the CAD estimation, are presented.

The prediction errors are very high for the complete models. Therefore, it is justified to reduce it to a smaller number of relevant parameters where the observation matrix is well conditioned, to allow an adequate identification of the parameters.



(a) (b)

Figure 9. Error E_p. 60% error in CAD. (a) 1- 83 relevant parameters (b) 1-30 relevant parameters.

The prediction errors of models T1-T4, when considering different percentages of error in the CAD estimation, present very close values around the optimal number of base parameters. This number is between 20 and 30 base parameters. T5 model, due to its bad conditioning, is the model with the highest errors when few base parameters are used.

Table 3 presents the prediction errors E_p of validated models on the validation trajectory when 20 relevant parameters are used. It is confirmed that the smallest prediction error is reached for the obtained models by transfer of inertial properties. For these models, the one with the lowest error is the T4 model. However, the differences are not significant, so that the error levels reached in the models derived from the inertial transfers are observed to depend neither on the transfer sequence used nor on the number of selected convergence solids.

Table 3. Variation of the prediction errors in the estimated generalised forces for the validation trajectory.

20 relevant parameters	T1	T2	T3	T4	T5
Error E_p, %	16.25	16.83	16.26	12.71	31.50
κ of TV	$3.7x10^{+3}$	$7.6x10^{+2}$	$2.5x10^{+3}$	$8.2x10^{+3}$	$6.7x10^{+16}$

Figure 10 presents the estimated generalised forces using 20 relevant parameters with a 20% error in the CAD parameters because this level of error is an expected value in a CAD approximations. It is realised that in all the models obtained by transfer of inertial properties, and reduced using the dynamic contribution index, they can adequately estimate the generalised forces for a VT validation trajectory.

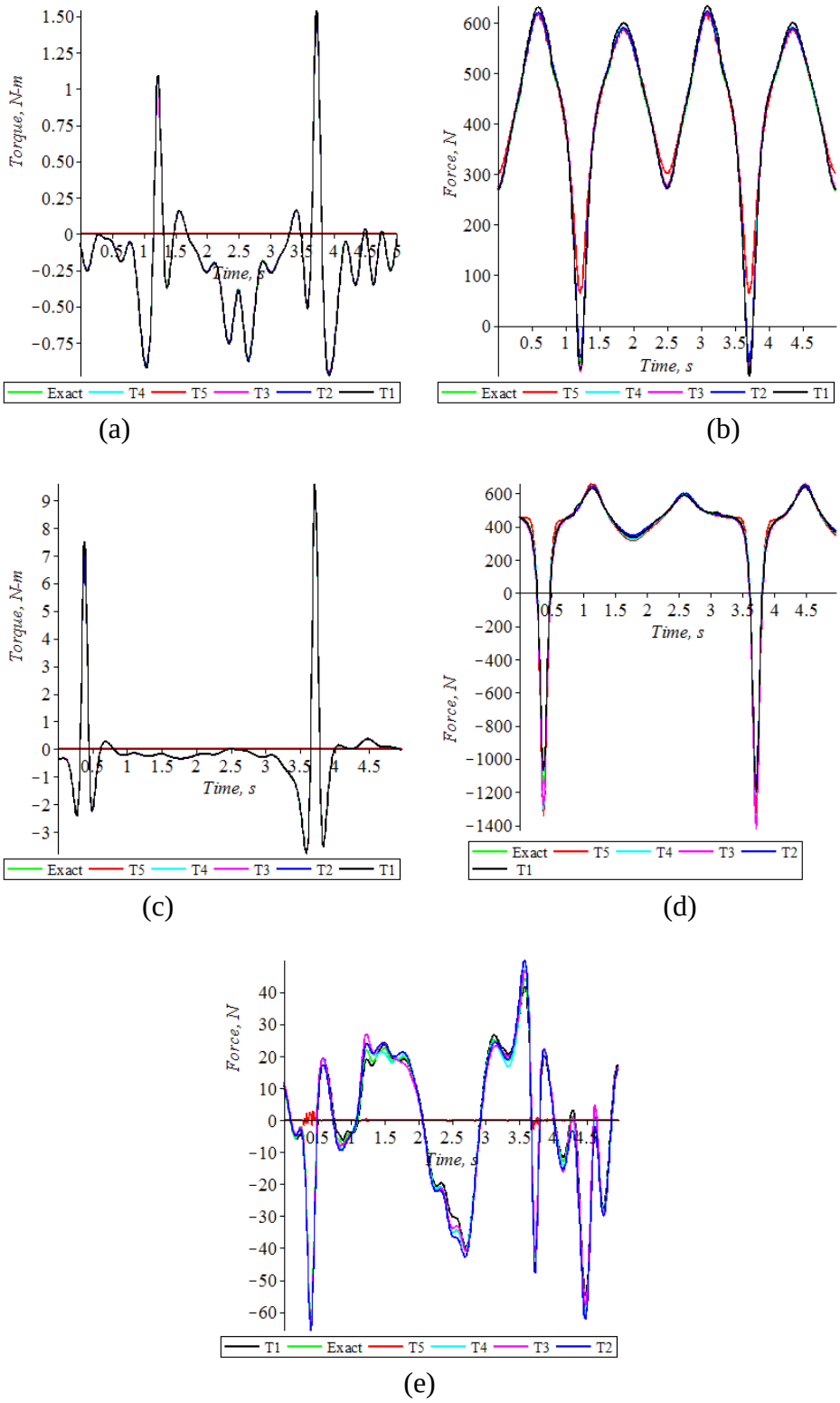


Figure 10. Generalised forces. (a) Right wheel, (b) right lower strut, (c) steering, (d) left wheel, (e) left lower strut

CONCLUSION

The work carried out was aimed at developing a new methodology for the identification of dynamic parameters in LMM. An algorithm was applied to obtain the linear equations of movement with respect to the parameters to be identified.

The application of the concept of transfer of the inertial properties allows obtaining the symbolic expressions of a set of base parameters, from which an identification model can be built. In comparison with standard numerical procedures such as the SVD, having the symbolic expressions of the base parameters allows obtaining the number of independent parameters, dependent only on the topology of the mechanism and independent of the movements or trajectories applied to it. The size of the base parameter vector is defined according to the number of transfers of inertial properties that can be made. This calculation is done in a symbolic way and the rank is determined unambiguously. Thus, the number of base parameters is independent of the precision handled by the numerical calculation program. This acquires importance given the bad numerical conditioning that characterises the LMM. A set of base parameters derived from inertial transfers allows obtaining not only the size of the model but also the symbolic relations of linear dependence between the parameters. In standard procedures, the selection of dependent parameters is made with the only condition that a maximum rank system is obtained. It does not consider how close to the singularity the matrices that define the linear dependencies are.

The dynamic contribution index can be calculated when the symbolic expressions of the base parameters are known. Therefore, it can be used to reduce the models derived from transfers of inertial properties. Although the dynamic contribution index depends on the initial estimation of the parameters, the ordering used to reduce the model is not significantly modified with the errors that such estimation could present. This ensures a valid reduction criterion even when using coarse CAD model estimates.

The estimations made with the complete model in base parameters present higher prediction errors than the reduced models. A relevant model that satisfactorily predicts the generalised forces of the mechanical system must have a minimum number of parameters that guarantee a low prediction error and a maximum number of parameters limited by low conditioning of the observation matrix. The models obtained by transfer of inertial properties and reduced by the index of dynamic contribution allow estimating with low errors the generalised forces of a low mobility system. The models with relevant parameters derived from different transfers of inertial properties have similar behaviours in the region of the optimal number of relevant parameters. In these models, for the cases studied, it was observed that neither the transfer sequence nor the amount of convergence solids influence the determination of a better model.

A novel methodology has been developed that allows the identification of dynamic parameters in low mobility mechanical systems, required for the construction of error-tolerant dynamic LMM models with few relevant parameters, with the final aim of accurately reproduce the dynamic behaviour of those low mobility mechanical systems.

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