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RESEARCH ARTICLE

Robust H∞ Control Design for Improving Handling and Ride Comfort in Semi-Active Suspension Systems

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ABSTRACT - This study investigates the application of robust H∞ control design for semiactive suspension systems. The goal is to achieve a balance between ride comfort and handling. A quarter-car model is used to simulate the system's dynamics. The findings demonstrate that the robust H∞ control approach with μ-synthesis offers significant advantages compared to traditional passive control and nominal H∞ control methods. When compared to the passive system, the robust H∞ controller with μ-synthesis results in a 50% reduction in body displacement (from 0.04 meters to 0.02 meters) during a simulated road bump. It also achieves a 25% reduction in peak body acceleration (from 4 m/s² to 3 m/s²) and a 37.5% reduction in suspension deflection (from 0.04 meters to 0.025 meters). These improvements translate to a smoother ride with less body movement and improved handling due to better tire contact with the road. The μ-synthesis method specifically addresses uncertainties like passenger weight and road conditions. This leads to more consistent performance in real-world driving scenarios. Overall, this study highlights the effectiveness of robust H∞ control design in achieving a well-balanced suspension system that enhances both ride comfort and handling.

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1. INTRODUCTION

Suspension systems for cars play a significant role in handling and passenger comfort. "Handling" refers to the ability of a vehicle to respond accurately to driver inputs, especially during maneuvers such as cornering, accelerating, and braking. Parameters that judge excellent handling include the vehicle's stability, steering precision, and response time. On the other hand, "ride comfort" pertains to how well a vehicle absorbs road irregularities to provide a smooth ride for passengers [1, 2]. This is often evaluated through parameters such as body displacement, acceleration, and suspension deflection. The primary objective of a well-designed suspension system is to decrease the effect of road irregularities on a car's chassis, increasing the comfort of passengers [3]. It also helps maintain tires in the best possible contact with the pavement, which is essential for mobility and safe handling. Traditionally, the goals of passive suspension systems have been met by mechanical parts such as dampers and springs. Their restricted performance stems from the need to balance handling and ride comfort. If the suspension is softer, the ride is smoother, but handling may suffer. On the other hand, a stronger suspension improves handling but decreases comfort. Semi-active suspension systems provide an alternative to overcome these limitations [4, 5]. These systems use adjustable dampers that provide real-time adjustments for their damping properties. Therefore, suspension management may be approached with greater sophistication, allowing researchers to adapt handling and ride comfort to specific driving situations.

1.1 Limitations of H∞ Control Designs

The H∞ control designs, while effective in providing robust performance in the presence of system uncertainties and external disturbances, have certain limitations that need to be addressed. One of the main disadvantages of traditional H∞ control designs is their inherent conservatism. These designs often result in overly conservative control laws because they are tailored to handle the worst-case scenarios. This conservatism means that the control system is designed to perform well under the most extreme conditions, which can lead to suboptimal performance under normal operating conditions. As a result, the handling and ride comfort of the vehicle might not be fully optimized, as the control system is not taking advantage of the full potential of the suspension system [6, 7].

Another significant limitation is the complexity and computational intensity of H∞ control designs. The mathematical formulations and algorithms involved are often intricate and require substantial computational resources, which can make real-time implementation challenging [8]. This complexity can also lead to difficulties in tuning and maintaining the control system, as the high computational demands may exceed the capabilities of standard automotive control hardware [8, 9].

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1.2 Introduction of Robust H∞ Control Designs

Active suspension technologies have come a long way in the last several years. Using robust H∞ control designs is one interesting way to improve their capabilities further. The ultimate objective of this sophisticated control approach is to maximize ride comfort and handling in semi-active suspension systems [6, 7]. These systems deliver greater performance under a range of driving circumstances by utilizing advanced algorithms. Semi-active suspension systems with strong H∞ control dynamically respond to changing road conditions and driving dynamics, in contrast to typical passive suspensions, which have limited flexibility. As a result, passengers receive more managed and enjoyable rides. The ability of a robust H∞ control design to balance handling accuracy with ride comfort is one of its main advantages[8]. The control system can precisely adjust suspension settings, such as stiffness and damping rates, to efficiently reduce vibrations and body motions while maintaining the best possible tire contact with the pavement. As a consequence, there are fewer pitch and roll movements, improving ride comfort for passengers. This improves comfort levels without sacrificing speed or agility [8, 9]. Strong H∞ control improves the stability and equilibrium of vehicles, whether they are navigating rough terrain or making fast turns. This guarantees a more pleasurable driving experience and gives the driver greater confidence [8, 10].

To improve handling and ride comfort in current automobiles with active suspension systems, a robust H∞ control design is necessary. Semi-active suspension systems may efficiently adjust to changes in vehicle load, road conditions, and external forces by applying approaches such as μ-synthesis or combined H2/H∞ control [11]. This guarantees reliable performance in a variety of driving scenarios [12]. Robust control facilitates smooth integration with existing automobile layouts and electrical systems, in addition to enhancing the dependability and predictability of the suspension system. This makes it possible to adapt dynamically to shifting driving scenarios. Furthermore, the use of a strong H∞ control architecture drives innovation in active suspension technology, hence advancing research and development efforts that focus on enhancing system performance and efficiency. This effort improves vehicle dynamics and ride comfort by stretching the limits of control theory and engineering, influencing the direction of mobility and transportation in the future. In the end, it ensures that every person has a smoother, safer, and more comfortable driving experience [12]. These drawbacks necessitate the exploration of more refined control approaches that can address these issues while maintaining robust performance.

1.3 Optimizing Ride Comfort and Handling Precision in Semi-Active Suspension Systems through Robust Control Design

In response to these limitations, robust H∞ control designs have been developed. These advanced control strategies aim to reduce the conservatism inherent in traditional H∞ designs by incorporating additional information about the system's dynamics and uncertainties. By doing so, they achieve a better balance between robustness and performance, enhancing both handling and ride comfort in various driving conditions. Robust H∞ control designs also leverage modern computational techniques to simplify implementation and improve real-time applicability. The complex balancing required in the design of semi-active suspension systems is underscored by the challenges posed by passenger biodynamics. This complex balancing involves the intricate process of optimizing multiple, often conflicting objectives within a suspension system, such as maximizing ride comfort while maintaining vehicle handling and stability. Traditional passive suspension systems struggle to achieve this balance due to their fixed mechanical properties, resulting in trade-offs where improving one aspect (e.g., ride comfort) typically compromises another (e.g., handling). This highlights the need for advanced control strategies, such as robust H∞ control, which are essential for effectively managing these trade-offs and enhancing the performance of semi-active suspension systems [13]. The goal of these systems is to maximize ride comfort and maneuverability. The location of passengers to the center of gravity of the vehicle presents challenges that require complicated solutions. Depending on the passenger's distance from the center of gravity, distinct effects are caused by roll, pitch, and vertical motions. Furthermore, pain increases when a person's resonance frequency increases with the motion of the vehicle [14]. This means that vibrations caused by passenger movement must be well managed. A thorough strategy is needed to address these issues. This entails combining sophisticated control techniques with knowledge of human physiology. This makes it possible for suspension characteristics to be dynamically adjusted to reduce discomfort [15, 16].

Robust control design techniques, such as μ-synthesis and robust H∞ control, present potential solutions for semi-active suspension systems [17]. These methods reduce the uncertainty caused by differences in passenger biodynamics. Engineers can create controllers that dynamically adjust to fluctuations by taking oscillations in biodynamic parameters into explicit consideration and rephrasing the control issue within the framework of robust control. For passengers, this improves stability and comfort during the voyage. Strict analysis and testing are used in performance evaluation to determine how well these strategies improve handling and ride comfort[18, 19]. This offers insightful information for ongoing innovation and application. The pursuit of a superior driving experience has driven advancements in suspension technology. Semi-active suspensions offer a practical balance between adaptability and efficiency[20, 21]. However, challenges persist in achieving optimal performance. This is particularly true with balancing ride comfort and handling precision amidst real-world uncertainties[22, 23]. Robust H∞ control design has emerged as a powerful tool for addressing these challenges. It offers a systematic framework for optimizing suspension performance and enhancing the overall driving experience for occupants across diverse driving conditions [24, 25]. An approach to overcoming the challenge of creating practical control schemes for semi-active suspension systems in real-world scenarios is through robust H∞

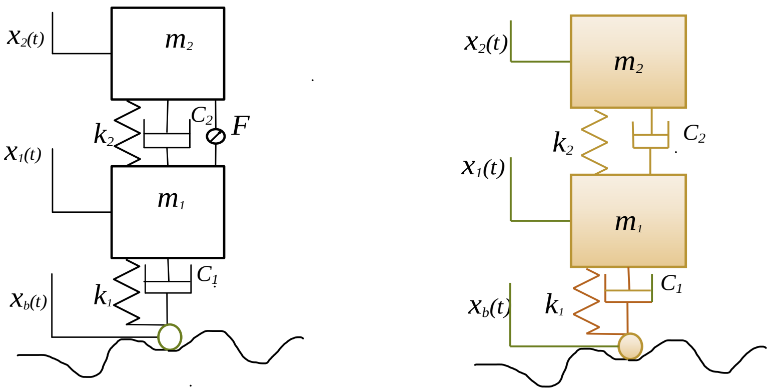
control design. These situations frequently incorporate unknowns related to road characteristics, vehicle loads, and driver behavior, in addition to possible errors in the system model. Robust H∞ control accounts for these uncertainties when designing the control strategy, seeking to provide the best possible handling and comfort across a variety of conditions. By combining the robustness of H∞ control with the adaptability of semi-active dampers, this method can significantly enhance the overall driving experience, offering precision handling alongside a comfortable ride. The driving force behind this work is the development of a control strategy that leverages the adjustable nature of semi-active dampers and the robust design of H∞ control. This method can provide a smooth ride coupled with accurate handling, significantly improving the driving experience. To further refine the controller, μ-synthesis can be employed to address structured uncertainties, such as variations in vehicle dynamics.

To further improve performance, μ-synthesis, a robust control design technique, can be integrated to handle structured uncertainties, such as variations in vehicle parameters. μ-synthesis minimizes the impact of these uncertainties by determining the worst-case system gain. This allows for the design of a controller that guarantees stability and performance despite these variations. The resulting semi-active suspension system demonstrates improved reliability and adaptability to changing driving conditions. The motivation behind this work stems from the need to address the inherent trade-offs in traditional passive suspension systems, which struggle to balance ride comfort and handling due to their fixed mechanical properties. Semi-active suspension systems with the ability to adjust damping properties in real time offer a promising solution by dynamically optimizing performance based on driving conditions. However, these systems must contend with various uncertainties, such as changes in passenger weight, road conditions, and vehicle dynamics. By employing a robust H∞ control design, this study aims to enhance the adaptability and effectiveness of semi-active suspensions, ensuring consistent performance and improved driving experience. The robust H∞ control strategy, particularly the μ-synthesis method, is chosen for its ability to handle these uncertainties, ultimately contributing to advancements in vehicle dynamics and providing a smoother, safer, and more comfortable ride for passengers.

2. METHODS AND MATHEMATICAL MODELING OF QUARTER-CAR SUSPENSION SYSTEMS

Advances in suspension technology have been driven by the quest to find a balance between passenger comfort and vehicle handling. Conventional passive suspensions prioritize either comfort or handling with their fixed spring and damper settings. On the other hand, semi-active suspensions offer greater control by using a feedback-controlled hydraulic actuator. This allows engineers to dynamically adjust the balance between comfort and handling based on driving conditions [26]. This research investigates the application of H∞ control design in semi-active suspension systems, which bridge the gap between passive and active systems. A simplified quarter-car model (Figure 1) is used to illustrate the concept. The model considers the car body (mass m_2) and the wheel assembly (mass m1) connected by a passive spring *(k_2)* and damper ($C_{1,2}$). These components represent the standard suspension elements. Additionally, a spring *(k_1)* models the compressibility of the tire, and variables account for body travel *(y(t))*, wheel travel *(x(t))*, and road disturbances *(x(b))*. The key element in a semi-active system is the controlled force *(F)* applied between the body and the wheel. This force, influenced by feedback control, represents the active component that allows for real-time adjustments based on the H∞ control strategy.

H∞ control design is a powerful technique that considers uncertainties inherent to real-world driving scenarios, such as variations in road profiles, vehicle loading, and driver behavior. By considering these uncertainties, the H∞ control strategy aims to achieve robust performance. This results in a suspension system that maintains a desirable balance between ride comfort and handling, even under varying conditions. This approach utilizes the adjustability of the controllable force *(F)* in semi-active dampers to significantly improve the overall driving experience.



(a) Semi-Active suspension (b) Passive suspension

Figure 1. Quarter-Car model suspension system

$$the \sum F_x = ma_x$$
$$m_2\ddot{x}_2 = -k_2(x_2 - x_1) - C_2(\dot{x}_2 - \dot{x}_1) + F \tag{1}$$

By solving for $\ddot{x}_2 = \ddot{x}_3$, where $\dot{x}_2 = x_1$

$$\ddot{x}_2 = \frac{1}{m_2}\left[-k_2(x_2 - x_1) - C_2(\dot{x}_2 - \dot{x}_1) + F\right] \tag{2}$$

$$m_1\ddot{x}_1 = -k_1(x_1 - x_b) - C_1(\dot{x}_1 - \dot{x}_b) - k_1(x_1 - x_b) + C_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + F \tag{3}$$

By solving for $\ddot{x}_1 = \ddot{x}_4$, where $\dot{x}_3 = \dot{x}_4$

$$\ddot{x}_1 = \frac{1}{m_1}\left[-k_1(x_1 - x_b) - C_1(\dot{x}_1 - \dot{x}_b) - k_1(x_1 - x_b) + C_2(\dot{x}_2 - \dot{x}_1) + k_2(\dot{x}_2 - \dot{x}_1) + F\right] \tag{4}$$

Finally, by manipulating the initial differential equations obtained from Newton's law and incorporating the chosen state variables, we arrive at a system of first-order differential equations. This system is arranged in a specific matrix format known as the state space equation. This equation provides a powerful tool for analyzing the system's behavior and designing control strategies, such as H∞ control, to achieve a desired balance between ride comfort and handling.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + D \end{aligned} \tag{5}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_2}{m_2} & \frac{-C_2}{m_2} & \frac{k_2}{m_2} & \frac{C_2}{m_2} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_1} & \frac{C_2}{m_1} & \frac{(-K_2 - k_1)}{m_1} & \frac{-C_2}{m_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m_2} \\ 0 & 0 \\ \frac{k_1}{m_1} & \frac{-1}{m_1} \end{bmatrix} u \tag{6}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_2}{m_2} & \frac{-C_2}{m_2} & \frac{k_2}{m_2} & \frac{C_2}{m_2} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_1} & \frac{C_2}{m_1} & \frac{(-K_2 - k_1)}{m_1} & \frac{-C_2}{m_1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m_2} \\ 0 & 0 \\ \frac{k_1}{m_1} & \frac{-1}{m_1} \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{-k_2}{m_2} & \frac{-c_2}{m_2} & \frac{k_2}{m_2} & \frac{c_2}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix} \tag{7}$$

2.1 Robust H∞ Control for Reliable Comfort and Handling in Semi-Active Suspensions

When designing control systems, such as those for semi-active suspensions, it is crucial to obtain an accurate mathematical model of the physical system. However, real-world systems are often complex, exhibiting nonlinear behavior and distributed parameters. These complexities can make it difficult to analyze and use the model for control design [26]. This is where the concept of model simplification comes into play. The goal is to create a linear, constant-coefficient model that adequately captures the essential dynamics of the suspension system. This simplified model should retain the inherent characteristics of the actual system while being easier to work with for control design purposes.

Traditional control design approaches, such as those based on frequency response analysis, incorporate margins (phase and gain) to account for these modeling errors. However, state-space approaches, which rely on the system's differential equations, do not explicitly incorporate such margins in the design process.

2.2 Design of Robust H∞ Control for Semi-Active Suspensions

Robust control theory recognizes the inherent uncertainties and errors in the models used for control design. In the case of semi-active suspension systems, these uncertainties can include variations in passenger weight distribution, road conditions, or even slight differences in actuator behavior[27]. H∞ control, a specific approach within robust control theory, takes these uncertainties into account during the design process. The main objectives of H∞ control are as follows:

a) Robust Stability: The control system should remain stable even in the presence of these uncertainties. This ensures that the suspension continues to function effectively and maintains vehicle control under different driving conditions.
b) Robust Performance: Despite these uncertainties, the control system should still exhibit the desired performance characteristics. In the case of semi-active suspensions, this means achieving a good balance between ride comfort (minimizing passenger acceleration) and handling (ensuring good road contact and vehicle response).

Robust control theory involves both frequency-domain and time-domain analysis methods. However, understanding the core concepts is essential for appreciating the value of H∞ control in designing reliable control systems for semi-active suspensions. By incorporating uncertainties into the design process, H∞ control offers a powerful approach for achieving superior handling and passenger comfort while maintaining stability under real-world driving conditions. Uncertainties in the Actuator Model**:** The hydraulic actuator used for active suspension control connects the body mass (m_2) and the wheel assembly mass (m_1). The actuator dynamics are approximated as a first-order transfer function with a time constant of 1/60 seconds (represented by 1/(1+s/60)). The actuator also has a maximum displacement limitation of 6 cm.

The H∞ norm, denoted for scalar, Φ (s), || Φ ||∞, gives a maximum value of $|\Phi(jw)|$, which is said to be a norm that measures the worst-case amplification of a system across all frequencies. This worst-case scenario considers not only the nominal system behavior (represented by the mathematical model) but also potential uncertainties such as variations in passenger weight, road conditions, or even slight differences in actuator performance.

In robust control theory, the magnitude of a transfer function is quantified using the H∞ norm. Let us assume that we have a transfer function Φ (s), which is proper and stable. (Note: A transfer function Φ (s) is termed proper if its infinity H∞ norm is bounded and finite. If Φ (∞) = 0, it is referred to as strictly proper.) The H∞ norm of Φ (s) is defined as follows:

$$\|\Phi\|_\infty = \bar{\sigma}\left[\Phi(jw)\right] \tag{8}$$

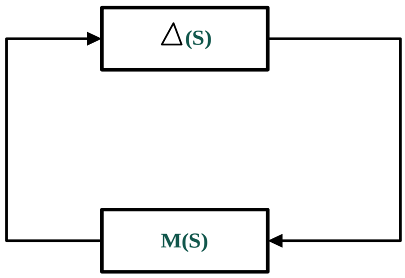


Figure 2. Closed-loop model of the plant

In Figure 2, $\Delta(s)$ and $M(s)$ are stable and proper transfer functions.

Theorem: The small-gain theorem states that if

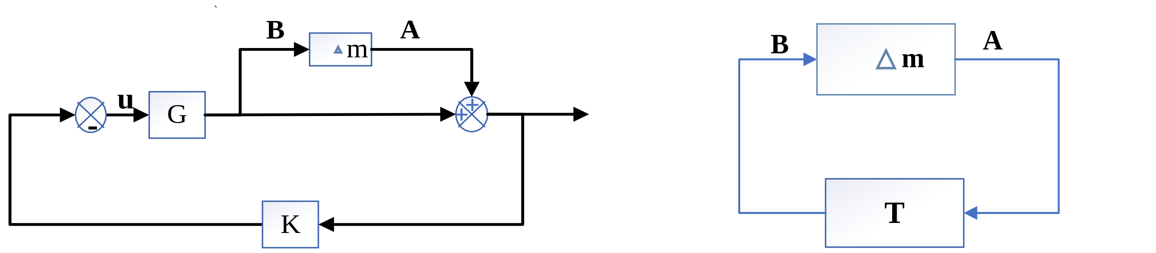
$$\|\Delta(s)M(s)\|_\infty \leq 1$$

Then, this closed-loop system is stable. That is, if the H∞ norm of $\Delta(s)M(s)$ is smaller than 1, this closed-loop system is stable. This theorem is an extension of the Nyquist stability criterion.

To assess robust stability, unstructured uncertainty errors in some instances, are regarded as multiplicative.

$$\tilde{G} = G(1+\Delta_m) \tag{9}$$

In this model, $\tilde{G}$ represents the true plant dynamics, while G denotes the model plant dynamics (Figure 3).



(a) Unstructured multiplicative uncertainty plant model (b) Successive modifications of the plant model

Figure 3. Block diagram of the generalized plant model with unstructured multiplicative uncertainty

$$(1+KG)^{-1}KG = T \tag{10}$$

Applying the small-gain theorem to the system, we obtain the condition for stability to be

$$\left\|\Delta_m T\right\|_\infty < 1 \tag{11}$$

where $\Delta_m = W_m$; therefore

$$\left\|W_m T\right\|_\infty < 1 \tag{12}$$

By equating equations (6) and (8), we obtain

$$\left\|\frac{W_m K(s)G(s)}{1+K(s)G(s)}\right\|_\infty < 1 \tag{13}$$

We design a controller (K(s)) for a semi-active suspension system, represented by a stable plant model (G(s)) as in Figure 4. A simple solution might be to set the controller to zero (K(s) = 0). This would technically satisfy a specific design constraint (inequality, as shown in equation (9). However, a zero controller would not be beneficial. This essentially means that no control action is applied to the suspension. This means that the controlled suspension should effectively minimize the impact of road disturbances (inputs) on the car's body movement (outputs). We need to find a better transfer function for the controller (K(s)) to achieve a desirable outcome.

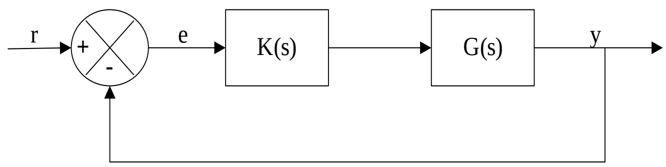


Figure 4. Modified material of the plant

$$\lim_{t\to\infty}\left[r(t)-y(t)\right] = \lim_{t\to\infty} e(t) \to 0 \tag{14}$$

Therefore, the transfer function Y(s)/R(s)

$$\frac{Y(s)}{R(s)} = \frac{KG}{1+KG}$$

$$\frac{E(s)}{R(s)} = \frac{R(s)-Y(s)}{R(s)} = 1-\frac{Y(s)}{R(s)} = \frac{1}{1+KG}$$

$$\frac{1}{1+KG} = s$$

The relationship between body position (W_{xb}), suspension deflection (W_{sd}), and wheel position (W_{xw}) ($W_{xw} = W_{xb} - W_{sd}$) creates a natural trade-off. Any attempt to reduce body movement at low frequencies (less than 5 rad/s) through control will likely increase suspension deflection, and equation (15) indicates a generalized plant model.

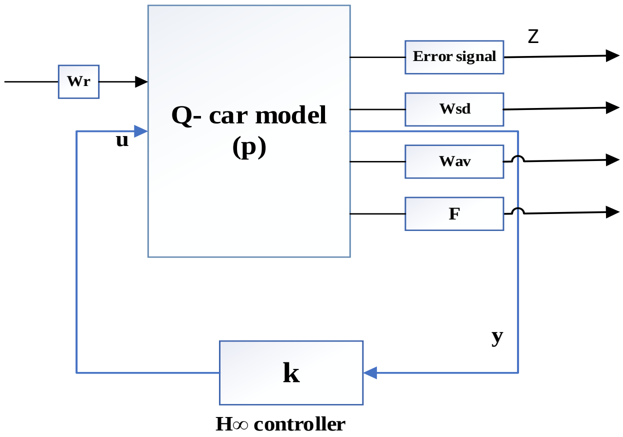


Figure 5. Block diagram of the generalized Q-car model with unstructured multiplicative uncertainty

$$\begin{bmatrix} Z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \begin{bmatrix} w_r(s) \\ u(s) \end{bmatrix} \tag{15}$$

In a semi-active suspension system, the relationship between the control signal applied (u(s)) and the system's output (y(s)) is defined by the controller's transfer function, K(s). This means that the control signal is essentially determined by multiplying the system output by K(s).

$$\begin{aligned} u(s) &= K(s)y(s) \\ Z(s) &= \Phi(s)w_r(s) \end{aligned} \tag{16}$$

Therefore, by equating the above equation, we obtain

$$\Phi(s) = p_{11} + p_{12}K(s)\left[I - p_{22}K(s)\right]^{-1} p_{21} \tag{17}$$

Note that if we choose the generalized plant P matrix as

$$p = \begin{bmatrix} 0 & W_m G \\ I & -G \end{bmatrix} \tag{18}$$

Note that the controlled variable z is related to the external disturbance w_r by

$$p = \begin{bmatrix} W_r & -W_r G \\ I & -G \end{bmatrix} \tag{19}$$

Now, by equating equations (13) and (14), we obtain the following:

$$p = \begin{bmatrix} W_r & -W_r G \\ 0 & W_m G \\ I & -G \end{bmatrix} \tag{20}$$

The design process utilizes a mathematical construct called the generalized plant. This generalized plant, defined by equation (15), incorporates the original system dynamics of the suspension along with the uncertainties arising from real-world driving conditions. Overall, this design process highlights the use of robust control theory in designing controllers for semi-active suspensions. By constructing a generalized plant that incorporates uncertainties, the design process can ensure both robust stability and minimize the impact of disturbances on the controlled variable (smoother ride). The specific equations and Figures 2-5 directly explain the core concepts and provide valuable insight into this design approach.

3. SIMULATION RESULTS AND DISCUSSION

This section examines different control strategies for a semi-active suspension system, which are analyzed using MATLAB Simulink 2023a. The main objective is to enhance both ride comfort and handling. Three distinct methods are compared: passive control (no active intervention), H∞ synthesis (idealized model), and robust H∞ with μ synthesis (considering uncertainties). The robust H∞ with the μ synthesis method provides a significant advantage by explicitly considering real-world variations, such as passenger weight and road conditions. This consideration allows for an optimized control strategy that minimizes discomfort caused by disturbances while still maintaining excellent handling. Ultimately, this approach results in a well-balanced suspension system, leading to a smoother and more controlled driving experience.

Table 1. Quarter-car model suspension parameters

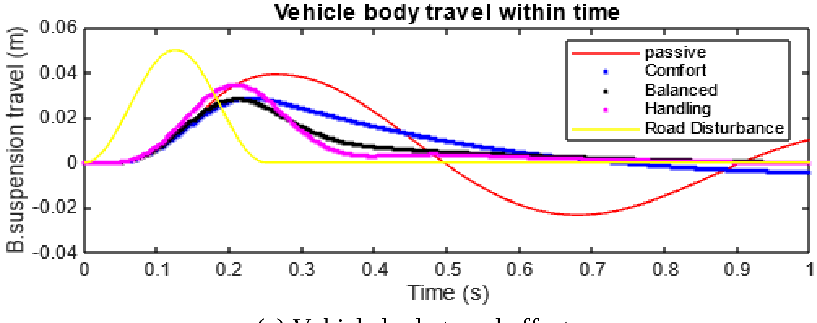
Parameters	Value	Unit
Body mass (m_2)	423.5	kg
Unspring mass (m_1)	56	kg
Suspension stiffness (k_2)	31000	N/m
Suspension damping (c_2)	1500	N·s/m
Tire stiffness (k_1)	225000	N/m

As shown in Figure 5, the road disturbance (W_r) is modeled as a normalized signal (d_1) shaped by a weighting function (W_{dr} = 0.07), reflecting a chosen emphasis on this disturbance source. A constant weight of 0.06 denotes a broadband road imperfection with a magnitude of 6 cm. The sensor noise in both measurements is represented as a normalized signal (d_2 for W_{sd} and d_3 for W_{av}) shaped by individual weighting functions (d_2 = 0.01 for W_{sd} and d_3 = 0.5 for W_{av}), indicating the intensity of the broadband noise for each sensor (0.02 and 0.4, respectively). Our control objectives, viewed as a disturbance rejection strategy, aim to minimize the combined impact of these disturbances (d_1, d_2, and d_3) on a weighted combination of control effort (u), suspension travel (W_{sd}), and body acceleration (W_{av}). The H∞ norm quantifies the impact of disturbances, focusing on minimizing it from the disturbance inputs (d_1, d_2, and d_3) to the error signals (e_1, e_2, and e_3), likely representing deviations from the desired values of control effort, suspension travel, and body acceleration. The controller endeavors to achieve optimal performance (minimized peak gain) by rejecting the influence of road disturbances and sensor noise on the system's behavior while also considering control effort and maintaining the desired suspension and body acceleration characteristics.

In the design of robust H∞ control for semi-active suspension systems, the selection of weighting factors is crucial to achieving the desired balance between ride comfort and handling. These factors determine the emphasis placed on various performance metrics. For the Comfort Condition, the weighting factor (β) is set to 0.01 to prioritize the minimization of body acceleration, ensuring a smoother ride by reducing the vibrations transmitted to passengers and enhancing comfort during normal driving conditions. In the Balanced Condition, the weighting factor (β) is set to 0.5 to provide an equilibrium between ride comfort and handling by compromising between minimizing body acceleration and suspension deflection, aiming to ensure a comfortable ride while maintaining adequate handling performance. For the Handling Condition, the weighting factor (β) is set to 0.99 to prioritize the minimization of suspension deflection, thereby improving the vehicle's handling characteristics and maintaining tire-road contact and vehicle stability, especially during aggressive driving maneuvers or on uneven road surfaces.

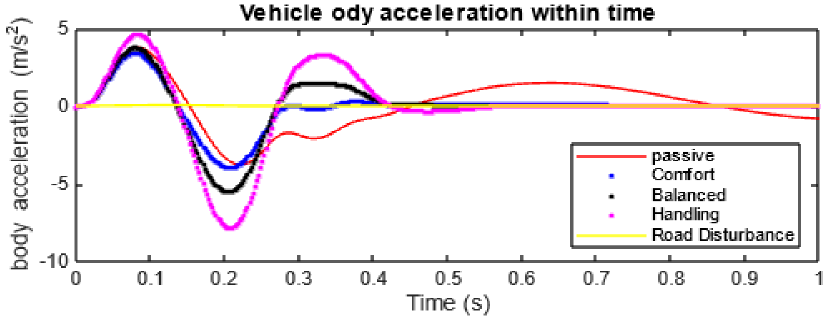
3.1 Time Domain Simulation Responses of Nominal H∞ Control

Figure 6 shows the results obtained from simulations involving a passive suspension system and two control methods for a semi-active suspension system: passive and H∞ synthesis. Two key parameters influence passenger comfort: car body motion and acceleration. The figures compare the performance of the passive suspension system (no control) against the two semi-active control methods (passive and H∞ synthesis). In the simulation, a road disturbance with specific characteristics is incorporated. A time vector (t) is defined, and a road disturbance signal is created accordingly. This disturbance initiates at zero and persists for 1 second, the duration defined by t, with a sampling time of 2.5 milliseconds (0.0025). The initial 101 samples, corresponding to the first 0.2525 s, simulate a gradual transition from zero to a peak value. This transition is achieved through a cosine function with a frequency of 8 Hz. The peak value of the disturbance is set at 0.025 meters, indicating a bump with a height of 2.5 centimeters.

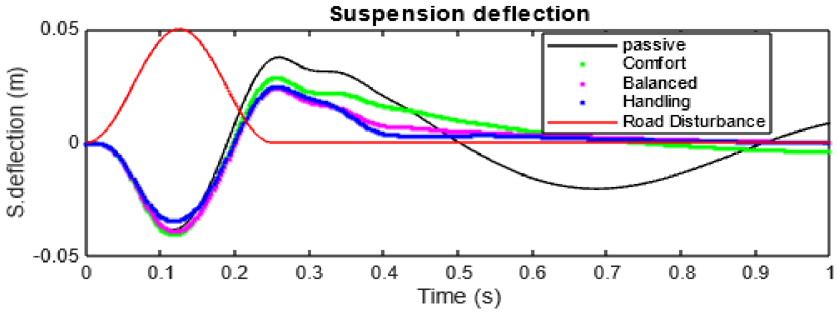


(a) Vehicle body travel effect

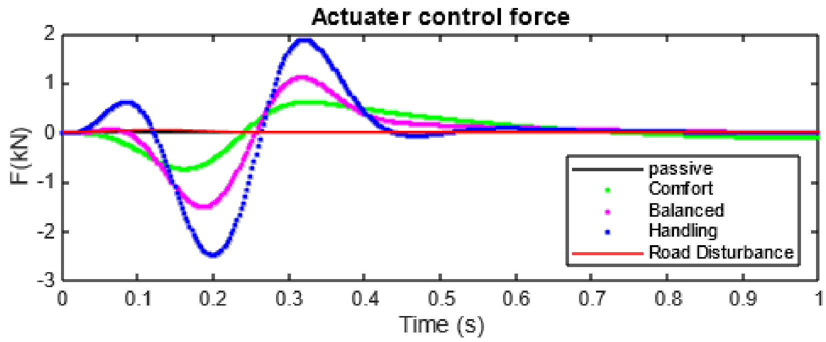
Figure 6. Time-domain responses of the nominal H∞ control



(b) Vertical acceleration



(c) Suspension deflection



(d) Controller effort actuator force

Figure 6. (cont.)

Figure 6 illustrates the performance of different suspension systems during a 1-second simulation. Figure 6(a) focuses on body travel. The passive system exhibits a peak displacement of 0.04 m and shows fluctuations throughout the simulation. In contrast, the comfort and balanced systems achieve a significantly smoother ride with a smaller peak displacement of only 0.02 meters. The comfortable and balanced systems also return to zero displacements much faster, reaching equilibrium at approximately 0.65 seconds. While the handling system exhibits a higher peak point compared to the comfort and balanced systems, it reaches zero displacements the quickest at approximately 0.35 seconds. This trade-off highlights the focus of each system: comfort and balance prioritize reduced body movement for a smoother ride, while handling prioritizes faster settling time for better control.

Figures 6(b) and 6(c) examine the suspension deflection and vertical acceleration responses of a semi-active suspension system to a road disturbance. The responses are presented for four configurations: a passive system (no control) and three H∞ controllers with varying weighting factors. The designed controllers achieve closed-loop H∞ norms of 0.94, 0.67, and 0.89, indicating good disturbance rejection performance. These values were obtained by designing and tuning H∞ controllers by varying weighting factors, which are discussed in section 3.0. Notably, all active control strategies significantly reduce both suspension deflection and vertical acceleration compared to the passive system, especially below the critical rattle space frequency of 25 rad/s. This translates to a smoother and more controlled ride.

Figure 6(b) illustrates the vertical acceleration response of the system to the road disturbance. The passive system experiences the greatest vertical acceleration throughout the simulation, as shown by its larger fluctuations. All three H∞ controllers (comfort, balance, handling) achieve a reduction in vertical acceleration compared to the passive system. The comfort controller and balanced controller both return to zero acceleration in 0.42 seconds. The handling controller exhibits a slightly slower return to zero at 0.432 seconds. The vertical acceleration of the passive system remains nonzero within the displayed timeframe.

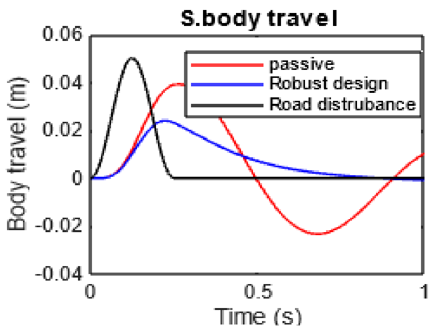
Figure 6(c) illustrates the suspension deflection response of the system to the road disturbance. The passive system experiences the most significant deflection, which is evident from its larger fluctuations throughout the simulation. All three H∞ controllers (comfort, balance, handling) achieve a reduction in suspension deflection compared to the passive system. The comfort controller prioritizes minimizing deflection, achieving a return to zero in 0.58 seconds. The balanced controller offers a compromise, reaching zero in 0.42 seconds. The passive system exhibited sustained deflection throughout the observed timeframe.

Figure 6(d) illustrates the above scenarios, where the passive controller, depicting no active force, is expected to show a flat line at zero (kN). The comfort-oriented controller might manifest as a curve fluctuating around a zero baseline, with smaller peak magnitudes than others, indicating efforts to minimize suspension movement. The balanced controller could exhibit a compromise between comfort and handling, with peak magnitudes potentially higher than those for comfort but lower than those for handling. In contrast, the handling-oriented controller may demonstrate larger fluctuations, potentially with higher peak magnitudes, prioritizing road handling over minimizing suspension movement.

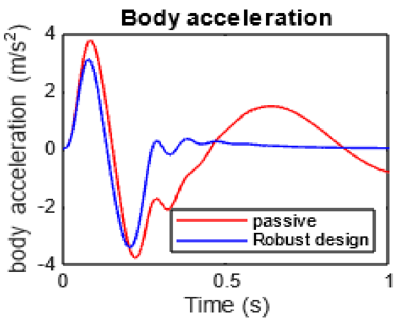
These observations suggest a trade-off between ride comfort and handling performance. The comfort controller prioritizes minimizing both suspension deflection and vertical acceleration, potentially at the expense of requiring more control effort from the actuator. The balanced controller provides a compromise between these two objectives.

3.2 Time Domain Simulation Responses of Robust H∞ Control using μ-synthesis

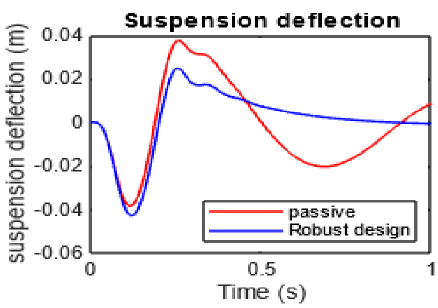
In recognition of the limitations of the nominal actuator model, this section focuses on robust H∞ control design using μ-synthesis. The objective of this method is to achieve consistent performance in the presence of uncertainties. By utilizing the μ function, a robust controller (K_{rob}) is synthesized for the balanced performance scenario (β=0.5), resulting in an optimal robust performance of 0.906. Simulations using K_{rob} for a road bump yield similar responses to those observed with the balanced H∞ controller, indicating that the robust controller maintains performance while reducing the impact of model uncertainty. These findings translate to a more consistent ride quality in real-world driving scenarios.



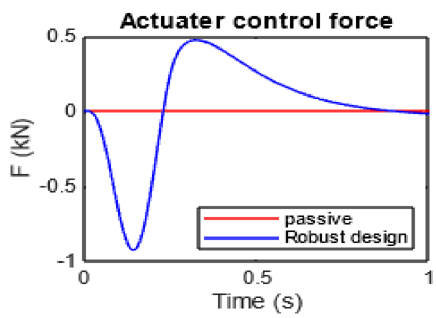
(a) Body travel



(b) Comfort



(c) Handling



(d) Actuator control force

Figure 7. Time-domain responses of the robust H∞ μ-synthesis

The μ-synthesis design process involves simulating the system's response using the newly designed robust controller. This allows for a comparison of the performance between the controlled system (closed-loop) and the uncontrolled system (passive) when subjected to road disturbances.

Figure 7 depicts a simulation running for 1 second. In Figure 7(a), which focuses on body travel, the uncontrolled system (red line) experiences a peak displacement of 0.04 meters and exhibits fluctuations throughout the timeframe. In contrast, the controlled system (blue line) experiences a smaller peak displacement of only 0.02 meters and returns to zero at approximately 0.7 seconds. This signifies a significant reduction in body movement, leading to a smoother ride. Similarly, Figure 7(b) examines body acceleration. The peak body acceleration experienced by passengers in the uncontrolled system can reach 4 meters per second squared and fluctuates throughout the simulation. The robust controller reduces this peak value to 3 meters per second squared. Additionally, the controlled system returns to zero body acceleration at approximately 0.4 seconds, offering a more comfortable ride. Figure 7(c) focuses on suspension deflection, which represents the extent to which the suspension compresses or extends. In the uncontrolled scenario, the peak deflection reaches 0.04 m and fluctuates throughout. The robust controller has the potential to bring this down to 0.025 meters, with the deflection returning to zero at approximately 0.3 seconds. This ensures that the tire maintains better contact with the road for improved handling.

A key difference between the passive and closed-loop systems lies in the control force. As shown in Figure 7(d), the uncontrolled system has no active control, resulting in a constant zero control force (represented by the flat red line). The robust controller, however, introduces a control force (represented by the blue line) that varies over time. This force can reach a peak value of approximately 0.6 kN, actively counteracting road disturbances and enhancing the overall suspension performance.

4. CONCLUSIONS

The investigation of different control strategies for a semi-active suspension system, conducted through MATLAB Simulink 2023a, reveals promising insights into enhancing both ride comfort and handling. When comparing passive control, H∞ synthesis, and robust H∞ with μ synthesis, the robust method has an advantage. This approach explicitly addresses real-world uncertainties such as passenger weight and road conditions, offering an optimized control strategy that minimizes discomfort from disturbances while ensuring excellent handling. The resulting suspension system achieves a well-balanced performance, resulting in a smoother and more controlled driving experience. Through comprehensive simulations and analyses, it is evident that the robust controller effectively maintains performance despite model uncertainties, resulting in a more consistent ride quality in real-world driving scenarios. Overall, this research study emphasizes the importance of robust control design in achieving reliable and effective suspension system performance under varying conditions.

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CONFLICT OF INTEREST

We declare that there are no conflicts of interest.

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