$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where $P_t$ is the closing price at time $t$, and $P_{t-1}$ is the closing price at time $t-1$.

According to Dowd [16], given a random variable $X$, VaR is defined as in equation (2).

$$\mathrm{VaR}(X; p) = F^{-1}(p) \tag{2}$$

for all $p \in (0, 1)$, where $F^{-1}$ is the quantile function.

### Non-parametric methods

The non-parametric approach is a method that uses historical data to evaluate VaR. It does not make any assumption about the past data and it mainly relies on the historical simulation method. Dowd [16] stated that the short-term VaR forecasting will not change significantly but it will ensure that the recent trend is similar to the previous trend to the greatest extend. One of the reasons that historical simulation was commonly used is because it still gives a relatively accurate result although some of the problems have been ignored.

1. Basic Historical Simulation (HS) Method

The basic historical simulation method is widely used in the financial and non-financial institutions because of its simplicity. The purpose of this method is to estimate the VaR from the past data without making any assumption. The return series will be calculated and sorted in descending order, then VaR will be estimated by determining its position using $N(1-\alpha)+1$ where $N$ is the total observation and $\alpha$ is the significance level.

2. Age-Weighted (AW) Historical Method

In this method, each observation will be given different weight. The most recent observation will be given a heavier weight than the previous observation because the probability of the recent loss happening again was higher. Age-weighted method involves a decaying factor, $\lambda$ where it's value is in the range between 0 and 1. The value of $\lambda$ indicates the decaying process of an asset. The closer $\lambda$ is to 0, the faster the decay process of an asset, and the slower the decay process of an asset when the value of $\lambda$ is close to 1. The weight of each observation was introduced by Boudoukh et al. [17] as shown in equation (3).

$$w(t) = \frac{\lambda^{t-1}(1-\lambda)}{1-\lambda^n} \tag{3}$$

where $\lambda$ is the decaying factor $(0 < \lambda < 1)$, $t$ is the time period, and $n$ is the total number of observation. Boudoukh et al. [17] recommended using $\lambda = 0.98$. The summation of the weights for the observations is equal to one. After calculating the $w(t)$ of every data, the cumulative value of each observation will be calculated. Finally, based on the selected confidence level, interpolation is performed to obtain the VaR.

### Parametric methods

The parametric approach is the opposite way compared to the non-parametric approach where it makes an assumption that the return of assets follows a probability distribution. The parametric approach is also known as the analytic or correlation method. Ruppert [18] claims that it is the simplest method to estimate VaR by using parametric method, especially when there are large numbers of assets in the portfolio, instead of only one asset. In this study, we will implement four parametric methods.

1. Normal Distribution

Normal distribution is symmetrical around its mean and has small tails. The normal distribution is widely used because only two variables are needed to calculate VaR. These are the mean $\mu$ ($\mu \in \mathbb{R}$) and standard deviation $\sigma$ ($\sigma > 0$). The probability distribution function (PDF) of normal distribution is shown in equation (4).

$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right] \tag{4}$$

Normal VaR can be estimated by using equation (5).

$$\mathrm{VaR} = \mu \quad - \tag{5}$$

where $\sigma$ is the $\sigma$ -score corresponding to the 95% confidence level [16].

2. Student’s t-distribution

Student’s t-distribution adapts to the fat tail in the sample by adjusting the degree of freedom, $\nu$. Hence, student’s t-distribution is usually used to fit the financial return because it often produces value far from the average. When the number of degrees of freedom goes towards infinity the Student’s t-distribution goes towards being normally distributed [19]. The PDF of student’s t-distribution is shown in equation (6).

$$f(x|\nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi\nu}\left(1+\frac{x^2}{\nu}\right)^{\frac{\nu+1}{2}}} \tag{6}$$

The student’s t VaR can be estimated by using the formula below.

$$\text{VaR} = \mu - \sigma t_{\sigma,\nu} \tag{7}$$

where $t_{\sigma,\nu}$ is the $t$-score corresponding to the 95% confidence level and degree of freedom, $\nu$.

3. Generalized Extreme Value (GEV) Distribution

Generalized extreme value distribution is the limiting model for maximum and minimum values of a data set. This is very important in risk modelling, because the probability of occurrence of rare event is always low [20]. GEV distribution is a family of continuous probability distributions developed from the extreme value theory. It unites the distribution of Gumbel, Fréchet, and Weibull which are also known as Type I, Type II, and Type III distribution. GEV distribution contains 3 parameters, namely, location ($\mu \in \mathbb{R}$), scale ($\sigma > 0$), and shape ($\xi \in \mathbb{R}$) parameters. In this study, the block-maximum method with one day block size was implemented. The PDF of GEV distribution is shown below.

$$f(x;\mu,\sigma,\xi) = \begin{cases} \frac{1}{\sigma}(1+\xi\frac{(x-\mu)}{\sigma}\exp\left(-\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}\right) & \text{if } \xi \neq 0 \text{ and } 1+\xi\frac{(x-\mu)}{\sigma} > 0 \\ \frac{1}{\sigma}\exp\left(-\frac{(x-\mu)}{\sigma}\right)\exp\left(-\exp\left(-\frac{(x-\mu)}{\sigma}\right)\right) & \text{if } \xi = 0 \end{cases} \tag{8}$$

The GEV VaR can be estimated by using the equation below.

$$\text{VaR} = \mu - \frac{\sigma}{\xi} + \frac{\sigma}{\xi}(-\ln(p))^{-\xi} \tag{9}$$

4. Variance Gamma (VG) Distribution

VG distribution is a distribution that can accommodate data sets with heavy tails and high centres, and it is more peaked than normal distribution [21]. VG distribution is a flexible distribution and having different appearance depends on its parameter value. It has 4 parameters, which are location ($\mu \in \mathbb{R}$), scale ($\sigma > 0$), shape ($\lambda > 0$), and asymmetry ($\alpha \in \mathbb{R}$) parameters. The PDF of VG distribution is shown in equation (10).

$$f(x;\mu,\sigma,\lambda,\alpha) = \sqrt{\frac{\pi}{2}}\,\frac{\lambda^{\frac{1}{\lambda}}\exp\left(\frac{\alpha(x-\mu)}{\sigma^2}\right)}{\sigma\Gamma\left(\frac{1}{\lambda}\right)}\left(\frac{|x-\mu|}{\sqrt{\alpha^2+\frac{2\sigma^2}{\lambda}}}\right)^{\frac{1}{\lambda}-\frac{1}{2}} K_{\frac{1}{\lambda}-\frac{1}{2}}\left(\frac{|x-\mu|\sqrt{\alpha^2+\frac{2\sigma^2}{\lambda}}}{\sigma^2}\right) \tag{10}$$

where $-\infty < x < \infty$, $\Gamma(\cdot)$ is a gamma function and $K_\eta(w)$ is a modified bessel function of the third kind with order $\eta$. $K_\eta(w)$ for $\eta \in \mathbb{R}$ and $w > 0$ is given by equation (11).

$$K_\eta(w) = \frac{1}{2}\int_0^\infty z^{\eta-1}\exp\left(-\frac{w}{2}\left(z+\frac{1}{z}\right)\right)dz \tag{11}$$

The VG VaR equation is very complicated and it is usually computed by using statistical software.

### Parametric estimation method

In this study, we have chosen the maximum likelihood estimation method to estimate the parameters. This is because it is asymptotically normal and allows simple approximation of the standard error and confidence interval.

The likelihood principle proposed that given a statistical model, all the evidence related to model parameters in the sample are included in the likelihood function. It returns the maximum likelihood estimator to estimate an unknown parameter, $\theta$.

Let $X_1, X_2, X_3, \dots, X_n$ be $n$ independent and identically distributed observations which having a same density function $f(x|\theta)$, where $\theta$ is an unknown constant in a parameter space, $\Theta$.

Given the data $s = (x_1, x_2, x_3, \dots, x_n)$, the likelihood function of $\theta$ is defined in equation (12).

$$L_n(\theta \mid s) = \prod_{i=1}^{n} f(x_i \mid \theta) \tag{12}$$

The maximum likelihood estimator $\hat{\theta}_n(s)$ for $\theta$ is a value in the parameter space $\Theta$ which maximizes the likelihood function, such that

$$L_n(\hat{\theta}_n(s) \mid s) = \max_{\theta \in \Theta} \prod_{i=1}^{n} f(x_i \mid \hat{\theta}) \tag{13}$$

The standard method to obtain the maximum likelihood estimator, $\hat{\theta}_n(s)$ is to find the root of the following log-likelihood function:

$$l_n^{(1)}(s \mid \hat{\theta}) = \frac{d}{d\theta} \ln L_n(\hat{\theta} \mid s) \tag{14}$$

### Christoffersen backtesting method

VaR models are valuable if they can foresee the future risk accurately. Hence, the models should be backtested with proper method to ensure the model accuracy [22]. The backtesting method used in this study is Christoffersen conditional coverage test which is a joint test of conditional coverage and independence by Christoffersen [23].

A $\text{VaR}_{t+1}^{p}$ measure promise that the actual loss will not exceed $\text{VaR}_{t+1}^{p}$ forecast $p * 100\%$ of the time. According to Christoffersen [23], the hit sequence of VaR violations, $\{I_{t+1}\}$ is defined as:

$$I_{t+1} = \begin{Bmatrix} 1, \text{if } r_{t+1} > \text{VaR}_{i+1}^{p} \\ 0, \text{if } r_{t+1} < \text{VaR}_{i+1}^{p} \end{Bmatrix} \tag{15}$$

where $p$ is defined as the probability of actual loss exceed predicted loss. The hit sequence return 1 when the actual return is smaller than the VaR value predicted, and vice versa.

When performing a backtest, it is necessary to construct a sequence $\{I_{t+1}\}_{t=1}^{T}$ across $T$ days indicating when the past violations occur. For violation prediction, the hit sequence should be unpredictable and distributed independently over time $T$ as a Bernoulli variable that takes value of 1 with probability $p$ and value 0 with probability $(1-p)$. The Bernoulli distribution is given as follow.

$$f(I_{t+1}; p) = (1-p)^{1-I_{t+1}}\, p^{I_{t+1}} \tag{16}$$

1. Unconditional Coverage Test

Unconditional Coverage test is used to determine whether the fraction of violations, $\hat{\pi}$ is significantly different from the coverage rate, $p$. The null and alternative hypothesis is shown as:

$$\begin{aligned} H_0 &: p = \pi \\ H_1 &: p \neq \pi \end{aligned} \tag{17}$$

Likelihood for null hypothesis $L(p)$ and alternative hypothesis $L(\hat{\pi})$ are necessary to perform before the test was conducted.

$$L(p) = \prod_{t=1}^{T} f(I_{t+1}; p) = (1-p)^{T_0}\, p^{T_1} \tag{18}$$

$$L(\hat{\pi}) = \prod_{t=1}^{T} f(I_{t+1}; \hat{\pi}) = (1-\hat{\pi})^{T_0}\, \hat{\pi}^{T_1} \tag{19}$$

where $T_0$ and $T_1$ are the number of zeros and ones in the sample respectively, and $\hat{\pi}$ is the fraction of violations in the sample, which is computed as $\frac{T_1}{T}$.

The unconditional coverage hypothesis using the likelihood ratio test is then compute as:

$$LR_{UC} = -2\ln\frac{L(p)}{L(\hat{\pi})} \tag{20}$$

Apart from that, $p$-value associated with the test statistic can also computed by using $1 - F_{\chi_1^2}(LR_{UC})$ where $p$-value is defined as the probability of incorrectly reject a true null hypothesis, and $F_{\chi_1^2}(\cdot)$ denotes the cumulative distribution function of a $\chi^2$ variable with one degree of freedom. The null hypothesis is rejected if $p$-value is less than the desired significance level.

2. Independence Test

The objective of independence test is to ensure every violation is independent to each other. Hence it is necessary to assume that the violation is time-varying and described as a first-order Markov sequence with transition matrix of $\pi_1$.

$$\pi_1 = \begin{pmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{pmatrix} \tag{21}$$

where $\pi_{ij} = P(I_{t+1} = j | I_t = i)$. The first-order Markov sequence must satisfy Markov properties, that is, only today's results will affect tomorrow's outcome, which means only $I_t$ matters on $I_{t+1}$.

Given a likelihood sample of $T$ observations, the likelihood function of the first-order Markov process is

$$L(\pi_1) = (1-\pi_{01})^{T_{00}}\, \pi_{01}^{T_{01}}\, (1-\pi_{11})^{T_{10}}\, \pi_{11}^{T_{11}} \tag{22}$$

where $T_{ij}$ is the number of observations with a $j$ following an $i$ with $i, j \in \{0,1\}$.

The maximum likelihood estimates for $\pi_{01}$ and $\pi_{11}$ are as follows.

$$\hat{\pi}_{01} = \frac{T_{01}}{T_{00} + T_{01}} \tag{23}$$

$$\hat{\pi}_{11} = \frac{T_{11}}{T_{10} + T_{11}} \tag{24}$$

The estimated transition matrix, $\hat{\pi}_1$ is given as

$$\hat{\pi}_1 = \begin{pmatrix} 1-\hat{\pi}_{01} & \hat{\pi}_{01} \\ 1-\hat{\pi}_{11} & \hat{\pi}_{11} \end{pmatrix} = \begin{pmatrix} \dfrac{T_{00}}{T_{00}+T_{01}} & \dfrac{T_{01}}{T_{00}+T_{01}} \\ \dfrac{T_{10}}{T_{10}+T_{11}} & \dfrac{T_{11}}{T_{10}+T_{11}} \end{pmatrix} \tag{25}$$

If each violations are independent to each other, $\hat{\pi}_{01} = \hat{\pi}_{11} = \hat{\pi}$. Hence, the transition matrix is shown as below under independence assumption.

$$\hat{\pi} = \begin{pmatrix} 1-\hat{\pi} & \hat{\pi} \\ 1-\hat{\pi} & \hat{\pi} \end{pmatrix} \tag{26}$$

A likelihood ratio test $LR_{IND}$ is used to test the independence hypothesis, $\pi_{01} = \pi_{11}$.

$$LR_{IND} = -2\ln\left[\frac{L(\hat{\pi})}{L(\hat{\pi}_1)}\right] \sim \chi_1^2 \tag{27}$$

3. Conditional Coverage Test

The conditional coverage test will be continued when VaR violations are independent to each other and the expected number of violations is correct. The likelihood ratio test statistic is shown as follows.

$$\begin{aligned} LR_{CC} &= -2\ln\left(\frac{L(p)}{L(\hat{\pi}_1)}\right) \sim \chi_2^2 \\ &= -2\ln\left(\frac{L(p)}{L(\hat{\pi})}\cdot\frac{L(\hat{\pi})}{L(\hat{\pi}_1)}\right) \\ &= -2\ln\left(\frac{L(p)}{L(\hat{\pi})}\right) - 2\ln\left(\frac{L(\hat{\pi})}{L(\hat{\pi}_1)}\right) \\ &= LR_{UC} + LR_{IND} \end{aligned} \tag{28}$$

Therefore, the joint test of conditional coverage can be calculated by adding unconditional coverage and independence test [24].

## RESULTS AND DISCUSSION

### Data and empirical findings of stock returns

In this study, the daily closing stock prices is collected from Investing.com. The collected data covered 2516 observations from 4 January 2010 to 31 December 2019. Daily stock prices were converted into daily log-return series using R package *quantmod* [25] to determine the behavior of the stock returns. Table 1 presented descriptive statistics of stock returns.

**Table 1.** Descriptive summary statistics of daily log-return.

| Stock | Mean | Standard Deviation | Skewness | Kurtosis | Median | Minimum | Maximum |
|---|---|---|---|---|---|---|---|
| AAPL | 0.0009 | 0.0159 | -0.2619 | 7.9707 | 0.0009 | -0.1319 | 0.0850 |
| GOOGL | 0.0006 | 0.0150 | 0.7706 | 16.7712 | 0.0005 | -0.0875 | 0.1506 |
| MSFT | 0.0006 | 0.0140 | -0.1109 | 10.7640 | 0.0003 | -0.1210 | 0.0994 |

From Table 1, it can be seen that the mean daily log-return of three stocks is positive, which indicates the corresponding stock gives a slightly increasing positive return. The standard deviation of the stocks is less volatile, and the returns are stable because their standard deviation is relatively small. In addition, the skewness of AAPL and MSFT is negative, which means that their return distribution has a long left tail, while the income distribution of GOOGL has a long right tail because of the positive skewness coefficient. Apart from that, the large kurtosis value indicates that the return series of these three stocks have a leptokurtic distribution with a fatter tail compared to normal distribution.

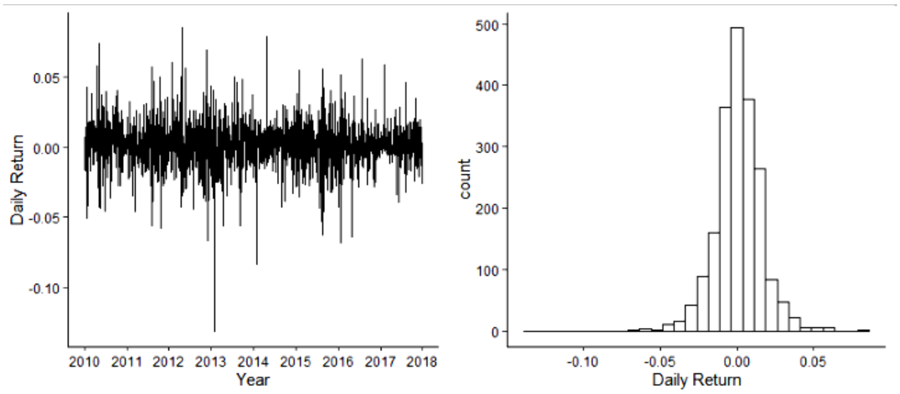


**Figure 1.** Time series plots of AAPL daily return (left) and its histogram (right).

In Figure 1, the return of AAPL shows a highly fluctuating pattern. In 2013, the stock return of AAPL fell about 26% due to the increase of the android market eroded the iOS dominance. From the histogram, we can see that the stock return of AAPL are quite symmetrical and have a fatter left tail. The fatter left tail means AAPL stock has a large number of negative returns.

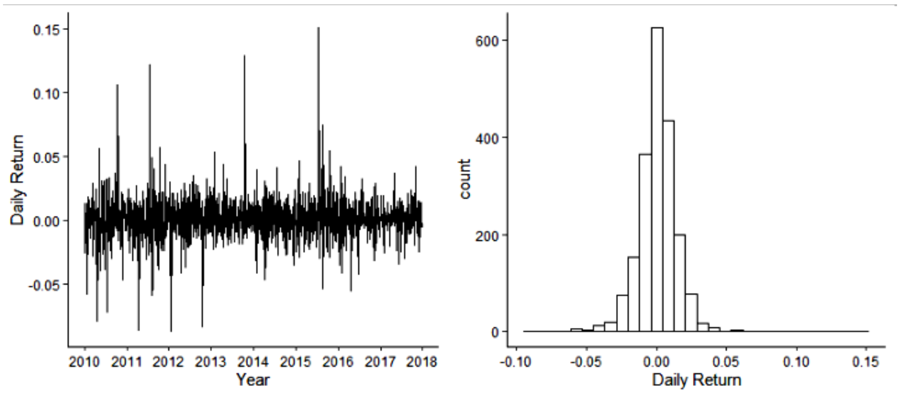


**Figure 2.** Time series plots of GOOGL daily return (left) and its histogram (right).

In Figure 2, the return of GOOGL shows a high volatile fluctuation pattern. In August 2015, after Alphabet announced its reorganization, the stock return of GOOGL soared. From the histogram, we can see that the stock return of GOOGL is asymmetric and had a fatter right tail. The fatter right tail means that GOOGL stock gives a large amount of profit.

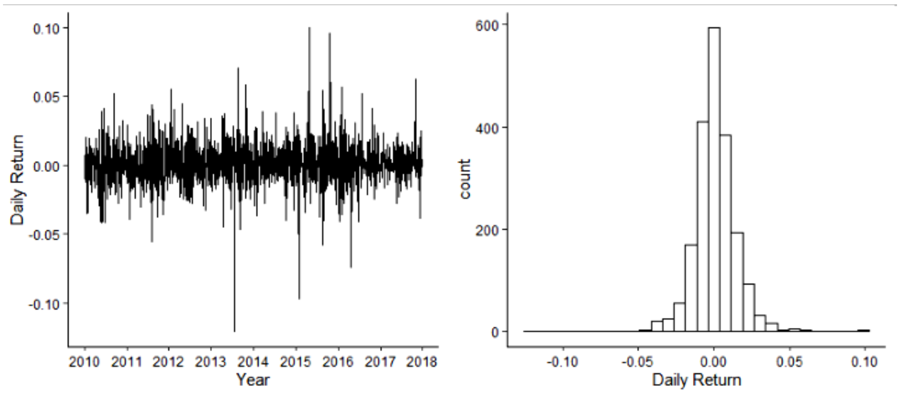


**Figure 3.** Time series plots of MSFT daily return (left) and its histogram (right).

In Figure 3, the return of MSFT shows a high volatile fluctuation pattern after 2013. MSFT experienced an unprecedented drop as it's share drops 12%. Microsoft's results showed a \$900 million loss due to unsold Microsoft's Surface RT. From the histogram, we can see that the stock return of MSFT was fairly symmetrical and have a fatter left tail. The fatter left tail means that MSFT stock gives a large number of negative returns.

**Table 2.** Shapiro-wilk test.

| Company | $W_{stat}$ | p-value |
|---|---|---|
| AAPL | 0.95392 | $2.2 \times 10^{-16}$ |
| GOOGL | 0.89383 | $2.2 \times 10^{-16}$ |
| MSFT | 0.93421 | $2.2 \times 10^{-16}$ |

The normality test has become a standard feature in statistical work. For continuous data, the normality test is very important in determining the measure of central tendency and statistical methods for data analysis. In this study, the Shapiro-Wilk test will be used to check the normality assumption of the data and the result is shown in Table 2. Since the p-value of AAPL, GOOGL and MSFT are very small and less than significance level of 0.05, the null hypothesis was rejected, which means that the negative log-return data of three companies are not normally distributed. The results of Shapiro-Wilk test can also be supported by using Q-Q plot (Figure 4).

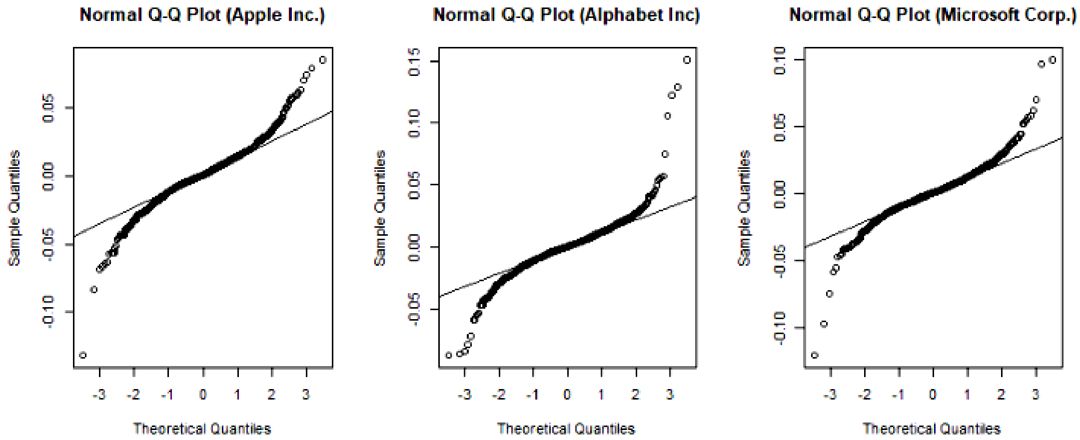


**Figure 4.** QQ-Plot of (a) AAPL, (b) GOOGL and (c) MSFT.

## Parameter estimation

In this section, the parameters of the selected distribution were estimated via maximum likelihood estimation. The selected distributions are implemented by using the R packages *fitdistrplus, QRM, ismev, evd* and *VarianceGamma* [26 – 30]. Table 3, 4 and 5 showed the estimated parameters from AAPL, GOOGL, and MSFT stock returns with the normal distribution, student's t-distribution, GEV distribution, and Variance Gamma distribution.

**Table 3.** The estimated parameters of the three distributions for AAPL stock return.

| Model | Parameters | | | |
|---|---|---|---|---|
| Normal | μ | σ | - | - |
| | 0.000905 | 0.015938 | | |
| Student's t | μ | σ | ν | - |
| | 0.001049 | 0.011376 | 3.909471 | |
| GEV | μ | σ | ξ | - |
| | -0.005444 | 0.018050 | -0.195952 | |
| VG | μ | σ | α | λ |
| | 0.001571 | 0.015027 | -0.000670 | 0.347793 |

**Table 4.** The estimated parameters of the three distributions for GOOGL stock return.

| Model | Parameters | | | |
|---|---|---|---|---|
| Normal | μ | σ | - | - |
| | 0.000602 | 0.014947 | | |
| Student's t | μ | σ | ν | - |
| | 0.000718 | 0.009748 | 3.543739 | |
| GEV | μ | σ | ξ | - |
| | -0.005878 | 0.017371 | -0.105881 | |
| VG | μ | σ | α | λ |
| | 0.000968 | 0.13420 | -0.000367 | 0.313369 |

**Table 5.** The estimated parameters of the three distributions for MSFT stock return.

| Model | Parameters | | | |
|---|---|---|---|---|
| Normal | μ | σ | - | - |
| | 0.000608 | 0.014028 | | |
| Student's t | μ | σ | ν | - |
| | 0.000464 | 0.00694 | 3.754225 | |
| GEV | μ | σ | ξ | - |
| | -0.005222 | 0.016958 | -0.158615 | |
| VG | μ | σ | α | λ |
| | -0.000257 | 0.013006 | 0.000866 | 0.314193 |

From Table 3–5, the return series of AAPL, GOOGL, and MSFT are compressed at the centre near to zero because the value of location and scale parameters of each distribution are approximately zero. For Student's t-distribution, the degree of freedom indicates the heaviness of the tail of the t-distribution. Since the degree of freedom of the return series of AAPL, GOOGL, and MSFT are small, it indicated that the tails of the return series for t-distribution is higher than the tails of the normal distribution. For GEV distribution, the shape parameter governed the type of distribution and the shape of the tail. The tail of GEV distribution is thicker when it has a higher shape parameter value. Since the shape parameter of the three stock returns are negative value, this means that the return series are distributed as Weibull distribution. For VG distribution, the asymmetry parameter ($\alpha$) and shape parameter ($\lambda$) will change the shape of the distribution. A positive $\alpha$ value will skew the distribution to the right and vice versa. The peakedness of VG distribution will increase when $\lambda$ decreases or approaches to zero. Since the return series of AAPL and GOOGL have negative $\alpha$ and a small $\lambda$, this indicates that the return series have a high peak and skewed to the left. In contrast, the return series of MSFT is skewed to the right due to its positive $\alpha$ value.

### Value-at-Risk

In this section, VaR of the daily log-return series for selected stocks are computed by using basic historical simulation method, AW historical method, normal distribution, student's t-distribution, GEV distribution, and VG distribution. 95% VaR are calculated for 2012 business days in the time period 01/01/2010 to 31/12/2017. The result is shown in Table 6.

**Table 6.** A 95% VaR estimates of selected methods.

| | Non-Parametric Method | | Parametric Method | | | |
|---|---|---|---|---|---|---|
| | HS | AW-HS ($\lambda = 0.98$) | Normal | Student's t | GEV | VG |
| AAPL | -2.50 | -1.91 | -2.53 | -2.34 | -2.75 | -2.39 |
| GOOGL | -2.20 | -1.30 | -2.40 | -2.09 | -2.61 | -2.15 |
| MSFT | -1.99 | -1.11 | -2.25 | -2.06 | -2.55 | -2.05 |

For the basic historical simulation method, the VaR at a 95% confidence interval is the daily return located at the 5% percentile of the daily return series. At the confidence level of 95%, the lowest risk value is -1.99% in Microsoft Corporation. At the same level of reliability, the highest risk value has achieved stocks of Apple Inc -2.50%. The reliability level of 95% can be interpreted as that the stock return will not fall below the calculated risk value within 100 working days for 95 working days.

For AW historical method with 95% confidence level, Microsoft Corporation has the lowest risk, followed by Google Inc. and Apple Inc. This is because the lowest risk value is -1.11% of Microsoft Corporation, while the highest risk value is -1.91% of Apple's stock at the same reliability level. The maximum loss of AAPL shows that Apple is the most risky stock because it has the highest risk value, which means that the loss would not exceed 1.91% for 95 days of 100 days.

For normal distribution approach at the level of 95% confidence, Microsoft Corporation is again the share which is expected to experience the least loss. Microsoft Corp. has the lowest risk value -2.25% while at the same level of reliability highest risk value has achieved stocks of Apple Inc -2.53%. This shows that analysts have 95% confidence that the daily loss of AAPL shares will not exceed -2.53%. For example, when an investor invests 1000 ringgit in AAPL stock, there is 95% confidence that the investor will not experience the most serious daily loss, that is, the loss of no more than 25.30 ringgit.

Apart from that, Microsoft Corporation has the lowest loss of -2.06% at a 95% confidence level compared to Apple Inc and Google Inc when student's t-distribution was used. From the result, we can see that Apple Inc is the top risky share, which it's worst daily loss was expected at not more than -2.34%. Among other companies, Google Inc ranked second risky share with 2.09% loss.

Unlike the other distributions, the GEV distribution is mainly concentrated on the tail data. At 95% confidence level, Microsoft is the lowest risk stock (-2.55%), followed by Google Inc (-2.61%) and Apple Inc (-2.75%). The minimum loss of MSFT indicates that MSFT is the least risky stock, because its risk value is the smallest, which means that the loss would not exceed 2.55% for 95 days of 100 days.

For VG distribution, AAPL remains the most risky share which will bring the biggest loss of 2.39%, while MSFT remains as the least risky share. It can also be easily understood by using an example of, the investor will not lose more than RM23.90 if he or she makes an RM1000 investment on AAPL stock, but the investor will not lose exceed RM20.50 if he or she makes an RM1000 investment on MSFT.

In comparison for all companies, it can be observed that the 95% VaR estimates by using AW historical method are relatively low compared with the basic historical method, normal distribution, student's t-distribution, GEV distribution, and VG distribution. Next, the stock of MSFT gives the lowest VaR value compared to the other stocks at 95% confidence level no matter using which methods, followed by GOOGL and AAPL. In fact, MSFT shares will be recommended to investors because of its lower risk level.

### Backtesting

In this section, the VaR from 2018 to 2019 is tested by using the stock returns from 2017 to 2018 to ensure its accuracy. The estimated VaR is then compared with the real return. The backtest procedure will be conducted by using the unconditional coverage test, independence test, and conditional coverage test. The results were discussed in the sections below.

### VaR-breaks observations

At 95% confidence level, the number of 95% VaR produced by each distribution is compared to the expected number of the violation. The result is shown in Table 7.

**Table 7.** Expected and actual number of 95%-VaR obtained by each model.

| | AAPL | GOOGL | MSFT |
|---|---|---|---|
| Trading Days | 479 | 479 | 479 |
| Expected $X_{VaR}(0.05)$ | 23 | 23 | 23 |
| **Basic Historical Simulation Method** | | | |
| Actual $X_{VaR}(0.05)$ | 27 | 24 | 28 |
| **AW Historical Method** | | | |
| Actual $X_{VaR}(0.05)$ | 5 | 3 | 4 |
| **Normal** | | | |
| Actual $X_{VaR}(0.05)$ | 28 | 28 | 32 |
| **Student's t** | | | |
| Actual $X_{VaR}(0.05)$ | 32 | 35 | 36 |
| **GEV** | | | |
| Actual $X_{VaR}(0.05)$ | 27 | 25 | 30 |
| **VG** | | | |
| Actual $X_{VaR}(0.05)$ | 28 | 24 | 29 |

From Table 7, it can be observed that almost all distribution underestimates the VaR, as the actual VaR exceedance is higher than expected VaR violation, except AW historical method. At the confidence level of 95%, AW historical method has overestimated the VaR value. This is because the actual VaR-breaks were extremely low compared to the expected VaR exceedance.

### Unconditional coverage test

The unconditional coverage test is then conducted to determine whether the fraction of violation $\hat{\pi}$ is significant different from the coverage rate, *p*. The result is shown in Table 8. From the table, the low p-value indicates that the corresponding distribution does not perform well at a 95% confidence level. For nonparametric method, the historical method performs well on three data sets, because it gives a high p-value. For the parametric method, when compared with normal distribution and student t- distribution, GEV and VG distribution generally perform relatively well.

**Table 8.** Unconditional coverage test for 95%-VaR.

| | AAPL | GOOGL | MSFT |
|---|---|---|---|
| **Basic Historical Simulation Method** | | | |
| $LR_{UC}$ | 0.393 | 0.00011 | 0.685 |
| *p-value* | 0.531 | 0.992 | 0.408 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **AW Historical Method** | | | |
| $LR_{UC}$ | 23.013 | 30.386 | 26.445 |
| *p-value* | 0 | 0 | 0 |
| *Conclusion* | **Reject** | **Reject** | **Reject** |
| **Normal** | | | |
| $LR_{UC}$ | 0.685 | 0.685 | 2.588 |
| *p-value* | 0.408 | 0.408 | 0.108 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **Student's t** | | | |
| $LR_{UC}$ | 2.588 | 4.727 | 5.566 |
| *p-value* | 0.108 | 0.030 | 0.018 |
| *Conclusion* | Fail to reject | **Reject** | **Reject** |
| **GEV** | | | |
| $LR_{UC}$ | 0.393 | 0.048 | 1.495 |
| *p-value* | 0.531 | 0.827 | 0.221 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **VG** | | | |
| $LR_{UC}$ | 0.685 | 0.00011 | 1.053 |
| *p-value* | 0.408 | 0.992 | 0.305 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |

### Independence test

The VaR violations dependence is examined using independence test. The result is shown in Table 9.

**Table 9.** Independence test for the 95%-VaR.

| | AAPL | GOOGL | MSFT |
|---|---|---|---|
| **Basic Historical Simulation Method** | | | |
| *$LR_{IND}$* | 3.276 | 0.494 | 2.861 |
| *p-value* | 0.070 | 0.482 | 0.091 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **AW Historical Method** | | | |
| *$LR_{IND}$* | 0.106 | 0.038 | 2.350 |
| *p-value* | 0.745 | 0.845 | 0.125 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **Normal** | | | |
| *$LR_{IND}$* | 2.861 | 0.084 | 4.514 |
| *p-value* | 0.091 | 0.772 | 0.034 |
| *Conclusion* | Fail to reject | Fail to reject | **Reject** |
| **Student's t** | | | |
| *$LR_{IND}$* | 1.524 | 0.154 | 2.784 |
| *p-value* | 0.217 | 0.695 | 0.095 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **GEV** | | | |
| *$LR_{IND}$* | 3.276 | 0.357 | 4.274 |
| *p-value* | 0.070 | 0.550 | 0.039 |
| *Conclusion* | Fail to reject | Fail to reject | **Reject** |
| **VG** | | | |
| *$LR_{IND}$* | 2.861 | 0.494 | 4.791 |
| *p-value* | 0.091 | 0.482 | 0.029 |
| *Conclusion* | Fail to reject | Fail to reject | **Reject** |

From Table 9, the non-parametric method performed well for the three data sets. For the parametric method, the distributions work well in AAPL and GOOGL data sets. However, under the normal distribution, GEV distribution and VG distribution of MSFT data set, the null hypothesis is rejected. This implies that these distributions do not adapt sufficiently and efficiently enough to large losses of MSFT, and the risk of bankruptcy probably occurred in the shortest time.

### Conditional coverage test

Finally, the result of conditional coverage test, which is a joint test of unconditional coverage test and independence test, is shown in Table 10.

**Table 10.** Conditional coverage test for the 95%-VaR.

| | AAPL | GOOGL | MSFT |
|---|---|---|---|
| **Basic Historical Simulation Method** | | | |
| *$LR_{CC}$* | 3.669 | 0.494 | 3.547 |
| *p-value* | 0.160 | 0.781 | 0.170 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **AW Historical Method** | | | |
| *$LR_{CC}$* | 23.119 | 30.424 | 26.512 |
| *p-value* | 0 | 0 | 0 |
| *Conclusion* | **Reject** | **Reject** | **Reject** |
| **Normal** | | | |
| *$LR_{CC}$* | 3.547 | 0.769 | 5.934 |
| *p-value* | 0.170 | 0.681 | 0.051 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **Student's t** | | | |
| *$LR_{CC}$* | 4.113 | 4.881 | 7.443 |
| *p-value* | 0.128 | 0.087 | 0.024 |
| *Conclusion* | Fail to reject | Fail to reject | **Reject** |
| **GEV** | | | |
| *$LR_{CC}$* | 3.669 | 0.405 | 5.768 |
| *p-value* | 0.160 | 0.817 | 0.056 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |
| **VG** | | | |
| *$LR_{CC}$* | 3.547 | 0.494 | 5.845 |
| *p-value* | 0.170 | 0.781 | 0.054 |
| *Conclusion* | Fail to reject | Fail to reject | Fail to reject |

## CONCLUSION

This study aimed to estimate the Value-at-Risk (VaR) of the selected stock of the mobile phone industry, and identify the stock with the lowest VaR which is worth investing in. Based on the analysis, certain distributions were chosen to fit.

the selected mobile phone stock under NASDAQ, which are AAPL, GOOGL, and MSFT. In this study, non-parametric VaR methods (i.e. historical and AW historical method) and parametric VaR methods (i.e. normal, student's t, GEV, and VG distributions) are used to measure the VaR of AAPL, GOOGL, and MSFT. Next, the VaR of the AAPL, GOOGL, and MSFT were then estimated and compared. Among the non-parametric and parametric VaR methods, AW-HS produces the smallest VaR which are -1.91%, -1.30%, and -1.11% for AAPL Inc, GOOGL Inc, and MSFT Corporation respectively. From the result of VaR estimates, it was shown that AAPL is the riskiest share to invest because it has the highest VaR estimates for each distribution. In contrast, MSFT is the most stable stock. The lowest VaR estimates for each distribution of MSFT shows that it will give the least loss to the investors. For VaR estimates, backtest play an important role to examine the model accuracy so that the model can adapt in any situation. From the conditional coverage test, it can be observed that the historical method performed well among the non-parametric method, and normal, GEV, and VG distribution also performed well among parametric distribution.

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