Alternative scheme for frequency response function measurement of experimental-analytical dynamic substructuring

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ABSTRACT

The accuracy of the predicted dynamic behaviour of an assembled structure using the frequency based substructuring (FBS) method is often found to be diverged from the experimental counterparts. The divergence which has become the paramount concern and major issue for structural dynamicists is because of the unreliable experimental frequency response function (FRF) data of the interfaces of substructures, arising from the limited resources of appropriate excitation points and accelerometer attachments in the vicinity of the interfaces. This paper presents an alternative scheme for FRF measurement of the experimental FRF data of substructures. In this study, an assembled structure consisting of two substructures were used, namely substructure A (Finite element model) and substructure B (Experimental model). The FE model of substructure A was constructed by using 3D elements and the FRFs were derived via the FRF synthesis method. Specially customised bolts were used to allow the attachment of accelerometers and excitation to be made at the interfaces of substructure B, and the FRFs were measured by using impact testing. Both substructures A and B were then coupled by using the FBS method and the coupled FRF was validated with the measured FRF counterparts. This work revealed that the proposed scheme with specially customized bolts has successfully predict the lower modes of the structure, specifically for the 1st, 2nd, 3rd and 4th modes.

Keywords: Frequency based substructuring, modal testing, structural dynamics.

INTRODUCTION

The understanding of the dynamic behaviour of engineering structures has always been a paramount consideration for structural engineering applications. In most cases, the structures are usually very complex and consist of a large number of components of which some of whose dynamic behaviour is very difficult to be determined by the finite element method [1, 2]. For the past few decades, the dynamic behaviour of an assembled structure consisting of several components, has been efficiently and economically predicted by using the dynamic

substructuring methods [3–6]. One of the most popular dynamic substructuring methods used is frequency based substructuring (FBS).

This method has the capability and versatility to calculate the FRF of an assembled structure (coupled FRF) by combining the analytical, derived and measured, FRFs from the substructures. The equations for the coupled FRF are derived by balancing the forces and enforcing continuity at the interface which is thoroughly explained in [7]. For an instance, Law, Rentzsh and Ihlenfeldt [8–10] predicted the dynamic response of an operating machining tool by coupling the FRF of an operating machine with other several substructures via the FBS method. Another striking example is that the FBS method was used for predicting the dynamic behaviour of the isolation system for an operating machinery as demonstrated in [11, 12]. All the examples have demonstrated a strong link between efficiency and economics of the FBS method in the use of predicting the dynamic behaviour of a complex structure.

Attempts to use the FBS method for combining the experimental results with the numerical counterparts have been made by structural dynamicists for decades, however, the attempts have often found to be unsuccessful[13, 14]. The essential issue lies in the FBS method used in the preparation of experimental work as highlighted in [15]. The basic theory of the FBS method is that the FRF of all degrees of freedom (DOFs) at the interface connection is required for the coupling process [16] but measuring accurately all the translational and rotational FRFs at the interfaces of substructures is very challenging and problematic [17, 18]. A few approaches have been proposed to improve the FBS method such as the transmission simulator method [19], the variability improvement of key inaccurate node groups (VIKING) [20], inverse dynamic substructuring [21] and modal expansion methods [22, 23]. In addition, the use of rotational accelerometer has been used recently to measure the rotational responses which can be used to improve the predicted dynamic behaviour via FBS method [23, 24].

In some cases, the position or design of the interface of a structure itself is almost impossible to be excited in all directions. For instance, when the interface is located at the centre of a large flat span structure. This sort of scenario   was discussed at length in [25]. This situation has instigated a notion to the author to introduce an alternative approach for measuring the FRFs at the interface by attaching a specially customized bolt, which allows the dynamists to directly excite the interface of a structure [26]. This approach was slightly different to the transmission simulator approach, where the size and the mass of the bolt is very small in hence the additional decoupling process is needless.

This study is developed an alternative scheme for FRF measurement of experimental-analytical dynamic substructuring using the FBS method. Two identical beam substructures are used to analyse the capability of the proposed scheme for the FBS method. The specially customized bolts are used for the excitation and accelerometer attachment to the test substructure (substructure B). The coupled FRF between both substructures are then compared with the actual measured FRF of the assembled structure for evaluation purposes.

Physical Test Structure
The applicability of the proposed alternative scheme for FRF measurement for FBS is demonstrated on a simple steel beam structure which consists of two rectangular cross-section steel beams, namely substructures A (FE derived FRF) and B (Experimental FRF). Both substructures were 380mm long, 45mm wide and 6mm of thickness, and then combined

together by two bolted joints as shown in Figure 1. This simple configuration allowed an initial study of the efficiency and applicability of the proposed scheme for FRF measurement for a simple structure such as beams as demonstrated in the previous work [27–29]. The excitation frequency of interest for this case study was between 0 to 2000Hz.
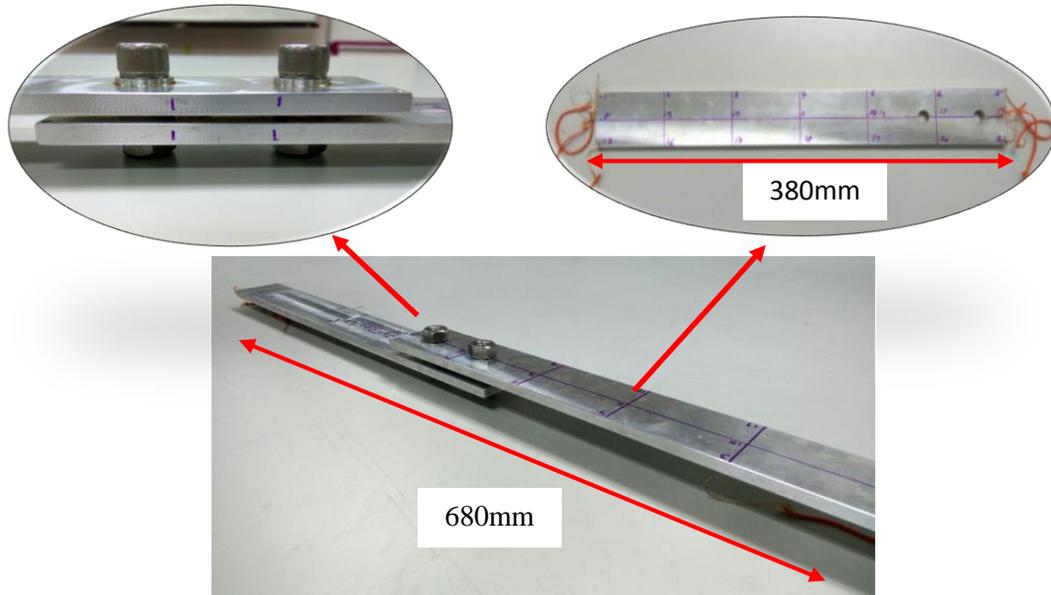

380mm

680mm

Figure 1. Test structure

Description of specially customised bolts
The basic theory of the FBS method describes that all the FRF matrix at the interface must be included for ensuring the successful implementation of FRF coupling [7]. In order to obtain a complete FRF matrix, all the translational and rotational responses must be measured [26]. Specially designed bolts are introduced to ensure that the accelerometer attachment and excitation at the interfaces can be successfully performed. The bolts are made of hardened steel and designed with a 12x12x12mm cube head and M10 full thread as shown in Figure 2(a). The specially customised bolts are attached to substructure B as shown in Figure 2(b).
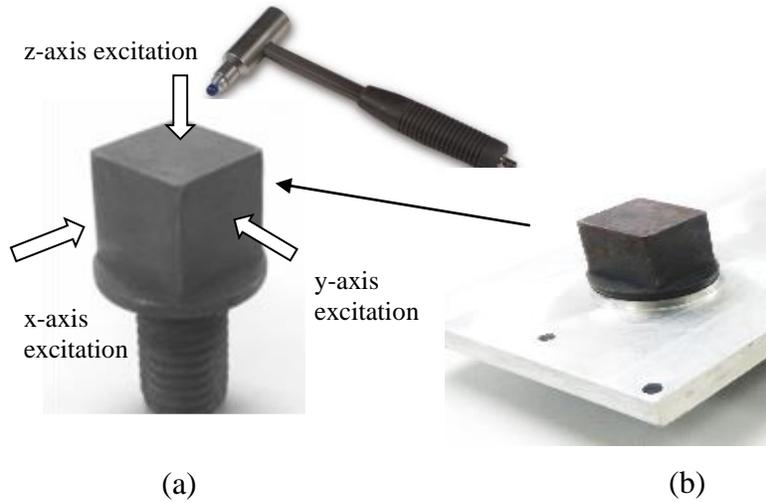
Figure 2. (a) Specially customised bolt with cube head (b) Attachment of specially customised bolts to substructure B

FINITE ELEMENT MODELLING FOR SUBSTRUCTURE A

As stated in the introduction section, the interface FRFs of substructure A were derived numerically by using the FRF synthesis method based on calculated modes. For this method, the synthesized FRF matrix $H_{syn}(\omega_k)$ and mode shapes are expressed by:

$$H_{syn}(\omega_k) = \sum_{i=1}^{N} \frac{\{\emptyset\}_i \{\emptyset\}_i^T}{\left(\omega_{n_i}^2 - \omega_k^2\right) + j2\xi_i\omega_k\omega_{n_i}} \tag{1}$$

where N is the number of calculated modes, $\{\emptyset\}_i$ is the ith mass normalised mode shapes, ω_{n_i} is ith natural frequency and ξ_i is the ith modal damping ratio. The substructure A was discretised into 26622 3D elements and the properties were defined as follows; density = 7850 kg/m^3, Young's modulus = 70GPa and Poisson's ratio = 0.33. Since only 3D elements were used for the modelling, therefore, only the translational FRFs were synthesized and used in the process of experimental-analytical FRFs coupling. Based on the experimental modal analysis from the next section of this paper, the synthesized FRFs were extracted up to 4000Hz, which are two times more than the frequency of interest of the assembled structure to minimise the truncation errors.

EXPERIMENTAL SET UP FOR EXPERIMENTAL SUBSTRUCTURE

The FRFs of substructure B were measured by using an impact testing method by referring to the previouse studie [30, 31]. The substructure was tested under free-free boundary conditions to minimise the effect of constraint to the structure. The interfaces of substructure B, which are in the vicinity of the two through holes, were used for the attachment of the two specially customised bolts to allow the impact excitation in x,y and z directions. Two

100mv/g tri-axial accelerometers were attached to the interfaces in order to measure the translational responses in all directions as shown in Figure 3. The frequency bandwidth for the measured FRFs was also between 0-4000Hz. The measured FRFs were compared with the synthesized FRFs, and the results of the comparison are discussed in the discussion section.
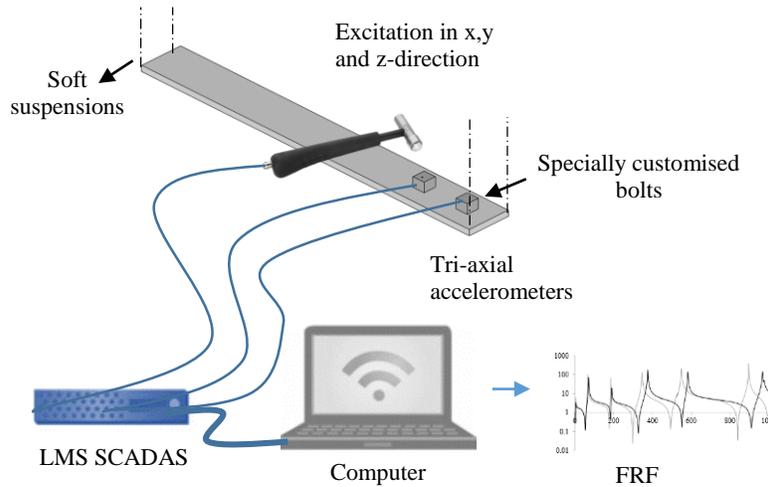

Figure 3. Experimental set-up for substructure B

The same method and procedure were used to measure the FRFs at the interface of the assembled structure consisting of substructures A and B as shown in Figure 4. The frequency bandwidth used for the assembled structure was between 0-2000Hz.
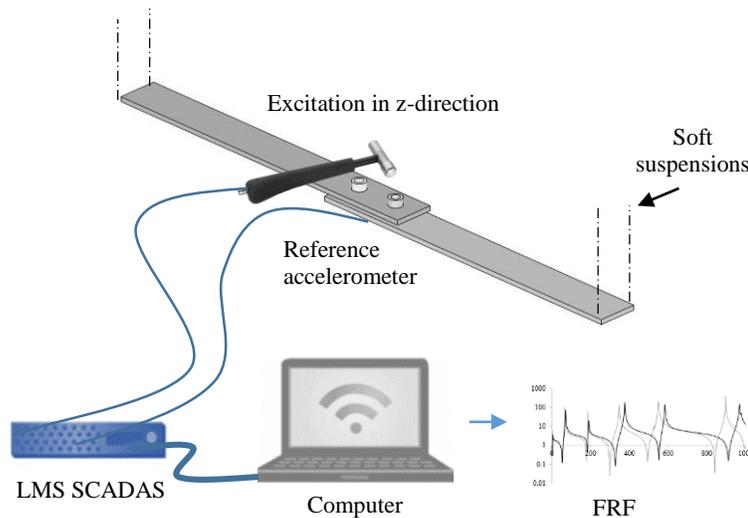

Figure 4. Experimental set-up for assembled structure.

FBS METHOD

The assembled structure (S) consisting of substructures (A) and (B) in which the DOFs are classified either as internal or coupling DOFs. The internal DOFs of both substructures are represented as subscript a and b, while the connection DOFs are described as subscript c. Both substructures are independent of one another before the coupling process is carried out. The c-DOFs must be the same for both substructures. The dynamic behaviour for each substructure in the frequency domain is:

$$\mathbf{X}_n = \mathbf{H}_{nn}\mathbf{F}_n \tag{2}$$

where $\mathbf{X}_n$ is the complex displacement matrix, $\mathbf{H}_{nn}$ is the complex admittance matrix in the form of displacement, force $\mathbf{F}_n$ is the applied force vector and the subscript n is the total number of DOFs for each substructure. Now, let the superscript S represent the coupled system where both substructures A and B are connected at DOFs c. The rigid connections between substructures A and B, compatibility implies that

$$\mathbf{X}^A{}_c + \mathbf{X}^B{}_c = \mathbf{X}^S{}_c \tag{3}$$

and the force equilibrium requires that

$$\mathbf{F}^A{}_c = \mathbf{F}^B{}_c = \mathbf{F}^S{}_c \tag{4}$$

The FRFs of the system can be defined as

$$\begin{Bmatrix} \mathbf{X}^S{}_a \\ \mathbf{X}^S{}_c \\ \mathbf{X}^S{}_b \end{Bmatrix}_n = \begin{bmatrix} \mathbf{H}^S{}_{aa} & \mathbf{H}^S{}_{ac} & \mathbf{H}^S{}_{ab} \\ \mathbf{H}^S{}_{ca} & \mathbf{H}^S{}_{cc} & \mathbf{H}^S{}_{cb} \\ \mathbf{H}^S{}_{ba} & \mathbf{H}^S{}_{bc} & \mathbf{H}^S{}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{F}^S{}_a \\ \mathbf{F}^S{}_c \\ \mathbf{F}^S{}_b \end{Bmatrix}_n \tag{5}$$

The equation above can be derived to obtain the coupled FRF. More detailed information regarding the derivation and explanation is available in [32, 33]. The derived FRFs from the FE model of substructure A and FRFs measured from substructure B were coupled by using a rigid type of coupling. The frequency bandwidth was set between 0-2000Hz and the coupled FRF was compared with the measured counterpart for the validation purposes.

RESULTS AND DISCUSSION

The accuracy of the predicted dynamic behaviour by using FBS method is highly dependent on the sufficient FRF in all DOFs at the coupling interfaces. The attempts to exclude one of the degrees of freedom during the process of coupling has usually led to completely different results [25]. Therefore, in order to obtain an accurate FRF prediction via the FBS method, all the DOFs at the interfaces need to be extracted and coupled. In this study, specially customised bolts attached to the two through holes of substructure B as shown in Figure 2(b) were used for the measurement of interface FRFs of the substructure. In order to validate the applicability of the proposed FRF measurement scheme, the measured interface FRFs of the substructure are validated with the FRFs derived from the FE model. The comparisons of the FRFs are presented in Figures 5, 6, 7 and 8 respectively.
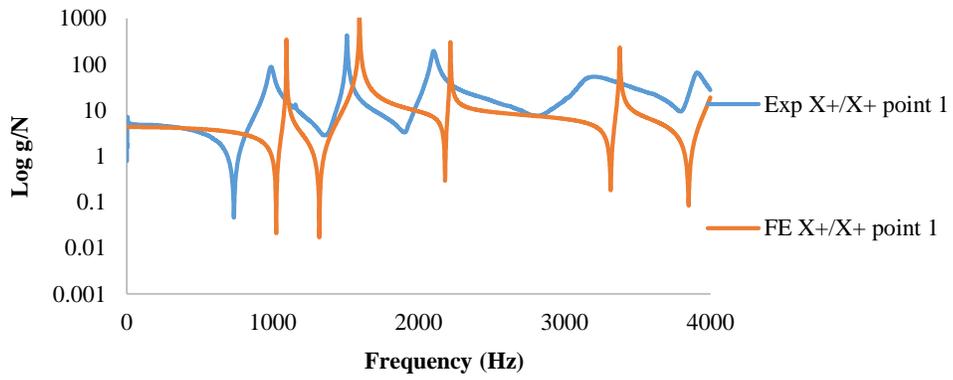
Figure 5. The comparison of the measured and FE FRF in x direction
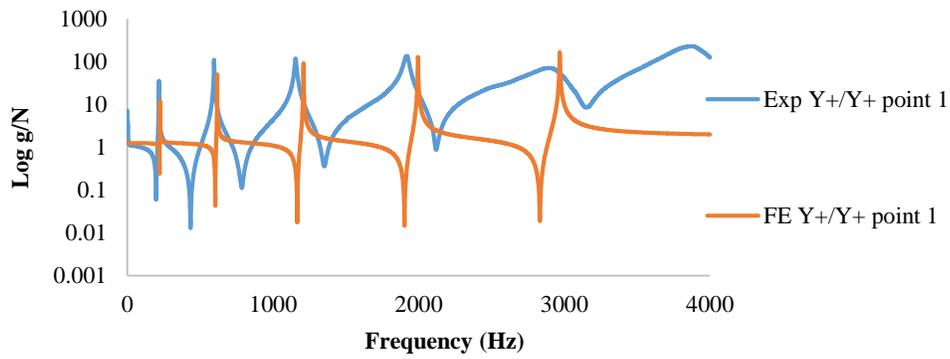

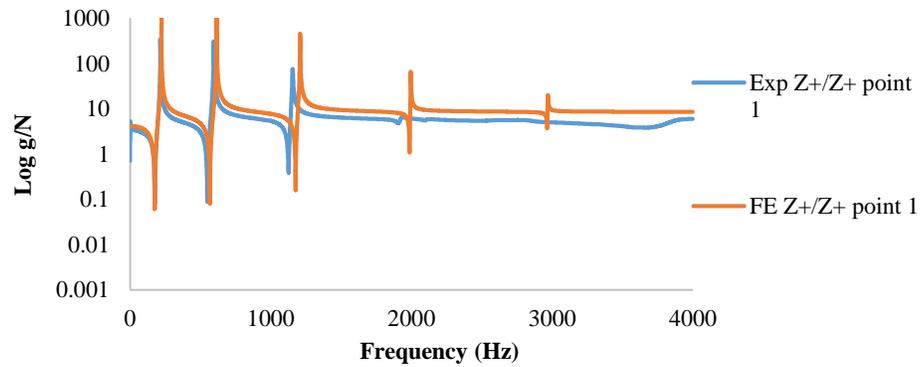Figure 6. The comparison of the measured and FE FRF in y direction

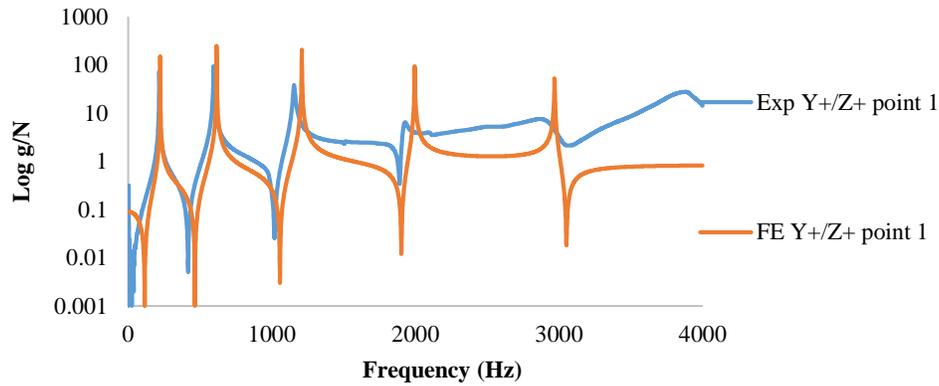Figure 7. The comparison of the measured and FE FRF in z direction

Figure 8. The comparison of the measured and FE FRF in cross-axis response between y and z directions.

From the figures presented (Figure 5,6,7 and 8), it can be clearly seen that the pattern and amplitude of the resonance as well as anti-resonance of the measured FRF are almost identical with the synthesized FE FRF in z direction as shown in Figure 7. It also can be observed that the cross-axis FRF between y and z directions is in relatively good agreement with the FEFRF, especially for the lower frequency as depicted in Figure 8. However, the measured FRFs in x and y directions are incapable of matching well with the FE counterparts, specifically for the anti-resonances of the high frequency. However, the correlations between the measured and FE FRFs in x and y directions as illustrated in Figures 5 and 6 were found to be poor, a result which must probably be attributed to suspension movement effects (dominated by z direction). In addition, the mode shapes of the substructure are more noticeable in z direction.   The sequence of events has led to the measured FRFs in x and y directions for the higher frequency becoming slightly inaccurate.

The FBS method was employed to predict the FRF of the complete assembled structure by coupling the measured FRFs of substructure B and FE FRFs of substructure A. The comparison between the coupled FRF obtained from the FBS method and the measured FRF of the assembled structure is presented in Figure 9. It was found that there are six modes of the measured FRF within the frequency bandwidth between 0-2000Hz. This achievement shows that all the six modes have been successfully calculated via the FBS method. Therefore, it is imperative to note that the proposed FRF measurement scheme offers a great capability in both enhancement and improvement to the current interface FRF measurement technique for the FBS method.
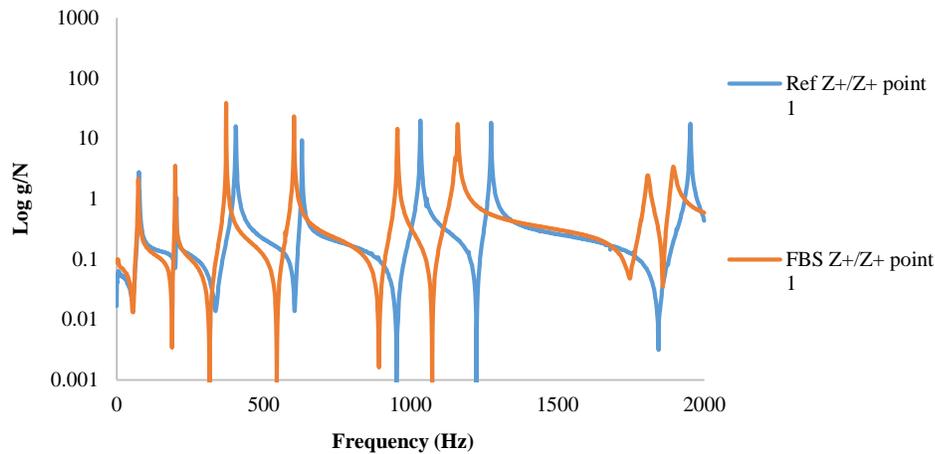
Figure 9. The comparison of two sets of the FRFs of the assembled structure obtained from the FBS method and from the experiment.

The relationship shown in Figure 9 indicates that there is a good match between the first and second resonant frequencies obtained from the FBS method and experimental counterparts. However, it is easily seen that the predicted FRF exhibits some quite significant differences in its resonance frequencies as the frequency increases. The comparison of the FRF reveals that the use of the specially customised bolts for the measurement of the interface FRFs of a substructure is more suitable for low resonant frequencies. Based on engineering judgement and observation, the discrepancies are most likely attributed to the effect of low accuracy of FRFs in x and y directions as mentioned before. In addition, the additional masses from the specially customised bolts have shifted more modes towards the low frequency segment. Furthermore, the discrepancies between the measured and coupled FRF might be due to invalid assumptions about the model properties of finite element models and the exclusion of rotational FRFs [33, 34].

CONCLUSIONS

The applicability of the proposed FRF measurement scheme by using specially customised bolts for the measurement of the interface FRFs of steel beam substructure is presented. All the translational FRFs at the interface of substructure B were successfully measured using the proposed measurement scheme. The 1st, 2nd, 3rd and 4th resonance peaks were accurately predicted by using the presented method. This achievement suggests that the FBS method has been successfully used for coupling and predicting experimental and analytical dynamic behaviour of an assembled structure, specifically for the lower modes. Therefore, it is worth noting that the proposed scheme has a great capability in both enhancement and improvement in the quality of measurement of the interface FRFs of substructures, especially for a complex structure with a large number of substructures where the measurement of the interface FRFs is usually very problematic and challenging.

ACKNOWLEDGEMENTS

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