

A comparative study of the Regula Falsi method, Newton's method, and the steepest descent method for solving nonlinear equations

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ABSTRACT - A numerical method has been introduced to help mathematicians to solve the functions. Numerical methods are powerful tools used to approximate solutions to equations that may not have exact solutions or are difficult to solve analytically. This study presents a comparative analysis of numerical methods that is the Regula Falsi method, Newton's Method, and Steepest Descent method. These methods are employed for solving nonlinear equations. All these methods will be compared and tested with eight different types of test functions including polynomials, exponentials, cubic, and trigonometric functions and also with different tolerance and initial guess. The performance of these methods was evaluated using performance profiles based on the number of iterations and CPU time. The results demonstrate that Newton's Method outperforms the other approaches, exhibiting the fastest convergence, and the least computational cost. Regula Falsi shows moderate performance, while Steepest Descent lags in efficiency due to its higher iteration count and CPU usage. The findings underscore the significance of selecting appropriate numerical techniques to optimize computational efficiency, with potential applications across diverse scientific and engineering disciplines.

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1. INTRODUCTION

In the 21st century, mathematics, science, and even art are accepted as fundamental to scientific computations. This was proven when various novel algorithms for solving numerical problems were developed to improve studies in mathematics and science. Moreover, this fundamental helps to solve many problems such as real-world algebra, geometry and calculus in engineering, biological, natural sciences, social sciences, and physical sciences [1]. Therefore, numerical iterative methods were chosen to solve the problems.

Optimization can be defined as the process of finding the best solution to a problem under specific conditions and criteria. Iterative numerical analysis is a mathematical approach that produces a sequence of progressively accurate approximate solutions for nonlinear equation problems. This technique has a long and storied history, with origins dating back to ancient times. Today, numerical iterative methods remain a vital focus of research and development as mathematicians and scientists strive to create new and more efficient methods for solving nonlinear equations. These methods are especially useful in situations in which it is difficult or challenging to find the exact roots of a polynomial by using analytical methods [2].

To evaluate the advantages and disadvantages of an iterative method, factors such as error, accuracy and the number of iterations will be assessed. Recently, several researchers have expressed that the iterative methods can be used for improving some iterative techniques in order to solve nonlinear examples in different fields such as pure science and engineering [3]. This research project focuses on solving nonlinear equation problems using numerical iterative methods. Essentially, this approach involves continuously refining the solution based on an initial estimate until a satisfactory result is obtained. For this research project, the Regula Falsi method, Newton's method, and the Steepest Descent method have been selected.

Nonlinear equations arise in various fields, including engineering, medicine, and robotics, necessitating efficient numerical methods for their solutions. The first step in solving such equations involves determining all solutions to the system's polynomial equations [4]. Several researchers have explored iterative methods to enhance the efficiency of solving nonlinear equations across disciplines such as pure science and engineering [5]. Iterative numerical methods are widely regarded as accurate and efficient, making them indispensable in scientific and engineering applications. This study compares three numerical methods Regula Falsi, Newton's method, and the Steepest Descent method to determine their effectiveness in solving nonlinear equations.

The Regula Falsi method, also known as the False Position method, is an iterative root-finding algorithm [6]. It is a bracketing method that selects two initial points with opposite function signs and applies linear interpolation to approximate the root. This technique combines elements of the bisection method and linear interpolation, offering faster convergence in many cases [7]. However, its convergence rate may slow when the root is near one end of the interval, as the method tends to favour the endpoint with the lower absolute function value. Despite this limitation, Regula Falsi method remains a fundamental numerical approach due to its simplicity and historical significance. The method is defined as:

$$x_{n+1} = x_n - f(x_n) \cdot \left(\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \right) \quad (1)$$

The Regula Falsi method operates by repeatedly narrowing down the interval in which the root of the function lies, using linear interpolation to approximate the root. Specifically, it begins with two initial guesses, a and b , where the function $f(x)$ has opposite signs ($f(a) \cdot f(b) < 0$), ensuring the existence of a root within $[a, b]$. The method then calculates the root of the line connecting the points $(a, f(a))$ and $(b, f(b))$, which serves as the new approximation for the root. Unlike the bisection method, which divides the interval into equal halves, Regula Falsi method uses the function values to guide the selection of the next interval, often leading to faster convergence [8]. However, while Regula Falsi method is generally more efficient than bisection, its convergence rate can slow down if the root is closer to one endpoint, as the method tends to favour the endpoint with the smaller absolute function value [8]. Despite these limitations, the Regula Falsi method remains a foundational approach in numerical analysis due to its simplicity and historical significance.

Newton's method, also known as the Newton-Raphson method, is one of the most widely used iterative techniques for solving nonlinear equations due to its rapid convergence and high accuracy [9]. It begins with an initial guess and refines it iteratively using the function's derivative. This method is highly efficient but requires a good initial guess; otherwise, it may diverge or yield incorrect results. Additionally, computing derivatives can be complex for certain functions. The Newton-Raphson formula is:

$$x_{n+1} = x_n - \left(\frac{f(x_n)}{f'(x_n)} \right) \quad (2)$$

Despite the fact that Newton's method is one of the simplest methods and generally converges, it also has some weaknesses, such as being sensitive to the initial estimate of the solution. Generally, it is difficult to choose a good initial condition for most large systems of nonlinear equation problems [10]. This study uses the Newton's method to compare with other methods and determine which is the most effective. The Newton's method is straightforward and intuitive when solving nonlinear equations.

The Steepest Descent method, proposed by Cauchy, is a fundamental optimization algorithm used for solving nonlinear equations [11]. It minimizes a function by moving in the direction of the negative gradient. The step size, also called the learning rate, determines how far the algorithm moves in each iteration. A well-chosen step size ensures steady progress toward the minimum, whereas an excessively small step size results in slow convergence, and a large step size may cause divergence. The method is defined as:

$$x_{n+1} = x_n - a(\nabla f(x_n)) \quad (3)$$

where $\nabla f(x_n) = 2 \cdot f(x_n) \cdot f'(x_n)$, and a is chosen to minimize $f(x)$.

Step size, a is also called the learning rate and determines how far to move along the direction of the negative gradient. The step size is chosen to minimize $f(x)$ along the direction of the gradient at each iteration, which ensures progress toward the minimum. The simplicity of this approach makes it an excellent starting point for understanding optimization and forms the basis for many advanced techniques [12].

Despite its straightforward concept, the Steepest Descent method has some limitations, particularly in real-world applications. One major drawback is its slow convergence when the function $f(x)$ has narrow and elongated contours, as seen in poorly scaled problems. This is because the method often oscillates and takes small steps in narrow regions, leading to inefficiency. Additionally, the method's performance heavily depends on the choice of the step size, a is too small, the algorithm converges very slowly, while if it is too large, it may overshoot the minimum or fail to converge altogether. Despite these issues, the Steepest Descent method theoretical framework is crucial for understanding the behaviour and convergence properties of more advanced optimization algorithms, such as Conjugate Gradient and Quasi-Newton methods, which are designed to address these deficiencies [13].

The performance of these methods depends on factors such as initial conditions, function characteristics, and computational efficiency. This study evaluates and compares their effectiveness using various test functions, tolerances, and initial guesses to determine the most suitable approach for solving nonlinear equations.

2. METHODOLOGY

To determine the most effective method for solving nonlinear equations, standard test functions will be used to evaluate the performance of each iterative method. A variety of function types, including polynomial, trigonometric,

exponential, and logarithmic functions, will be tested in this study. Each numerical method will be applied to these functions, allowing for a comparative analysis of their efficiency and accuracy. The implementation and graphical representation of these methods will be conducted using Maple 2024 software.

2.1 Numerical Algorithms

Table 1. Algorithm of the regula falsi method

No	Step
1.	Define the initial interval, $[x_0, x_1]$
2.	Apply Regula Falsi method: $x_{n+1} = x_n - f(x_n) \cdot \left(\frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \right)$
3.	Repeat steps 1 and 2 until convergence is achieved.
4.	Stopping criteria: If the maximum difference $\ x_{n+1} - x_n\ < \varepsilon$, print the output.

Table 2. Algorithm of Newton's method

No	Step
1.	Define the initial guess, x_0
2.	Apply the Newton method: $x_{n+1} = x_n - \left(\frac{f(x_n)}{f'(x_n)} \right)$
3.	Repeat steps 1 and 2 until convergence is achieved.
4.	Stopping criteria: If the maximum difference $\ x_{n+1} - x_n\ < \varepsilon$, print the output.

Table 3. Algorithm: Steepest Descent method

No	Step
1.	Define the initial guess, x_0
2.	Apply Steepest Descent method: $x_{n+1} = x_n - a(\nabla f(x_n))$ <p>where $\nabla f(x_n) = 2 \cdot f(x_n) \cdot f'(x_n)$,</p> <p>Where a is chosen to minimize $f(x)$</p>
3.	Repeat steps 1 and 2 until convergence is achieved.
4.	Stopping criteria: If the maximum difference $\ x_{n+1} - x_n\ < \varepsilon$, print the output.

2.2 List of Nonlinear Equations

In this research, all methods will be applied to functions that derive from a previous research paper. The choice of the initial guess depends on the characteristics of the function, the location of the root, and the specific method being used, without relying on the interval of the function.

The most effective method can be determined by testing it with standard test functions to investigate the effectiveness of the iterative method. A few different types of functions, such as polynomial, trigonometric, exponential, and logarithmic, will be used in this research. Each method will be applied to evaluate each function. Using different functions will produce different outcomes, thereby demonstrating the capability of the methods.

Table 4. Test function

No	Type of Function	Function	Roots
1.	Combination of trigonometry and polynomial	$f(x) = \sin^2(x) - x^2 + 1$	1.464491648
2.	Combination of exponential and polynomial	$f(x) = x^2 - e^x - 3x + 2$	0.257530285
3.	Combination of exponential, trigonometry and polynomial	$f(x) = e^{2+x+2} - \cos(x + 1) + x^3 + 1$	-1.000000000
4.	Exponential	$f(x) = (x + 2)e^x - 1$	-0.4428543636
5.	Trigonometry	$f(x) = \sin(x) + \cos(x) + x$	-0.456624704
6.	Combination of logarithm, polynomial and trigonometry	$f(x) = \log_e - x^3 + 2\sin(x)$	1.297997743
7.	Exponential	$f(x) = e^{\sin(x)} - x + 1$	2.630664124
8.	Polynomial	$f(x) = x^3 - x^2 - 1$	1.465571232

3. RESULTS AND DISCUSSION

After testing all eight functions, the results are analysed to achieve the study’s objective. The performance of each numerical method is assessed using two key criteria: the number of iterations and CPU time. The number of iterations represents the total steps required for the method to converge, while CPU time measures the computational effort needed to reach the solution. By evaluating these factors, the efficiency of each method can be compared to determine the most effective approach for solving nonlinear equations.

Performance Profile Based on Number of Iterations

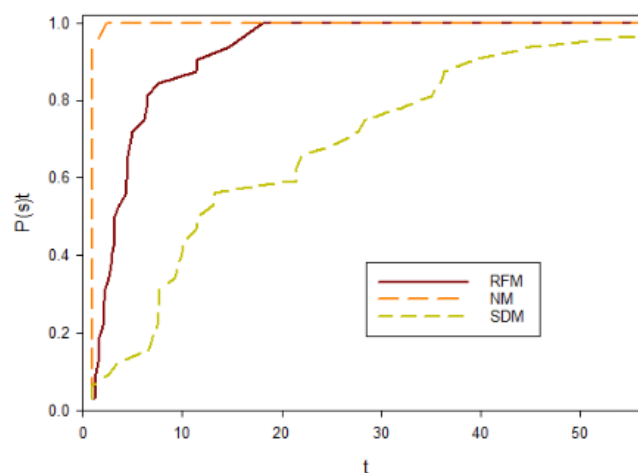


Figure 1. Performance profile of number of iterations

Figure 1 presents the performance profile comparing the Regula Falsi method (RFM), Newton method (NM), and the Steepest Descent method (SDM) based on the number of iterations required for convergence. From the graph, all methods successfully solve the given equations, as indicated by their curves, reaching the right side of the plot, attaining the value

of 1. Among the three methods, Newton's method demonstrates the highest efficiency, as it consistently requires the least number of iterations. This is evident from the topmost curve on the left side of the graph, which signifies that Newton's method outperforms the others for most test functions.

In comparison, the Regula Falsi method (RFM) requires more iterations than Newton method but still performs well. The Steepest Descent method (SDM), however, takes the highest number of iterations to converge, making it the least efficient in this category.

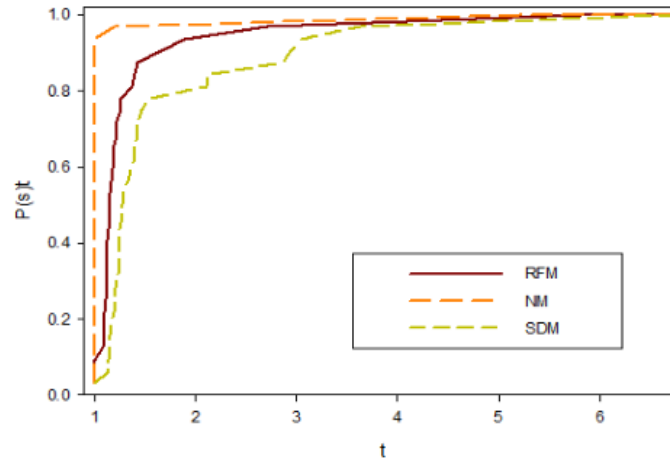


Figure 2: Performance profile of CPU time

Figure 2 illustrates the performance profile of the three methods concerning CPU time. Similar to the iteration-based analysis, all methods successfully solve the equations, as their curves reach the right side of the graph and attain a value of 1. Newton's method (NM) achieves the best performance, solving the highest proportion of problems with the least CPU time. The Regula Falsi method (RFM) follows closely behind, showing a rapid rise in performance but requiring slightly more computational time than Newton's Method. In contrast, the Steepest Descent method (SDM) is the least efficient in terms of CPU time. This method takes significantly longer to converge compared to the other two, making it less suitable for solving the test functions efficiently.

4. CONCLUSIONS

Newton's method demonstrated the fastest convergence and the lowest CPU time, making it the most computationally efficient among the three methods. However, each method has its own strengths and limitations. Newton's method can converge rapidly with an appropriate initial guess, but it may diverge if the initial approximation is not chosen carefully. The Regula Falsi method, on the other hand, guarantees convergence by bracketing the root, yet it is slower compared to Newton's method. The Steepest Descent method is more suitable for gradient-based optimization problems, but it is the least efficient in terms of both iterations and CPU time when applied to root-finding tasks. These findings highlight the importance of selecting the appropriate numerical method based on the specific requirements of the problem. The efficiency of each method depends on the nature of the function being solved, emphasizing the need for a situational approach in numerical computation.

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Muhammad Idham Fikri Faizal (Conceptualization; Formal analysis; Visualization; Writing - original draft; Writing - review & editing), Tasnim Atiqah Azizi (Methodology; Writing - original draft; Writing - review & editing, Resources), Ruhana Jaafar (Conceptualization; Visualization; Writing - original draft; Writing - review & editing; Supervision; Project

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DECLARATION OF ORIGINALITY

The authors declare no conflict of interest to report regarding this study conducted.

GENERATIVE AI DECLARATIONS

The authors claim that artificially intelligent-assisted technologies in the form of generative AI were not used to generate content, ideas, or theories. We have just utilized AI to enhance readability and refine the language. This was used with extreme human control and oversight. The authors take full responsibility for reviewing and approving the content.

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