اونيۜۏرسيتۑ مليسيا ڤهڠ
UNIVERSITI MALAYSIA PAHANG

ORIGINAL ARTICLE

Simulation and visualization of wave equation

N.A. Salahudin[1,*], N. Roslee[1], N. Zull Pakkal[1], S. Zokree[1] and H.F. Saipan@Saipol[2]

[1]Mathematical Sciences Studies, College of Computing, Informatics and Mathematics, Universiti Teknologi MARA (UiTM) Terengganu Branch, Kuala Terengganu Campus, 21080 Kuala Terengganu, Terengganu, Malaysia

[2]Department of Management of Technology, Malaysian Japan International Institute of Technology, Universiti Teknologi Malaysia, 54100 Kuala Lumpur, Malaysia

ABSTRACT – The partial differential equation (PDE) is significant in mathematics, physics, and engineering fields. The PDE model is one of the problems in a real-life situation that is complex to resolve and it involves many variables to be solved. The wave equation is one of the second-order partial differential equations that is commonly used in mathematics and physics. This equation was first studied in the 18th century by Euler, d'Alembert, Lagrange, and Laplace. In this paper, the finite difference method is used to solve the simple equation and compared with theoretical calculation. The results for the function is verified using Visual C++ programming. The values of error from both methods are compared, which are very small. The results show that the numerical calculation is as accurate as the theoretical method.

INTRODUCTION

According to [1], partial differential equations are used extensively in mathematics and science. They offer the fundamental mathematical basis for several major physical theories, including elasticity, hydrodynamics, electromagnetism, general relativity, and non-relativistic quantum mechanics. Partial differential equations have also been discovered to play an important role in pure areas of mathematics such as Complex Analysis, Differential Geometry, Topology, and Algebraic Geometry.

One of the most crucial equations in mechanics, mathematics, and physics is the wave equation. It depicts not only how strings and wires move, but also how fluid surfaces, like water waves move. Partial differential equations can be used to model the periodic process of wave motion. According to [2], the multiagent system (MAS) uses the wave equation in time-varying formation control to direct the agents to construct dynamical formations or track a target trajectory while retaining a formation.

Then, in 2021, [3] state that, it integrated the regular solution of the plane transverse electromagnetic wave mode of Maxwell's equations in the Chinese version of 2000 and the regular solution of the cylindrical transverse electromagnetic wave mode of Maxwell's equations in English version of 2011. It is used to correct electromagnetic wave equations of Maxwell's equation. [4] used the partial differential equation (PDE) model to develop in the bipolar coordinate system that finds applications in transport phenomena. The PDE taken is the Navier Stokes equation representing the steady flow of a viscous fluid past a pair of separated fluid spheres (termed as a two-sphere problem).

In many different fields, including physics, mathematics, and engineering, the wave equation has been applied. It is known as the prototype hyperbolic partial differential equation in mathematics. The study of chemical diffusion and other related processes leads to the diffusion equation, a generalized form of the wave equation. Additionally, it illustrates how waves (or variations in temperature) are distributed throughout time in a specific area. Contrarily, we have two initial conditions and two boundary conditions for hyperbolic equations. These equations result from convective problems in gas dynamics, wave mechanics, and vibrations.

Electrogravitodynamics (EGD) model in [5] employed wave equation. A formal structure for electrogravitodynamics (EGD) in a material environment with non-zero generating sources is presented from the development of wave equations in electromagnetism (EM) and gyrogravitation (GG) with respect to the transverse component fields and the pointing vector of each radiation. According to [6], a second order derivative in time and spectral fractional powers of orders (0, 1) of symmetric, coercive, linear, elliptic, second-order operators in confined domains are required for the solution of the evolution equation. A diffusion-wave equation with a starting point at negative infinity and a fractional derivative with respect to time, defined on an infinite interval by [7].

THEORITICAL CALCULATION

The general formula for the wave equation is stated as below:

$$\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2} \tag{1}$$

For this paper, we use simple wave equation to verify the theoretical solution with finite difference method. The displacement $y(x,t)$ of a string for $0 \le x \le 10$ and up to one half period of vibration. The function $y(x,t)$ satisfies the equation $\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$ and boundary condition $y(0,t) = 0$, $y(10,t) = 0$, $\frac{\partial y}{\partial t} = 0$ and $y(x,0) = \frac{1}{100}x(10-x)$.

The general solution for the equation are shown below:

$$V(x,t) = \sum_{n=0}^{\infty} [An\cos\frac{n\pi}{L}\alpha t + Bn\sin\frac{n\pi}{L}\alpha t]\sin\frac{n\pi}{L}x$$

$$V(x,t) = \sum_{n=0}^{\infty} [An\cos\frac{2n\pi}{10}t + Bn\sin\frac{2n\pi}{10}t]\sin\frac{n\pi}{10}x$$

$$Bn = 0$$

$$An = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

Bn is equal 0 because of the initial velocity, g(x) is 0. Therefore, the value of Bn will also be 0. The value of the boundary is replaced into the formula of An.

$$An = \frac{2}{10}\int_0^{10} \left(\frac{x}{100}\right)(10-x)\sin\frac{n\pi x}{10}dx$$

$$An = \frac{1}{5}\int_0^{10} \left(\frac{x}{100}\right)(10-x)\sin\frac{n\pi x}{10}dx$$

The above equation is after the simplified value of An and integrate the equation to get the answer of An.

$$An = \frac{8}{\pi^3} @ 0.258012275$$

$$U(x,t) = \frac{8}{\pi^3}\sum_{n=0}^{\infty} [\sin\frac{n\pi x}{10}\cos\frac{2n\pi t}{10}]$$

Therefore, the solution of wave equation stated as above and n is the value of λ which is λ = 1. So, the solution of the equation is as follow:

$$U(x,t) = \frac{8}{\pi^3}\sum_{n=0}^{\infty} [\sin\frac{\pi x}{10}\cos\frac{2\pi t}{10}]$$

By using the same equation, we solved numerically using finite difference method. The result for the numerical method is shown in Table 1.

Table 1. Numerical result generated

5	5	0	-0.16	-0.24	-0.24	-0.26	0
4	4	0	-0.12	-0.20	-0.20	-0.12	0
3	3	0	-0.04	-0.08	-0.08	-0.04	0
2	2	0	-0.04	0.08	0.08	0.04	0
1	1	0	0.12	0.20	0.20	0.12	0
0	0	0	0.16	0.34	0.24	0.16	0
$t = jk$	j i	0	2	4	6	8	10
	$x = ih$	0	1	2	3	4	5

SIMULATION

The wave equation is verified using C++ programming. The main window's display is controlled and updated often by `OnPaint()` function. Any function in the program that calls `InvalidateRec()` updates the display based on the current value of `fStatus`. When `fStatus=0` is used as the initial setting, menu items are first presented as shaded rectangles. Rectangular grids with uncalculated values make up the display when `fStatus=2`, and rectangular grids with calculated values make up the display when `fStatus=3`. The finite difference method is used to calculate the wave equation as shown in the algorithm below.

Input: i,j
Process:

for $i = 1$ to $i \leq n$

$$u_{i,1} = \frac{1}{2h^2}\left[2h^2 kuD_i + k^2\alpha^2 u_{i,0} - (-2h^2 + 2k^2\alpha^2 u_{i,0} + k^2\alpha^2 u_{i+1,0}\right]$$

for $j = 1$ to $j \leq m$

    for $i = 1$ to $i \leq n$

$$u_{i,j+1} = \frac{1}{h^2}\left[-h^2 u_{i,j-1} + k^2\alpha^2 u_{i-1,j} - (-2h^2 + 2k^2\alpha^2 u_{i,j} + k^2\alpha^2 u_{i+1,j}\right]$$

Simulation result

The visual interface consists of an input region which contains alpha, matrix, value of x, t and *compute* button. It also consists region for display the result in rectangular grids as its basic components. Figure 1 shows the interface output of the wave equation while Figure 2 shows the output of the wave equation. User need to fill in the input region according to the related function before compute the equation. After all the input are filled, user can compute the equation using *compute* button. An empty grid region will be appeared below the input region. User need to fill the boundary condition before click the *compute* button to display the numerical result.
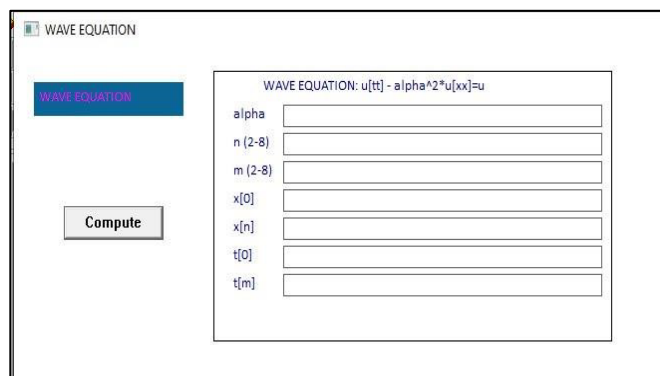
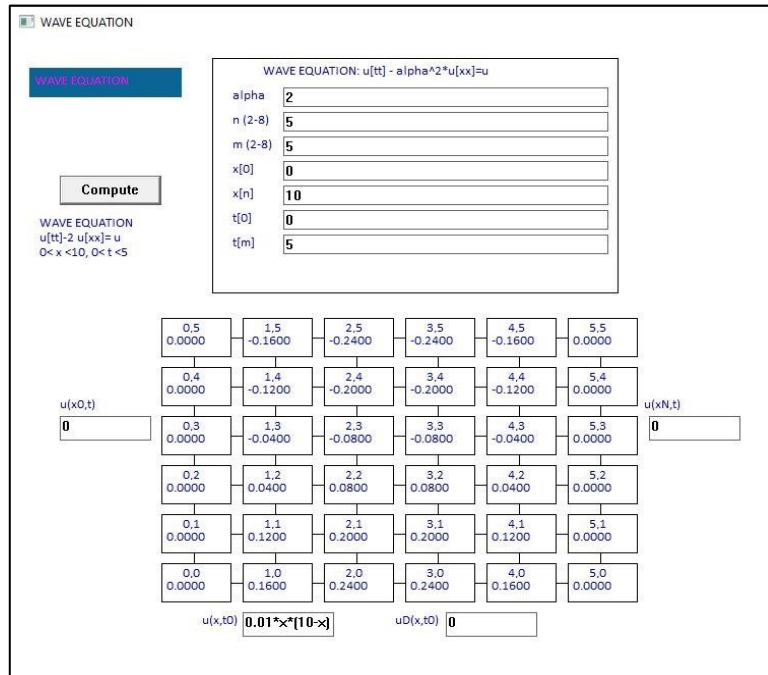**Figure 1.** The Visual interface of the wave equation

Figure 2. The output of wave programming

RESULTS AND DISCUSSION

The results for both methods are shown in Table 2. It is included the results of numerical calculation and theoretical calculation. It also consists of the errors value from both methods. The calculation to finds the errors are using the following formula,

$$| \, error \, | \; = \; | \, u(x,t) \, | \; - \; | \, u(i,j) \, |$$

Table 2. Results of the problem using numerical and theoretical methods

i	j	x_i	t_j	$u(i,j)$ (numerical solution)	$u(x,t)$ (exact solution)	$\lvert \, error \, \rvert$
0	0	0	0	0.0000	0.0000	0.0000
0	1	0	1	0.0000	0.0000	0.0000
0	2	0	2	0.0000	0.0000	0.0000
0	3	0	3	0.0000	0.0000	0.0000
0	4	0	4	0.0000	0.0000	0.0000
0	5	0	5	0.0000	0.0000	0.0000
2	0	2	0	0.1600	0.1517	0.0083
2	1	2	1	0.1200	0.1227	0.0027
2	2	2	2	0.0400	0.0469	0.0069
2	3	2	3	-0.0400	-0.0469	0.0069
2	4	2	4	-0.1200	-0.1227	0.0027
2	5	2	5	-0.1600	-0.1517	0.0083
4	0	4	0	0.2400	0.2454	0.0054
4	1	4	1	0.2000	0.1985	0.0015
4	2	4	2	0.0800	0.0758	0.0042
4	3	4	3	-0.0800	-0.0758	0.0042
4	4	4	4	-0.2000	-0.1985	0.0015
4	5	4	5	-0.2400	-0.2454	0.0054
6	0	6	0	0.2400	0.2454	0.0054

i	j	x_i	t_j	$u(i,j)$ (numerical solution)	$u(x,t)$ (exact solution)	$\lvert error \rvert$
6	1	6	1	0.2000	0.1985	0.0015
6	2	6	2	0.0800	0.0758	0.0042
6	3	6	3	-0.0800	-0.0758	0.0042
6	4	6	4	-0.2000	-0.1985	0.0015
6	5	6	5	-0.2400	-0.2454	0.0054
8	0	8	0	0.1600	0.1517	0.0083
8	1	8	1	0.1200	0.1227	0.0027
8	2	8	2	0.0400	0.0469	0.0069
8	3	8	3	-0.0400	-0.0469	0.0069
8	4	8	4	-0.1200	-0.1227	0.0027
8	5	8	5	-0.1600	-0.2454	0.0083
10	0	10	0	0.0000	0.0000	0.0000
10	1	10	1	0.0000	0.0000	0.0000
10	2	10	2	0.0000	0.0000	0.0000
10	3	10	3	0.0000	0.0000	0.0000
10	4	10	4	0.0000	0.0000	0.0000
10	5	10	5	0.0000	0.0000	0.0000
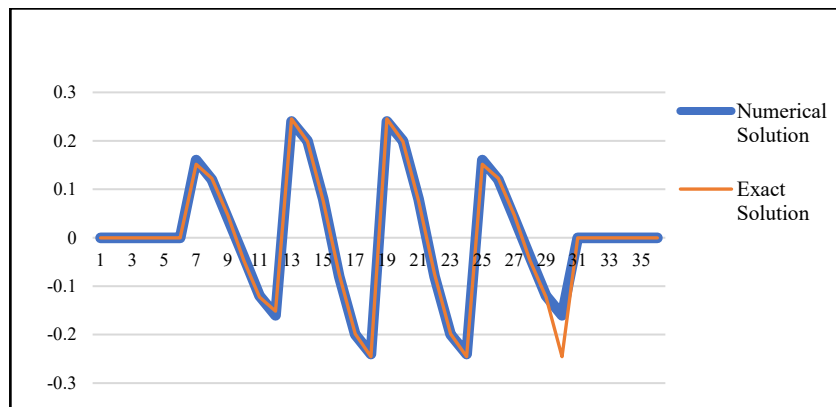
Figure 3. Comparison between numerical solution and exact solution

From the results in the Table 2, the value of errors from both methods are very small which means the calculation of the numerical method is accurate as the calculation of the theoretical method. The comparison of the error is shown in the Figure 3. The finite difference method can help other reseachers in solving the wave equation and this method is more easier to calculate the solution for the wave equation. The error values by using both approaches are minimal which is less than 0.01. It shows that both methods are accurate in solving the equation but it is more convenient using numerical method because it is easier to solve the equation. This programming is user friendly for the user in the mathematical field. The user can use the programming to solve the complex equation that related to the wave equation.

REFERENCES

[1]   T. Gowers, *The Princeton Companion to Mathematics,* New Jersey, United States: Princeton University Press, 2008, pp. 141-168.

[2]   Q.J. Zhang, and Y. Ding, "Wave equation-based time-varying formation control of Multiagent   Systems," IEEE Transactions on Control Systems Technology, vol. 26, no. 5, pp. 1578–1591, 2018.

[3]   X.D. Dongfang, "Dongfang Modified Equations of Electromagnetic Wave," Mathematics & Nature., vol. 1, no. 1, pp. 008-1 – 008-9, 2021.

[4]   R. Rani and T.S.L. Radhika, "On a PDE model for application in Industrial Engineering," The Journal of ZEPHYRUS, vol. 11, no. 5, pp. 153–168, 2021.

[5]   J. Alcubilla, "Wave Equations of Electrogravitodynamics," IOSR Journal of Applied Physics, vol. 12, no. 5, pp. 34–49, 2020.

[6] Banjai and E. Otárola, "A PDE approach to fractional diffusion: A space-fractional wave equation," Numerische Mathematik, vol. 143, no. 1, pp. 177–222, 2019.

[7] Pskhu and S. Rekhviashvili, "Fractional diffusion–wave equation with application in electrodynamics," Mathematics, vol. 8, no. 11, p. 2086, 2020.